To
Sir Isaac Newton, Kt.
President,
And to the
Council and Fellows
Of the
Royal Society
Of
London
Instituted for the
Advancement of Natural Knowledge;
This
Twenty Ninth Volume
Of
Philosophical Transactions
Is
Humbly Dedicated,
By
Edmond Halley, R. S. Secr.
To the Right Honorable
Edward Hawkesworth, Esq:
President of the Society
Fellow of King's College London
This Volume of Transactions
Is respectfully dedicated
By C. A. V. Newcomen
PHILOSOPHICAL TRANSACTIONS.

For the Months of January, February and March, 1714.

The CONTENTS.

I. THE Preface to the Reader, giving an Account of the Publisher's Design and Method, and inviting the curious Observers of the Phenomena, the diligent Inquirers into the Powers and Operations of Natural Agents, and the happy Inventors of new Discoveries, to contribute towards the carrying on these Publications with Success, by generously communicating their Observations, Discoveries and Inventions to the Publisher.

Experimentalis Professore Plumianno, & R. S. S.

III. An Extract from the Acta Eruditorum, for the Month of March, 1713. p. 111. Being a Dissertation concerning the dreadful contagious Distemper, seizing the Black Cattle in the Venetian Territories, and more especially about Padua.

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THE Philosophical Transactions; (begun to be Published by that indefatigably diligent Promoter of Learned Correspondence, Mr. Oldenburgh, the first who attempted any thing of this Nature) having been sufficiently known to the Curious, and always acceptable to the Learned, when due care has been taken in the choice of the Collections so recommended to the inquisitive and intelligent Reader; a long Preface must appear wholly unnecessary.

However, it may not be amiss to inform the Learned and Ingenious, of some Particulars of the present Publisher's Design: Which is;

First, To present the Publick with such short Tracts, as might otherwise be lost to Posterity, if printed in single Sheets or Pamphlets.

Secondly, To give the Extracts, for the most part, of the Material, omitting the Preambles and Conclusions, and the useless parts of such Letters as Correspondents shall please to honour him with, relating to the Improvement of Natural Philosophy, Mathematicks, or Mechanicks.

Thirdly, To give an exact account of such Experiments, as having been made before the Illustrious Royal Society, they shall please to order or permit the Publication of.

Fourthly,
Fourthly, To give, as Opportunity offers, the Abstracts or Accounts of such Books, relating to the Ends of the Royal Society's Institution, as the Authors shall please to communicate themselves; which shall be distinguished from others: or of such as are with difficulty to be procured in England, or extraordinary in their kind; upon which there shall sometimes be added some Remarks.

To this Undertaking the Publisher invites and desires all real Lovers of Knowledge to contribute their Assistance, by communicating their Observations, Discoveries, or Inventions; which shall not only be faithfully and duly Published, with due Acknowledgment; but some Returns made them, by Informations of what shall be thought acceptable to the Gentlemen, who shall think fit to entertain a Philosophical Correspondence: Who are desired, in any Tract or Discourse they shall so communicate, to omit all Personal Reflections; for if such should happen to be inserted, the Publisher shall take the liberty of leaving them out; it being his Opinion, that Disputes on Philosophical Subjects, may be managed with the utmost Candor, Respect and Friendship by the Disputants, whose only Aim ought to be the Search of Truth.

The Gentlemen willing to encourage this Undertaking, are desired to direct their Letters, To be left at the Royal Society's House, in Crane-Court, in Fleetstreet, London, for the Secretary of the ROYAL SOCIETY.
LOGOMETRIA

Auctore

ROGERO COTES,
Astr. & Ph. Exp. Professore PLUMIANO, & R. S. S.

Eruditissimo Viro

EDMUNDO HALLEIO,
Societatis Regalis Secretario S. P.

Mitto tibi, hortatu Illustriissimi Presidis NEWTONI, quae aliquot abhinc annis conscripseras de Rationibus dimetiendis. Tu vero, quem & Ipse dudum in eodem Argumento præclare versatus fueris, pro solito tuo candore, tentamen hoc qualecunque benigne accipies. Vale.

AGITUR in hoc Tractatu de Mensuris Rationum. Hæ Mensurae sunt quantitates cujusque generis, quarum magnitudines magnitudinibus rationum sunt analogæ. In dato itaque Systemate, rationis ejusdem cadem est mensura, duplicatae dupla, triplicatae tripla, subduplicatae subdupla, sesquiplilicatae sesquialtera: denique quocunque modo per compositionem vel resolutionem auctæ vel diminutæ rationis, similiter
acuta est vel diminuta mensura. Equalitatis ratio nullam habet magnitudinem, quia nullam addita vel detracta mutationem inducit; rationes quae dicuntur majoris & minoris inaequalitatis contrarias habent magnitudinem suarum affectiones, quoniam in compositione & resolutione contraria semper efficient: itaque si mensura rationis quam habet terminus major ad minorem positiva censeatur, mensura rationis quam habet terminus minor ad majorem erit negativa, mensura vero rationis inter aequales terminos nullius erit magnitudinis. Porro diversa mensurarum orientur Systemata, prout modis diversis exponitur analogia illa determinata & mutabilis quae est inter magnitudines rationum. Inde vero patet, exhiberi possit numero infinita Systemata, minuendo vel augendo Systematis cujusvis dati mensuras omnes in eadem data quacunque proportione, aut etiam pro mensuris adhibendo quantitates diversi generis. In tanta autem varietate confusionem aliquam oboriri necesse est, ni probe constiterit ad quodnam Systema referenda sint mensurae singulae de quibus contingat sermonem institui. Huic male remedium optime parari potest si mensura datae alijus rationis, qua commodissima videbitur, pro Modulo habeatur ad quem constanter in omni Systemate mensurae reliquarum rationum exigantur. Id enim si fiat, statim ex dato illo Modulo determinabitur Systema totum: nam ex mensuris constabunt quae Modulo erunt homogeneae, quaeque eo maiore habebunt magnitudines vel minores quo major ille fuerit vel minor, ut ita menfurandarum rationum invariata magnitudinem servetur analogia inter ipsas mensuras. Patebit igitur in sequentibus rationem quandam dari, dupli inter & tripli rationes intermediam, ad rationem vero tripli aliquanto propius accedentem, quae propopsito noftra non immerito apropriima judicetur, liquidem ipsa rei natura hujus usum suadere ac non incertis indiciiis efflagitare quodammodo videatur. Hanc ego, ex officio ejus defumpto nomine, Modularem Rationem appellabo; quo autem pacto ipsa fit accuratius definienda, ostendentur inferius, nunc enim de Logarithmis paucius sunt addenda.

Logarithmi sunt rationum mensurarum Numerales: solent autem in Canonem sic disponi, ut singulis numeris naturali ordine crescentibus, & in serie continua positis adscribatur Logarithmus, non quidem ipsius numeri uti Vulgo dicitur, sed rationis quam habet numerus ad Unitatem. Exinde vero rationis per quocunque terminos designatae facilis est inventio Logarithmi. Nam cum ratio antecedentis ad consequentem sit excessus rationis antecedentis ad Unitatem supra.
fupra rationem consequentis ad Unitatem: Logarithmus ejus similiter erit excessus Logarithmi rationis quam habet antecedens ad Unitatem; sicque Logarithmus consequentis; neutiquam enim duplicat loquendi modus jam a multis annis receptus, si recte intelligatur. Exinde porro peregrgium enascitur compendium ad operationes Arithmeticas. Datis enim duobus quibuscunque numeris in se multiplicandis, si quaeratur numerus ex multiplicatione producet; quoniam rationes numerorum datorum ad Unitatem, consequentiam simul addita rationem producit ad Unitatem, & rationem componendarum mensurar simul addita consequentia rationis compositae mensuram: Logarithmus producit aequabitur Logarithmis numerorum datorum simul summatis. Ad eundem modum si quaeratur numerus ex divisione ortus; quoniam ratio divisoris ad Unitatem & ratione dividendi ad Unitatem deducit relinquit rationem quoti ad Unitatem: habebitur quoti Logarithmus subducendo Logarithmum divisoris è Logarithmo dividendi. Et eodem argumento, si quaeratur dati cuiusvis numeri quaelibet potestas; quoniam ratio dati numeri ad Unitatem per Indicem potestatis multiplicata rationem efficit quam habet numeri potestas ad Unitatem, & mensura prioris rationis multiplicata per eundem Indicem efficit pariter mensuram rationis posterioris: Logarithmus potestatis aequabitur Logarithmo numeri dati per Indicem potestatis multiplicato. Et similiter Logarithmus cuiuslibet radicis numeri dati aequabitur Logarithmo numeri dati per Indicem radicis divisio. Igitur ope Canonis peragetur inventio potestatum & radicum per multiplicationem & divisionem, multiplicatio autem & divisio per additionem & subductionem. Ceterum de hisce vulgo notis Logarithrorum usibus non est mei instituti suffus differere: missis ergo ambagi- bus, ad alia nunc me confero & rem ipsam protinus aggradior.
Propositio I.

Invenire Mensuram Rationis cujuscunque propositae.

Proponatur Ratio inter $AC$ & $AB$, cujus Mensuram oportet invenire. Terminorum differentia $BC$ divisa concipiatur in particularis innumeris quam minimas $PQ$, atque ratio inter $AC$ & $AB$ in totidem rationes quam minimas inter $AQ$ & $AP$: & si detur magnitudo rationis inter $AQ$ & $AP$, dividendo dabitur ratio quam habet $PQ$ ad $AP$; atque adeo data illa magnitudo rationis inter $AQ$ & $AP$, per datam quantitatem $\frac{PQ}{AP}$ exponi potest. Manente $AP$, augeri vel minui intelligatur particula $PQ$ in proportione quavis; & in eadem proportione augebitur vel minuetur magnitudo rationis inter $AQ$ & $AP$: capiatur particula dupla vel tripla, subduplica vel subtripla, & evadet ratio duplicata vel triplicata, subduplicata vel subtriplicata; etiamnum igitur exponetur per quantitatem $\frac{PQ}{AP}$. Sed &, assumpta determinata quavis quantitate $M$, exponi potest per $M \times \frac{PQ}{AP}$: erit ergo quantitas $M \times \frac{PQ}{AP}$ mensura rationis inter $AQ$ & $AP$. Hæc vero mensura diversam habebit magnitudinem, & ad Systema diversum accommodabitur, pro diversa magnitudine quantitatis assumptæ $M$, quæ adeo vocetur Systematis Modulus. Jam quemadmodum summa rationum omnium inter $AQ$ & $AP$ æqualis est propositæ rationi, quam utique habet $AC$ ad $AB$: ita summa mensurarum omnium $M \times \frac{PQ}{AP}$ (per Methodos fatis notas invenienda) æqualis erit ejusdem propositæ rationis mensuræ quaestæ. Q. E. I.

Corol. 1. Terminis $AP$, $AQ$ ita ad æqualitatem accedentibus, ut quam minima sit corundem differentia $PQ$: erit $M \times \frac{PQ}{AP}$ vel $M \times \frac{PQ}{AQ}$ æqualis mensuræ rationis inter $AQ$ & $AP$ ad Modulum $M$. Corol.
Corol. 2. Unde Modulus ille $M$ est ad mensuram rationis inter terminos $AQ$ & $AP$, ut terminorum aliter $AP$ vel $AQ$ ad terminorum differentiam $PQ$.

Corol. 3. Data ratione inter $AC$ & $AB$, datur summa omnium $\frac{AP}{AQ}$, & summa omnium $M \times \frac{AP}{AQ}$ est ut $M$. Itaque mensura data cujuscunque rationis est ut Modulus Systematis ex quo defumitur.

Corol. 4. Modulus ergo, in omni mensuram Systemate, semper æqualis sit mensura rationis cujuscumque determinata atque immutabilis: Quam proinde Rationem Modularem vocabo.

Scholium 1.
Problematis solutio per Exemplum illufrabitur. Sit $z$ quantitas determinata quævis & permanens, fit vero $x$ quantitas indeterminata fluxuque perpetuo variabilis, ejusque fluxio sit $\dot{x}$; & quadratur mensura rationis inter $z + x$ & $z - x$. Statuatur hæc ratio æqualis rationi inter $y$ & $x$, exponatur autem numerus $y$ per $AP$, fluxio ejus $\dot{y}$ per $PQ$, $x$ per $AB$: & ex Corollario primo colligetur fluxionem quæsitæ mensuræ rationis inter $y$ & $x$ esse $M \times \frac{y}{x}$. Reponatur jam pro $y$ valor ejus $\frac{z + x}{z - x}$, itemque pro $\dot{y}$ valoris fluxio

$$\frac{z \dot{x}}{z - x} : \&$$

et fluxio mensura evadet $2M \times \frac{z \dot{x}}{z - x} = 2M \times \frac{z \dot{x}}{z - x}$. 

Statuat $2M$ in

$$\frac{z \dot{x}}{z - x} = \frac{z}{3} \frac{x^2}{z} - \frac{z}{5} \frac{x^4}{z} - \&c.$$

Atque adeo mensura illa fit

$$2M \text{ in } \frac{z \dot{x}}{z - x} = \frac{z}{3} \frac{x^2}{z} - \frac{z}{5} \frac{x^4}{z} - \&c.$$

Unde patet Corollarium sequens.

Corol. 5. Si duarum quantitatum summa sit $z$ & differentia sit $x$; 
& sumatur $2M \frac{x}{z} = A$, $A \frac{x^2}{z} = B$, $B \frac{x^3}{z} = C$, $C \frac{x^4}{z} = D$, &c: Mensura rationis quam habet quantitas major ad quantitatem minorem, erit $A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D + \&c$.

Scholium 2.
Non absimili computo mensura rationis inter $x + y$ & $x$ erit $M$ in $v - \frac{1}{5}v^2 + \frac{1}{3}v^3 - \frac{2}{4}v^4 + \frac{1}{4}v^5 - \&c$. Unde si mensura illa vocetur $m$, erit $\frac{m}{M} = v - \frac{1}{5}v^2 + \frac{1}{3}v^3 - \frac{2}{4}v^4 + \frac{1}{4}v^5$, &c: ac proinde
inde \( \frac{mm}{MM} = v - \frac{1}{2} v^3 + \frac{1}{2} \frac{m}{M} \frac{mm}{MM} - \frac{1}{3} v^5 \), &c; similiterque \( \frac{m^3}{M^3} = v^5 - \frac{1}{2} v^4 - \frac{1}{3} \frac{m}{M} \frac{m^3}{M^3} - \frac{1}{5} v^5 \), &c; quinetiam \( \frac{m^4}{M^4} = v^4 - \frac{1}{2} v^3 \), &c; ac denique \( \frac{m^5}{M^5} = v^5 \), &c.

Ut igitur vicissim, ex data mensura \( m \), inventiatur ratio quam metitur; addendo æqualia æqualibus habebitur \( \frac{m}{M} + \frac{mm}{2 MM} = v \)

* - \( \frac{1}{6} v^5 + \frac{1}{2} \frac{m}{M} \frac{m}{M} \frac{mm}{MM} + \frac{1}{5} v^5 \), &c; atque iterum \( \frac{m}{M} + \frac{mm}{2 MM} + \frac{m^3}{6 M^3} \)

\( = v \ast \ast \ast - \frac{1}{6} \frac{m}{M} \frac{mm}{MM} + \frac{1}{5} v^5 \), &c; atque tandem \( \frac{m}{M} + \frac{mm}{2 MM} + \frac{m^3}{6 M^3} \)

\( + \frac{m^4}{24 M^4} = v \ast \ast \ast \ast - \frac{1}{120} v^5 \), &c; atque tarnum \( \frac{m}{M} + \frac{mm}{2 MM} + \frac{m^3}{6 M^3} \)

\( + \frac{m^4}{24 M^4} + \frac{m^5}{120 M^5} = v \ast \ast \ast \ast \ast \), &c; id est, \( \frac{m}{M} + \frac{mm}{2 MM} + \frac{m^3}{6 M^3} \)

\( + \frac{m^4}{24 M^4} + \frac{m^5}{120 M^5} + &c. = v \). Itaque ratio quæsita inter \( i + v \) & \( i \), est ea quam habet \( i + \frac{m}{M} + \frac{mm}{2 MM} + \frac{m^3}{6 M^3} + \frac{m^4}{24 M^4} + \frac{m^5}{120 M^5} \)

\( + &c. \) ad \( i \). Ponatur \( m = M \), igitur \( \frac{m}{M} = 1 \); & exinde Ratio Modularis erit ea quam habet \( i + \frac{1}{3} + \frac{1}{2} + \frac{5}{6} + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{3} + \frac{1}{2} \) &c. ad \( i \).

Eodem modo, si detur ratio inter \( i \) & \( i - v \), mensura hujus rationis erit \( M \) in \( v + \frac{1}{3} v^2 + \frac{1}{3} v^3 + \frac{1}{4} v^4 + \frac{1}{5} v^5 \), &c. Et vicissim si detur rationis mensura \( m \), ratio erit ea quam habet \( i \) ad

\( i - \frac{m}{M} - \frac{mm}{2 MM} - \frac{m^3}{6 M^3} - \frac{m^4}{24 M^4} - \frac{m^5}{120 M^5} \) &c. Ponatur \( m = M \), igitur \( \frac{m}{M} = 1 \); & exinde Ratio Modularis erit ea quam habet \( i \) ad

\( i - \frac{1}{3} + \frac{1}{2} - \frac{5}{6} + \frac{1}{2} + \frac{1}{3} + \frac{1}{2} \) &c. Ex his ce vero patet Corollariwm sequens.

Corol. 6. Exposito termino \( R \), si sumatur \( \frac{1}{2} R = A, \frac{1}{2} A = B, \frac{3}{4} B = C, \frac{1}{4} C = D, \frac{1}{4} D = E, \) &c. in infinitum; & capiatur \( S = R + A + B + C + D + E + \) &c.; Ratio Modularis erit ea quæ est inter terminum minorem exposi tum \( R \) & majorem inventum \( S \). Vel exposito termino \( S \), si sumatur \( \frac{1}{2} S = A, \frac{1}{2} A = B, \frac{3}{4} B = C, \frac{1}{4} C = D, \frac{1}{4} D = E, \) &c. in infinitum; & capiatur \( R = S - A + B - C + D - E + \) &c.; Ratio Modularis erit ea quæ est inter terminum majorem exposi tum \( S \) & minorem inventum \( R \). Porro eadem ratio est inter \( 2,718281828459 \) &c. et \( i \), vel inter \( i \) & \( 0,367879441171 \) &c.

Scho-
Scholium 3.

Si forte termini minores desiderentur, qui eandem proxime Rationem Modularem ita exhibeant, ut nulli ipsis non majores propius instituenda erit operatio ad modum sequentem. Dividatur terminus major 2,71828 &c. per minorem 1, vel etiam major 1 per minorem 0,367879 &c. & rursus minor per numerum qui reliquus est, & hic rursus per ultimum residuum, atque ita porro pergatur:

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&c. &c. &c. &c.

prodibunt quotientes 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1, 16, 1, 1, &c. His inventis, perficiendae sunt binae rationum columnae, quarum altera terminos continet rationem habentes vera majorem, altera terminos quorum ratio est vera minor; inde computationem ad rationibus 1 ad 0, o ad 1, quae remotissimae sunt a vera; inde autem exorfant deducendo ad rationes reliquas, qua.
quia continet ad veram propius accedunt. Multiplicantur itaque termini 1 & o per quotientem primum 2, & scribuntur facti 2 & o infra terminos o & i; & addendo probadit ratio 2 + o ad o + i, sive 2 ad i. Hujus termini multiplicatur per quotientem secundum 1, factique 2 & i addantur terminis i & o; & habebitur ratio 2 + i ad 1 + o, sive 3 ad 1. Hujus termini multiplicatur per quotidentem tertium 2, factique 6 & 2 addantur terminis præcedentibus 2 & 1; & habebitur ratio 8 ad 3. Hujus termini multiplicatur per quotidentem quartum 1, factique 8 & 3 addantur terminis præcedentibus 3 & 1; & habebitur ratio 11 ad 4. Hujus termini rursus multiplicatur per quotidentem sextum 4, factique 76 & 28 addantur præcedentibus 11 & 4, ad inveniendam rationem 87 ad 32, & sic porro perdendum quousque libuerit, transitu alternis facto in alteram columnam. Hisce peractis, habebatur rationes verae majores 3 ad i, 11 ad 4, 87 ad 32, 193 ad 71, 1457 ad 536, 23225 ad 8544, 49171 ad 18089, &c. Vera autem minores erunt 2 ad i, 8 ad 3, 19 ad 7, 106 ad 39, 1264 ad 465, 2721 ad 1001, 25946 ad 9545, &c. Atque haæ quidem sunt præcipua & primarum rationes, quibus ad rationem propositionem continue approquinatur.

Quod si exquiratur integra series rationum omnium vera majorem quæ ita dari possint, ut nulla minoribus terminis designata ratio vera major ad veram propius accedat; & similiter series integra rationum omnium vera minor quæ ita dari possint, ut nulla minoribus terminis designata ratio vera minor ad veram propius accedat: inter primarias illas modo inventas inferendæ sunt aliae secundariae rationes. Hæ vero locum habent ubi quotiens unitatem superat. Inveniuntur autem mutata multiplicationes, quæ supra per quotidentem facta est, in continuam additionem terminorum tot vicibus quot sunt unitates in quotiente. Sic quia quotiens primus erat 2, termini 1 & o bis addendi sunt terminis 0 & 1; & summæ dabunt rationes 1 ad i, 2 ad 1. Hi ultimi termini 2 & i, quia quotiens secundus erat 1, semel addendi sunt terminis 1 & o; & summæ dabunt rationes 5 ad 2, 8 ad 3. Hi termini 3 & i, quia quotiens tertius erat 2, bis addendi sunt terminis 2 & 1; & summæ dabunt rationes 5 ad 2, 8 ad 3. Hi ultimi terminis 8 & 3, quia quotiens quartus erat 1, semel addendi sunt terminis 3 & 1; & summæ dabunt rationem 11 ad 4. Hi termini 11 & 4, quia quotiens
tiens quintus erat 1, semel addendi sunt terminis 8 & 3; & summæ dabunt rationem 19 ad 7. Hi denique termini 19 & 7, quia quo-

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<td>19 7</td>
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Et sic porro procedere licebit quoque commodum videbiris. 

Hi tarnem operatione peracta, series integra rationum omnium vera majorum, erit 1 ad 0, 3 ad 1, 11 ad 4, 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32, &c. similiterque series integra rationum omnium vera minorum, erit 0 ad 1, 1 ad 1, 2 ad 1, 5 ad 2, 8 ad 3, 19 ad 7, &c.

Harum approximationum utilitas ad alia multa fece diffundit: quapropter earum inventionem aliquanto prolixius expositam dedi, per Methodum quae mihi simplicissima & facillima videtur. Idem argumentum paulo aliter pertractarunt Viri celeberrimi Wallijsus & Hugenius.
Propositio II.

Logarithmorum Canonem Briggianum construere.

Numerorum Compositorum Logarithmi derivantur ex Logarithmis Primorum componentium, per additionem solam; horum autem investigatio pluribus modis institui potest: Exemplum unicium appono.

Per Corollarium quintum Propositionis superioris, scribendo \( M \) pro \( N \), inventantur Logarithmi rationum inter \( 126 \) & \( 125 \), \( 225 \) & \( 224 \), \( 2401 \) & \( 2400 \), \( 4375 \) & \( 4374 \); qui vocentur respective \( p, q, r, s \) & Logarithmus denarii suo rationis decupli erit \( 239p + 90q - 63r + 103s \), sive \( 2302585092994 \) & c. Itaque cum Logarithmus Briggianus denarii fit \( 1 \), fiat (per Corol. 3. Prop. 1.) ut denarii Logarithmus modo inventus \( 2302585092994 \) & c, ad Modulum suum \( 1 \), ita denarii Logarithmus Briggianus \( 1 \), ad Modulum Briggianum, qui adeo erit \( 0,434294481903 \) & c. Ponatur ergo deinceps iste valor pro \( M \), & erunt \( MX \times 202p + 76q - 53r + 87s \), \( MX \times 167p + 63q - 44r + 72s \), \( MX \times 114p + 43q - 30r + 49s \) Logarithmi Briggiani numerorum \( 7, 5, 3 \). Logarithmus numeri \( 2 \) habetur, subducendo Logarithmum numeri \( 5 \) à Logarithmo numeri \( 10 \). Atque ita dantur & Modulus Briggianus & Logarithmi Primorum omnium qui sunt minores denario.

Logarithmi numerorum sequentium Primorum \( 11, 13, 17, 19, 23 \), & c. ita computari possunt. Quaeratur tum factus à numeris Primo proposto utrinque proxime adjacentibus, tum Primi ipsius quadratum, quod semper unitate factum illud superabit. Logarithmo rationis quadrati ad factum (per Corol. 5. Prop. 1. inveniendo) addatur ipsius facti Logarithmus, qui semper componetur ex datis Logarithmis Primorum qui proposto Primo sunt minores: & semifumma erit Logarithmus Primi quæsitus.

Corol. Canonis Briggiani Modulus est \( 0,43429444481903 \) & c.; Hujus vero Reciprocus est \( 2302585092994 \) & c.

Scholium.

Ad hunc itaque modum perfici posset Logarithmorum Tabula amplissima, qualis edita est à Briggio vel Viacco. Inventioni autem Numerorum & Logarithmorum sibi invicem congruentium, qui intermedii
termedii sunt & ultra Tabulae limites excurrunt, abunde sufficiet terminus primus Seriei quæ in Corollario quinto Propositionis praecedentis exhibetur.

Si dato Numero intermedio quaeratur ejus Logarithmus; pone \( a \) & \( e \) pro Numero intermedio proposto atque huic proximo tabulari, ita ut \( a \) designet majorem, \( e \) minorem; sit eorum summa \( z \), differentia \( x \); pone \( x \) pro Logarithmo rationis quam habet \( a \) ad \( e \), hoc est, pro excessu Logarithmi Numeri \( a \) supra Logarithnum Numeri \( e \): & erit \( \lambda = 2M \frac{x}{z} \) quamproxime.

Si quaeratur Numerus qui congruit Logarithmo intermedio; quoniam est \( \lambda = \frac{2MX}{z} \), vel \( \frac{2MX}{2e + x} \); erit \( x = \frac{\lambda}{M + \frac{\lambda}{2}} \) ad \( \frac{\lambda}{M - \frac{\lambda}{2}} \) quamproxime.

**Proposito III.**

*Systematis cujusvis Logometrici constructionem exponere per Canonem Logarithmorum.*

Cas. 1. Si detur, \( \varepsilon \) Systemate proposito, mensura rationis alicujus determinatae: rationis cujusvis oblate mensura, erit ad mensuram illam datam determinatae rationis, ut oblate rationis Logarithmus, ad Logarithnum rationis eujusdem determinatae.

Cas. 2. Si non detur, \( \varepsilon \) Systemate proposito, mensura rationis alicujus determinatae: inveniendus erit Modulus propositi Systematis, per Corollarium secundum Propositionis prima. Et mensura cujusvis oblate rationis, erit ad Modulum inventum, ut oblate rationis Logarithmus, ad Canonis Modulum.

Casus hujus ultimi habentur Exempla in sequentibus.

**Proposito IV.**

*Spatium quodvis Hyperbolicum quadrare per Canonem Logarithmorum.*

Sit Hyperbola quaævis \( ERSF \) centro \( A \), Asymptotis \( ABC \), \( AD \) descripta; \& quaeratur area \( BEFC \) quam claudunt rectæ \( BE, CF \) ad Asymptoton \( AD \) parallelæ. Compleatur parallelogrammum \( ABED \), \& ad hunc Modulum inveniatur (per Propositionem
fitionem tertiam) mensura rationis inter \( AC \& AB \) vel inter \( BE \& CF \): Dico mensuram inventam æqualem fore magnitudini areae quaestæ \( BEFC \). Nam divisa conciatur hujus areae basis \( BC \) in partulas innumeras quam minimas \( PQ \), ea lege, ut ubique detur ratio illa quæ est inter \( AQ \& AP \), & ducantur Asymptoto \( AD \) paralleæ \( PR, QS \). Quoniam itaque est \( AQ \) ut \( AP \); erit divisim \( PQ \) ut \( AP \), hoc est, ut \( PR \) reciproce. Unde data est area \( PRSQ \), quæ proinde potest haber pro mensura rationis data quæ est inter \( AQ \& AP \). Hujus autem mensuræ Modulus erit parallelogrammum \( ABED \), per Corol. 2.-Prop.1. Nam si compleatur æquale parallelogrammum \( APRT \); statim intelligetur, ita illud se habere ad aream \( PRSQ \), ut se habet \( AP \) ad \( PQ \). Similes ergo summæ areae atque rationum utrinque colligendo; area tota \( BEFC \) erit mensura rationis totius quæ est inter \( AC \& AB \), vel inter \( BE \& CF \), ad eundem Modulum \( ABED \).

Aliter. Sit rursus Hyperbola quævis \( AP \); centro \( C \) atque Asymptoto \( CB \) descripta; & quæatur area Sectoris cujuslibet \( CAP \), semidiametris \( CA, CP \) curvaeque \( AP \) interiecti. Producta semidiametro utravis \( CAQ \) ultra verticem \( A \), ducatur illius conjugata \( CR \); & ad ipsas \( CQ, CR \) ordinatim applicentur à puncto \( P \) rectæ \( PQ, PR \), quæ Asymptoto \( CB \) occurant in \( Z \& X \); deinde agatur \( AB \) quæ Hyperbolam tangat in \( A \), Asymptoton secet in \( B \) rectamque \( CP \) in \( D \); & Triangulo \( ABC \) existente Modulo, area quaestæ Sectoris \( CAP \) erit mensura rationis inter \( QZ \cdot QP \& AB \), five rationis inter \( AB \& QZ - QP \).
five subduplicate rationis inter $OZ + QP$ & $OZ - QP$, five-subs-
duplicate rationis inter $AB + AD$ & $AB - AD$; vel erit mensura-
rationis inter $RP + RX$ & $CA$, vel rationis inter $CA$ & $RP - RX$,
vel subduplicate rationis inter $RP + RX$ & $RP - RX$. Nam si
ducantur recta $AE$, $PF$ quæ fecent Asymptoton $CB$ in $E$ & $F$,
alterque Asymptoro parallela sint: æquales erunt hæ omnes rationes
rationi quam habet $AE$ ad $PF$, vel $CF$ ad $CE$; erit & sector $CAP$
area $EAPF$ æqualis; similiterque triangulum $ABC$ duplicato tri-
angulo $AEC$, five parallelogrammo Asymptotis & Hyperbolæ in-
scripto æquabitur. Quare patet propositum ex supra demonstratis.

Data vero per modum priorern area $BEFC$, vel per modum po-
steriorem area $CAP$; dabitur alia quavis area Hyperbolica ad arcum
$EF$, vel ad arcum $AP$ terminata: quippe quæ temper est area
modo inventæ & area alicujus rectilineæ vel summa vel diffe-
rentia. Q. E. I.

Scholium.

Hinc facilem habent solutionem Problemata omnia, quæcunque
pendent ab Hyperbolæ quadratura. Exemplum fatis luculentum:
pæbebit descensus gravium in Mediis, quorum resistentia est in du-
plicata ratione velocitatis corporis moti. Sit $V$ velocitas maxima
quam corpus in hujusmodi Medio, 
infinite descedendo, potest acquisi-
rere; $T$ dimidium temporis quo
corpus idem in codem Medio, 
vi sola ponderis sui relativi, ab-
que resistentia cadendo velocitatem 
ilam acquirat; $S$ spatium hocce 
casu descripturn; $R$ pondus rela-
tivum corporis in Medio resis-
tente: & quæratur spatium $s$ quod
 corpus descedens, tempore quovis$t$, described in Medio resistentente;
& resistentia $r$ quam patitur in fine
illius temporis; & velocitas $v$ ex
isto descensu acquisita.

Centro $D$, vertice $A$ describatur
Hyperbola æquilatera $AT$; cujus
una Asymptotorum est $DC$ & ad
verticem tangens $AC$ semiaxi $AD$ æqualis. Capiatur area $DAT$ ad
dimidium trianguli $DAC$ ut $t$ ad $T$, secetur $DT$ tangente $AC$ in $P$.
Sumantur profit
quam ad rmenfuram ad KN, erit pro ad ac *8
bere
tia denotantibus pundum Sc
dione analogia parallelogrammum per
mum redangulum FL, dignitatem bolam,
onis proportionalis V aequalis, vel
ad anguli AC, reda
tions: AB ad ter & CZ R.
Avam & GE Libet EF
fumendo fub fumatur ad, ita ut t fit mensura rationis in-
ter Ef & EF ad Modulum T, & bifeicitur Ff in G: erit GF ad
GE ut AP ad AC, hoc est, ut v ad V. Sumantur GE, GF,
GH continue proportionales: & erit GH
ad GE ut AK ad
AC, hoc est, ut r ad
R. Erit infuper EG
ad EH ut CA ad CK; unde cum fit s ad S ut area ABNK ad
rectangulum CKN, hoc est, ut mensura rationis inter CA & CK
vel inter EG & EH ad mensurae Modulam: erit s mensura rati-
onis inter EG & EH ad Modulum S, atque inde dabitur.

Ex hisce porro facillime se prodit, per unicam quamvis Hyper-
bolam, constructio non inconcina; quam & adscribere visum est ob
dignitatem Problemati. In recta quavis GE sumatur utcunque
punctum F inter E & G, & ab altera parte capiatur Gf ipsi GF
aequalis, & sint GE, GF, GH continue proportionales. Deinde
per puncta E, F, H, G, f ducantur sih invicem parallelae rectae ER,
FL, HM, GQ, fI, quas fecet Hyperbola quavis LMQL centro E, AYsymptotis ER, EG descripta, & compleatur parallelogram-
mum EGQR. Jam si fit t ad T ut area Hyperbolica LfI ad
parallelogrammum EQ: erit s ad S ut area MHGQ ad EQ; v ad
V ut GF ad GE; r ad R ut GH ad GE.

Libet & casum alterum adjicere ubi corpus ascendit; ne forte
analogia illa, quae inter utrumque fervari debet, in allata construc-
tione quodammodo perire videatur. Ergo eadem atque prius
denotantibus V, R, T, S, ponantur v & r pro velocitate & resisten-
tia sub ascensus initio, s pro spatlo quod corpus ascendendo descri-
bere posset antequam tota velocitatis amittatur, t pro tempore hujius
ascensus. Ad EG erigatur perpendicularis GO ipsi EG aequalis,
& sumendo puncta F, f, ad easdem distantias hinc inde ad puncto G,
jungantur \(OF, Os\) quibus occurrar in \(T & t\) circuli arcus \(TGt\)
centro \(O\) descriptus, & sint \(Gb, Gf, GE\) continue proportionales,
& ducatur ipsi \(ER\) parallela \(hm\) Hyperbolæ occurrentis in \(m\). De-

inde si \(t\) sit mensura anguli \(FOf\) ad Modulum \(T\), hoc est, si \(t\) sit
ad \(T\) ut arcus \(TGt\) ad radium \(OG\): erit \(s\) mensura rationis inter
\(Eb & EG\) ad Modulum \(S\), vel erit \(s\) ad \(S\) ut area Hyperbolica
\(mhGQ\) ad \(EQ\); & \(v\) erit ad \(V\) ut \(Gf\) ad \(GE\); atque \(r\) ad \(R\)
ut \(Gb\) ad \(GE\).

**PROPOSITIO V.**

*Logisticam describere per Canonem Logarithmorum.*

\(S\) i ad Logisticae \(BODG\) Asymptoton \(APCF\) ordinatim appli-
centur binae quævis rectæ \(AB, FG\) intercludentes Asymptoti
portionem quamvis \(AF\): erit illa portio mensura rationis quam ad
invicem habent ordinatae; haec utique est natura Curvæ notissima.
Integrum ergo & perfectum Systema Logometricum per hanc Li-
neam exhibetur: id quod etiam de Hyperbola dici potest per Prop-
ositionem præcedentem, de Spirali æquiangula per subsequentem;
nam omitto complures alias Figuras, quae & ipsae dudum sunt in Geometriam receptae. Itaque si detur Asymptoti positio & simul duo puncta per qua Curva transire debet, dabuntur puncta reliqua per casum priorum Propositionis tertiae. Quod si data positio Asymptoti, detur insuper Systematis Modulus atque unicum punctum per quod ducenda erit Curva; invententur puncta reliqua per Casum posteriorem Propositionis ejusdem. Iste vero Modulus quo pacto definiendus sit, & qualem habeat magnitudinem, jam oportet exponere.

Ducatur recta $BC$ qua Curvam tangat in $B$ & Asymptoton fecet in $C$. Dico primo, magnitudinem subtangentis $AC$ eandem permanere ubicunque sumatur punctum $B$. Intelligatur enim Ordinata $PQ$ vicinissima Ordinatæ $ARB$, recta vero $QR$ parallela Asymptoto $AC$, ac detur Ordinarum intervallum illud quam minimum $AP$. Ob datam igitur lineolam $AP$, dabitur ratio quam habet $AB$ ad $PQ$, & divisim ratio quam habet $AB$ ad $RB$, atque adeo (propter similia triangula $BAC$, $BRO$) ratio quam habet $AC$ ad $RQ$ tive $AP$, atque inde magnitudo ipsius $AC$.

Dico secundo, determinatam hanc & immutabilem subtangentem $AC$, esse Modulum ad quem exignexae sunt mensurae illæ interceæ $AF$. Patet hoc per Corollarium secundum Propositionis primæ: nam dum termini $AB$ & $PQ$ ad æqualitatem proxime accedunt, erit $AC$ ad $AP$, quæ metitur rationem inter $AB$ & $PQ$, ut terminus $AB$ ad terminorum differentiam $BR$. Unde data subtangente, facilis est descriptio Curvæ & solutio Problematum omnium quæ exhinc pendent.

Si Curva jam descripta habeatur, subtangentis magnitudo sic determinabitur. Producatur Ordinata quævis $CD$ ad $E$, ita ut $CE$ ad $CD$ rationem habeat Modularem, per Corollarium sextum Propositionis primæ definitam; & recta $EB$ quæ à puncto $E$ parallela ducitur Asymptoto, quæque Curvæ occurrit in puncto $B$, æqualis erit subtangenti qualitate.
Corol. 1. Area \( A\bar{B}I\bar{H} \), qua e inter \( \text{Curvam BDI} \) & Asymptotam ejus \( A\bar{C}H \) infinita versus \( HI \) extenditur, & ad alteram partem ab Ordinata \( AB \) terminatur, æqualis est parallelogrammo \( A\bar{B}\bar{C}E \) ab Ordinata eadem \( AB \) & subtangente \( AC \) comprehenso. Componuntur enim area & parallelogrammum ex elementis quæ sunt ut \( \bar{A}P \times \bar{A}B \) & \( \bar{A}C \bar{X} \bar{R}B \), quæque adeo æquantur propter analogiam inter \( \bar{A}P \) & \( RB \), \( AC \) & \( AB \).

Corol. 2. Atque hinc, ob datam subtangentis magnitudinem, area illa indefinita erit ut ordinata ad quam terminatur.

Scholium.

Hujus Propositionis usus per Exemplum declarabitur. Proponatur ad quamlibet altitudinem à superficie telluris, invenire densitatem Atmosphærae. Sit \( AB \) telluris superficies, & abinde fursum producatur perpendicularis \( AH \), atque ad hujus puncta singula ductæ concipientur Ordinata \( FG \), quæ sint ut Aeris densitates in locis \( F \); & Ordinatarum termini omnes \( G \) in Linea Logistica \( BDGI \) sit erunt. Patet hoc per Corollarium secundum hujus Propositionis. Nam area indefinita \( FG\bar{I}H \) est ut quantitas seu pondus Atmosphærae supra locum \( F \), & pondus illud est vis quæ comprimit Aerem in hoc loco, istsæ vero vis (uti docet Experientia multiplex) est ut Aeris compressi densitas \( FG \).

Itaque si quotlibet altitudines sumantur in Arithmetica progressione; densitates Aeris in his altitudinibus erunt in progressione Geometrica; & differentia binarum quærumvis altitudinum, erit mensura rationis quæ est inter densitates Aeris in illis altitudinibus.

Cessante vi gravitatis, ita jam per vim aliquam extraneam intelligentur Aeris facta compressio, ut eandem habeat ubique densitatem quam ad terre superficiem; & quantitas ejus, quæ modo erat exposita per aream indefinitam \( H\bar{A}B\bar{I} \), nunc per æquale rectangulum \( A\bar{B}\bar{C}E \) exhibebitur. Atmosphæra hujus homogeneæ altitude \( AC \), est ad altitudinem Hydrargyri in tubo Torricellii, ut gravitas Hydrargyri ad gravitatem Aeris; atque inde datur. Huic autem datae altitudini æquantur (per Corol. 1.) subtangens Curvæ \( BDGI \), atque adeo Modulus Syltematis mensurarum omnium \( AF \). Est ergo Logarithmus rationis inter densitates Aeris in binis quibusvis altitudinibus, ad Modulum Canonis, ut altitudinem earundem differentia, ad Atmosphærae prædictæ homogeneæ altitudinem illam datam \( AC \).

\[ \text{D} \]

\[ \text{Hæc} \]
Hac ita se habebit ex Hypothesi, quod vis gravitatis eadem sit ad omnes altitudines. Ceterum ex Philosophia Newtoniana constat eam diminui, in recessu a centro telluris, in duplicata ratione distantiae: conclusio itaque paulo aliter se habebit. Sit $S$, centro telluris, & $AB$ superficies ejusdem; sumatur ipsis $SF$, $SA$ tertia proportionalis $Sf$, erigatur ordinata $fg$ quae sit ut Aeris densitas in $F$; & Curva $Bgn$ quam punctum $g$ perpetuo tangit, erit eadem atque prius Logistica, sed inverso situ. Augeatur enim altitudo $AE$ partícula quam minima $FM$, capiatur $Sm$ ad $SA$ ut $SA$ ad $SM$, ducatur Ordinata $mn$ quae sit ut Aeris densitas in $M$; & erit $Sm$ ad $Sf$ ut $SF$ ad $SM$, & divisim $fm$ ad $FM$ ut $Sf$ ad $SM$, sitque ut $Sf$ ad $SF$, hoc est, ut $SAq$ ad $SFq$. Unde $fm$ est ut $SFq$ inversa & $FM$ directe, id est, ut gravitatio & moles Aeris inter $F$ & $M$ conjunctim; adeoque $fm$ quae sit ut Aeris densitas in $M$; & erit $Sm$ ad $Sf$ ut $SF$ ad $SM$, & divisim $fm$ ad $FM$ ut $Sf$ ad $SM$, sitque ut $Sf$ ad $SF$, hoc est, ut pressio illius in Aerem inferiorum: & summa similium omnium arearum infra $fg$ est ut summa pressionum omnium supra $F$; id est, ut Aeris in $F$ densitas $fg$; & summarum differentiae $fgmn$ ut densitatum differentia $fg$—$mn$. Detur lineola $fm$; & erit $fg$ ut area $fgnm$, adeoque ut $fg$—$mn$, atque inde (componendo) ut $mn$. Ergo data lineola $fm$ erit mensura datae illius rationis quae est inter $fg$ & $mn$: atque hinc potest Curvam $Bgn$ esse Logistica. Sed & eandem esse cum supra descripta Logistica, facile abinde colligitur, quod ordinatae basi $AB$ vicinissimae & ad aequalia intervalla quam minima dispositae, respective sint æquales in utraque Curva; ac proinde eadem curvatura, eadem inclinatio tangentis ad punctum $B$, eademque subtangentis magnitudo.
Ergo si distantiae $SF$ ad centro telluris, capiantur in Musica progressione; harum reciprociæ, nempe distantiae $Sf$, erunt in progressione Arithmetica; & Aeris densitates $fg$ erunt in progressione Geometrica.

Ad inveniendam itaque densitatem in loco quovis $F$, minuenda est altitudo $AF$ in ratione distantiae $SF$ ad telluris femidiametrum $SA$: & Logarithmus rationis inter densitatem Aeris in $A$ & $F$, erit ad Modulum Canonis, ut altitudo illa diminuta $Af$, ad Atmosphæra homogeneæ altitudinem $AC$.

Quæ supra demonstrata sunt, accurate obtinebunt, si modo Atmosphæra ex Aere pariter Elastico tota constet: rationes igitur allatae paululum conturbabunt admitti vaporese atque exhalationes, quibus etiam accedet Caloris Frigorisque diversæ temperies ad altitutines diversas.

**Propositio VI.**

Logarithmorum Canonem ad Spiralem Aequiangulam accommodare.

Æ Quiangula Spiralis appellatur Linea illa curva $ADE$, qua polo $P$ descripta, in eodem dato angulo secat exterens à polo radios $PA, PD, PE, &c.$

Si centro $P$ & intervallo quovis $PA$ describatur circulus $ABC$, qui radiis $PA, PD, PE$ occurrat in $A, B, C$. Dico interceptum arcum $BC$ mensuram fore rationis quam habet $PD$ ad $PE$, & interceptum arcum $AB$ mensuram rationis quam habet $PA$ ad $PD$. Dividatur ehis arcus $AB$ in particulas quam minimas & aequalis $QR$, & jungantur $PQ, PR$ fecantes Spiralem ad $S$ & $T$ in angulis datis $PST, PTS$: & ob datam particulam $QR$, dabitur angulus $QPR$, atque adeo species Figuræ $SPT$, & ratio laterum $PS, PT$ Data ergo particula $QR$ mensuratur erit rationis datae quam habet.
habet \( PS \) ad \( PT \); \& summa particularum, nemo arcus \( AB \), mensura, erit summa similitudinis rationum, hoc est, rationis quam habet \( PA \) ad \( PD \). Et eodem argumento, erit arcus \( BC \), mensura rationis quam habet \( PD \) ad \( PE \).

Ducatur \( AF \) Spiralem! tan-νομόνων, quorum maxime vocavit \( fumma \) gens ad Cirkull & Spiralis intersectionem \( A \), huic vero in \( F \) occurrat recta \( PC \), quae ad radium \( PA \) normalis erigitur; \& sub(tangens \( PF \) erit mensura\( ruit\) Modulus, per Corol. 2. Prop. 1. Nam si in recta \( PS \) sumatur \( PK \) ipsi \( PT \) aequali, \& jungantur puncta \( V, T \); similia erunt triangula \( PAF, PST \).

Unde \( PF \) est ad \( VT \) ut \( PA \) ad \( VS \), sed \& \( VT \) est ad \( QR \) ut \( PT \) ad \( PA \): ergo ex aequo perturbate, \( PF \) est ad \( QR \) quae metitur rationem inter \( PS \) ad \( PT \), ut terminus \( PT \) ad terminorum differentiam \( VS \).

Scholium.

Spiralem æquiangulum, ad Meridiana Nauticæ divisionem demonstrandum, feliciter adhibuit Geometra clarissimus Edmundus Halleyus. Sit \( acp \) pars octava Sphaæ terrestriæ, \( p.Polt. \)qua quadrans Equatoris, \( ap \) quadrans Meridiani; \& quæratur magnitudo rectæ, quæ propositum quemlibet hujus arcum designet in Planisphario. Per Equatoris \& Meridiani intersectionem \( A \), ducta intelligatur linea Helicoeides \( ade \) quæ fecet omnes Meridianos ad angulum semirectum, huic occurrat in \( d \) parallelus \( AEquatorii circulus \( gd \), per idem punctum \( d \) agatur Meridianus \( pdb \); \& longitudo intercepti arcus \( AEquatoris ab \), erit magnitudo Nautica quæsitā arcus \( ag \), Resolvatur enim arcus \( ag \) in particulas innumeræ, quant maximas \( gk \), ductar parallelus \( kmn \), secans Meridianum \( pdb \) in \( v \), Lineam \( ade \) in \( m \); \& arcus Meridianus \( pmh \) ascendent \( AEquatoris particulas \( bh \), quæ erit ad \( mn \), five huic (ob angulum semirectum \( m\overline{dn} \) æqualem \( m\overline{dn} \) vel \( gk \), ut peripheria \( AEquatoris \) ad peripheriam parallelis \( kmn \). Est ergo particula \( bh \) magnitudo Nautica, particula \( gk \), \& summa particularum omnium \( bh \), nemo, longitudo arcus \( ab \), magnitudo Naut-
tica summæ particularum omnium $g_k$, id est, arcus $ag$. Manente
jam $\overset{\text{Equatore}}{abc}$ vel $\overset{\text{ABC}}{abc}$, concipiatur Sphærica superficies in
plano ejus Stereographice depingi; & Polo $p$ occupante centrum $P$,
proscient Meridiani $pga$, $pdb$, $pec$ in totidem rectas $PA$, $PDB$,
$PEC$ à centro $P$ exeuntes, ita ut distantia abinde puncti cuiusvis $D$
vel $A$, tangens sit arcus
dimidiati $pd$ vel $pa$ quem
distantia illa repræsentat.
Linea vero Helicoideas
$ade$ convertetur in Spiralem æquiangulam $ADE$,
polo $P$ descriptam, & se-
cantem radios suos om-
nes ad angulum semi-
rectum. Hoc sicutem
postulat nota Lex hu-
jusce Projectionis, ut an-
guli omnes eandem in
Plano ac in Sphærica
superficie magnitudinem
servent. Arcus itaque
propositi $ag$ magnitudo
Nautica $ab$ vel $AB$, est
ad subtangentem $PF$ vel huic jam æquali Sphærae radium $PC$, ut
Logarithmus rationis inter $PA$ & $PD$, hoc est, inter tangentes di-
midiatorum arcuum $pa$ & $pd$, vel $pa$ & $pg$, ad Modulum Ca-
nonis.

Hinc quoniam longitudo Radii est ad longitudinem arcus minuti
unius primi; ut 3437746770784939 &c ad 1, & reciprocus Moduli Ca-
nonis est 2,30258592994 &c, atque hi numeri in se multipliati efficiunt 7915794467897819 &c; si magnitudo illa Nautica $AB$ in minutis primis
exhibenda sit, uti mos exigit; subducta tangent arteficiali di-
midiati arcus $pg$ a tangente artificiali dimidiati arcus $pa$, multiplice-
tur residuum per numerum 7915794467897819 &c, et factus dabit
partes Meridionales desideratas. Perinde vero se habebit conclusio,
five in $\overset{\text{Equatore}}{abc}$, five extra huac alibi ad utramvis partem loca-
tur punctum $a$.  

\[ S_{\text{ago}} \]
Scholium Generale.

In eum portissimum finem præcedentia conscripti, ut allatis aliquot Exemplis offendenterem, qua commodissima ratione Logarithmorum usus in Geometriam recipi, & ad resolutionem Problematum difficiliorum adhiberi possit. Vidiorem est hoc loco nonnullas adiere porro constructiones, eodem consilio effectas, quæ, mihi istic tractant subinde se æ obviam non invitæ dedere: ut ıtæ, ex uteriore speci mine, de præstantia Methodi hujus Logometricæ judicium feratur.

Parabolæ Apolloniana AP fit A vertex, F focus, AQ axis, PQ ordinatim applicata ad axem. Ducatur AL quæ bifariam fecet PQ in L, & productæ adjiciatur LM quæ fit mensura rationis inter LA + AQ & QL ad Modulum AF; & recta AM aequalis erit arci Parabolico AP.

Spiralis Archimedeæ PQ similem habet extensionem in rectam. Sit Q polus ejus, QP radius à polo ductus ad Curvæ quodlibet punctum P, & ad eum radium normalis QA. Ducatur LA parallela tangenti Spiralem in P, quæ radium PQ bifariam fecet in L; & ponendo AF ad QL ut QL ad QA, ipsi AL adjiciatur LM quæ fit mensura rationis inter LA + AQ & QL ad Modulum AF; & recta AM æqualibus Spiralis arci PQ.

Spiralis Reciproca AE fit A polus, AB radius primus & infinitus, CD asymptotos radio primo parallela ad distantiam AC; & invenienda proponatur hujusce Curvæ longitudo. Inter Spiralem illum vulgarem Archimedis atque hanc, quam Reciprocam appellem illam vulgarem Archimedis atque hanc, quam Reciprocam appellor, isthæe intercedit differentia, quod cum illius radii sint ut angles quos faciunt cum radio suo primo, hujus radii è contrario sunt.
funt reciproce ut idem anguli: eandem utique proportionem habet radius $AE$ ad radium $Ae$, quam habet angulus $eAB$ ad angulum $EAB$. Unde facile colligitur, si ad puncta $E$ & $e$ ducentur tangentes $EF$, $ef$, & ad radios $AE$, $Ae$ erigantur normales $AF$, $As$, fore normales istas sibi invicem & Asymptoti intervallo $AC$ æquales. Invenitur autem longitudo cujusvis arcus $Ee$, ponendo $LM$ mensuram rationis inter $AE$ & $EF-AF$ ad Modulum $AF$, & similiter $lm$ mensuram rationis inter $Ac$ & $ef-Af$ ad æqualem Modulum $Af$. Nam si tangentium differentiae $EF-ef$ adjiciatur mensurarum differentia $lm-LM$, aggregatum æquabitur arcui $Ee$.

Linea illa Logistica, cujus aligras expressuimus affectiones in Propositione quinta, non absimilem habet longitudinis suæ determinationem; quam & hoc loco apponam in eorum gratiam qui hujusmodi contemplationibus délectantur. Oblata sit igitur Logistica $EMem$, cujus Asymptotos $AFaf$: & quærat longitudo cujusvis arcus $Ee$. Demissis in Asymptoton perpendiculis $ELA$, $ela$, &
& ductis tangentibus $EF$, $ef$, capiatur $AL$ æqualis excessui quo tangens $EF$ superat subtangenterm $AF$, & simili ter $al$ æqualis excessui quo tangens $ef$ superat subtangenterm $af$: & actis $LM$, $lm$ Asymptoto paralleli, si tangentium differentia $EF$ $ef$ adjicatur parallelarum differentia $lm$ $LM$, aggregatum æquabirur arcui $Ee$.

Accedo ad Cissoidem Diocleam. Sit $A$ vertex ejus, $AB$ diamenter Circuli genitoris, $BC$ Asymptotos, $PQ$ perpendicularis in diametrum demissa, Cissoidi in $P$ & diametro in $Q$ occurrens. Agatur $AC$ quæ sect Asymptoton in $C$ ac faciat angulum $BAC$ qui fit recti pars tertia, sumptaque inter $BQ$ & $BA$ media proportionali $BD$ jungatur $CD$; denique per medium perpendicularum $PQ$ ducatur $AE$ recta, quæ occurrit Asymptoto in $E$: & Cissoidis arcus $AP$ æquabirur duplicato excessui rectæ $AE$ supra diametrum $AB$, & simul triplicata mensurae rationis inter $BA + AC$ $BD + DC$ ad Modulum $BC$.

Si Cissoidis area $APQ$ convertatur circum axem $AQ$; generabitur solidum cujus dimensio pendet a Logometria, & sic confruitur. Sint $AQ$, $AB$, $AR$, $AS$, $AT$ continue proportionalis; deinde ad Modulum $TS$ capiatur $QX$ mensura rationis inter $AB$ & $BQ$, & retro ponatur $XZ$ æqualis ipsi $SR$ una cum dimidio ipsius $RB$ ac triente simul ipsius $BQ$: & solidum Cissoidale axem habens $AQ$ basiæque semidiametrum $PQ$, æquabirur Cylindro cujus eadem est basis & cujus altitudo est $OZ$.

rectam $PD$ in $S$: & semissumma solidorum Conchoidalium quæ generantur ex conversione Figurarum $AEDC$, $aeDC$ circum axem $AaP$, erit ad sectorem Sphæræ genitum ex circuli sectore $PRS$ circum axem eundem converso, ut $3PC \times PD + PRq$ ad $PRq$. Eorundem vero semidifferentia Cy-lindro æquaturn, cujus basis est circulus diametro $Aa$ descriptus, & cujus altitudo est mensura duplicata rationis inter $PD$ & $PC$ ad Modulum $PC$.

Area vero Figuraræ totius $AEda$ æquaturn rectangulo cujus basis est $Aa$, & cujus altitudo $CM$ est mensura rationis inter $PD + DC$ & $PC$ ad Modulum $PC$. Quod si desideretur quadratura partium $AEDC$, $aeDC$; ductis ad axem normalibus $AF$, $af$, in regula $CD$ sumenda est $CN$ quæ sit anguli $CPD$ mensura ad eundem Modulum $PC$: & acta per punctum $M$ recta $FMf$ quæ par-allele sit rectæ jungenti puncta $P$, $N$, quæque occurrat normalibus in $F$ & $f$; erit area $AEDC$ æqualis Trapezio $AFMC$, & area $aeDC$ æqualis Trapezio $afMC$.

Hyperbolæ quadraturam in superioribus expostam dedi, eo modo, qui mihi vi-fus est ad propositum quam maxime accommodatus. Li-bet aliam constructionem hoc loco apponere, & simul ad-ijicere gravitatis centrum. Oblata fit porio interior $ADB$, intercluse curvæ $ADB$ & rectæ cuvis $AB$ ad diametrum $PQ$ paralleæ. A Figuraræ centro $C$ produca-tur diameter $CDE$, quæ basin $AB$ bifarium fecet in $E$; deinde si in diametro
produca fumantur CR ad CD, & CD ad CS, ut basis AB ad diametrum PQ, & ad Modulum CS fiat CN mensura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur area curvilineæ ADB.

Hujus autem areae centrum gravitatis Z invenietur, capiendo CZ ad CR ut 2 CR ad 3 EN.

Sit nunc oblata portio exterior APQB, interclusa curvis oppositis AP, BQ, diametro PQ, & rectae curvis AB ad diametrum illam paralleæ. Efto CD conjugatae semidiametri longitudo extra portionem oblatam APQB positæ, quæ producunt in contrariam partem centri C bifariam secet basilin AB in E. Deinde in diametro producunt si fumantur CR ad CD, & CD ad CS, & CS ad CT, ut basis AB ad diametrum PQ, ponuntur vero CR & CT ad eandem centri partem cum basi AB, & ad Modulunm CS, in contrariam centri partem, fumatur CN mensura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur area curvilineæ APOB.

Hujus autem areae centrum gravitatis Z invenietur, capiendo CZ ad CR ut 2 TR ad 3 EN.

Pergo ad superficies ab Hyperbola circum axes suos convoluta genitas. Sit AN Hyperbola descripta vertice A, centro C, Asymptote CB, foco F, semiaxe principali AC, semiaxe conjugato AB normali ad AC; & ad axis AC punctum quodvis X fit XN ordinatim applicata, quæ Hyperbolæ occurrat ad N.
In axe CA capiatur CE ad CA ut CA ad CF; & ad eundem axem erecta perpendiculari EZ, quæ Asymptoto occurrat in G, angulo CEZ inebribatur æqualis ipsi CX recta CZ, quæ porro producTa secat ordinatim applicatam XN ad O. Tum sumatur KL quæ sit æqualis exceffui quo XQ superat AB, atque LM quæ sit mensura rationis inter CZ + ZE & CG + GE ad Modulum CE: & superficies genita ex arcus AN conversione circum axem AX, erit ad Circulum semidiametro AB descriptum, ut exceffus KM quo KL superat LM, ad semidiametrum illam AB.

Sit rursus BN Hyperbola descripta vertice B, centro C, foco E semiaxe principali CB, semiaxe conjugato CA normali ad CB; & ad axis AC punctum quodvis X sit XN ordinatim applicata, quæ Hyperbola occurrat ad N. In axe CB capiatur CE ad CA ut CA ad CF, & jungatur EX. Tum sumatur KL quæ sit ad XC utXE ad CE, & LM quæ rationis inter EX + XC & CE mensura sit ad Modulum CE: & superficies genita ex arcus BN conversione circum axem CX, erit ad Circulum semidiametro CB descriptum, ut linearum KL & LM aggregatum KM, ad semidiametrum illam AB.

His addere licebit ab Ellipsis genitas superficies. Sit ANB Ellipsis descripta centro C, verticibus A & B, foco F, semiaxe principali CB, semiaxe conjugato CA; & ad axis CA punctum quodvis X sit XN ordinatim applicata, quæ Ellipsis occurrat ad N. In axe CB capiatur CE ad CA ut CA ad CF, & jungatur EX. Tum sumatur KL quæ sit ad XC utXE ad CE, & LM quæ rationis inter EX + XC & CE mensura sit ad Modulum CE: & superficies genita ex arcus BN
conversione circum axem $CX$, erit ad Circulum semidiametro $CB$ descriptum, ut linearum $KL$ & $LM$ aggregatum $KM$, ad semidiametrum illam $CB$. Ut hae ultima constructio locum habeat, oportet semiaxem $CA$ circa quem conversio facta est, minorem esse altero semiaxem $CB$; alter enim Moduli $CE$ quantitas $\frac{CAq}{\sqrt{CBq-CAq}}$ evadet impossibilis, & constructio illa Logometrica (quod in hujusmodi casibus fieri solet) convertet se in Trigonometricam, qualis illa est quae jam sequitur.

Sit $ANB$ Ellipsis descripta centro $C$, verticibus $A$ & $B$, foco $F$, semiaxie principali $CA$, semiaxie conjugato $CB$; & ad axis $CA$ punctum quodvis $X$ sit $XN$ ordinatim applicata, quae Ellipsis occurrat ad $N$. Angulo $CXB$ inscribatur recta $CE$, quae sit ad $CA$ ut $CA$ ad $CE$. Tum sumatur $KL$ quae sit ad $XC$ ut $XE$ ad $CE$, & $LM$ quae anguli $XEC$ mensura sit ad Modulum $CE$, hoc est, quae sit æqualis arcui cujus sinus est $XC$ ad radium $CE$: & superficies genita ex arcus $BN$ conversione circum axem $CX$, erit ad Circulum semidiametro $CB$ descriptum, ut linearum $KL$ & $LM$ aggregatum $KM$, ad semidiametrum illum $CB$. Posset hujus etiam superficie dimensio per Logometricam designari, sed modo inexplicabili. Nam si quadrantis circuli quilibet arcus, radio $CE$ descriptus, sinum habeat $CX$ finunque complementi ad quadrantem $XE$: sumendo radium $CE$ pro Modulo, arcus erit rationis inter $EX+XC\sqrt{-1}$ & $CE$ mensura duxta in $\sqrt{-1}$. Verum isthaec aliis, quibus operæ pretium videbitur, diligentius excutienda relinquo. Ceterum ex precedentibus intelligi potest, quanta sit cognatio inter angulorum atque rationum mensu-
mensuras, quamque levi mutatione in se invicem facillime convertantur pro variis ejusdem Problematis casibus. De Cubicarum æquatiorum radicibus dudum ab Analytis observatum est; vel eas exprimi posse per Cardani regulas, atque adeo per duarum medium proportionalium inventionem; vel per divisionem arcus circularis in tres æquales partes, si forte fuerint inexplicabiles per memoratas regulas. *Hoc animadvertit Cartesius, sed & ante Cartesium idem observavit Franciscus Vieta sub finem Supplementi Geometriae. Exhinc autem aperte colligitur, qualis sit ordo Naturæ transcuentes ad Anguli trifæctionem à trifectione Rationis.

Mirabilem illum Harmoniam ulterior declarare luctet. Exemplo desumpto ab eadem Figura circum axes suos convoluta. Sit igitur ABPQ Ellipsis, axis ejus major AB, minor PQ, centrum C, focus F. Hæc circum axem utrumvis convoluta Solidum generet, cujus particulae constantes ex materia homogenea, vires attrahentes habeant in duplicata distantiarum ratione decrecentes: & quotatur vis qua Solidum illud attrahit corpusculum quodvis, in ejus superf-

* Sublato etenim termino secundo, tres habentur Equationum casus. Hi vero refolvuntur ope trianguli rectanguli ABC, rectum habentis angulum ad A, in quo insuper triangulo temper data sunt duo latera.

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Caf. 1. Nam si sit \( x^3 + 3ax = \pm 2ab \): ponantur \( AB = a, AC = b \); & sumantur \( M \) & \( N \) binæ medii proportionales inter \( BC + AC \) & \( BC - AC \); & erit \( M - N \) radix unica possibile affirmativa, si habeatur \( + 2ab \); vel \( N - M \) radix unica possibile negativa, si habeatur \( - 2ab \).

Caf. 2. Si sit \( x^3 - 3ax = \pm 2ab \), exifente a minore quam \( b \): ponantur \( AB = a, BC = b \); & sumantur \( M \) & \( N \) binæ medii proportionales inter \( BC + AC \) & \( BC - AC \); & erit \( M - N \) radix unica possibile affirmativa, si habeatur \( + 2ab \); vel \( M - N \) radix unica possibile negativa, si habeatur \( - 2ab \).

Caf. 3. Denique si sit \( x^3 - 3ax = \pm 2ab \), exifente a maiorque quam \( b \): ponantur \( AB = b, BC = a \); & sumatur \( M \) sinus trientis angulorum summæ \( A + B \), atque \( N \) sinus trientis angulorum differentiae \( A - B \), existente radio \( 2BC \); & erunt \( M, N, M + N \) tres radices possibles, si habeatur \( + 2ab \); vel \( M, N, - M - N \) tres radices possibles, si habeatur \( - 2ab \).

Atque ita Problemata omnia Solida solutionem facilem recipiunt, vel per Canonem Logarithmicum, vel per Canonem Trigonometricum.
Hinc quoniam Solidum posterius medium est proportionale inter Solidum prius & Sphaeram priorem: vis Solidi posterioris in corpulculo ad A, erit media proportionalis quamproxime inter vires Solidi prioris & Sphæræ prioris in idem corpulculo ad A, si modo axes Ellipsois sint prope æquales. Itaque in hoc casu, ponendo CG medium proportionalem inter CF & 3FD, & capiendo CH ad 3FE ut CA ad CF; posteriors Solidi vires ad A & P, vel ad B & Q, erunt ad invicem quamproxime ut CG ad CH.

Id quod non inutile præbet compendium ad inventionem Figuræ Telluris, qualem eam subtiliter instituit celeberrimus Newtonus, summus ille Philosophæ sanioris Instrutor.

Consideratio virium centripetarum aliud porro mihi fuggerit Exemplum, in quo fatis ampla se prodit mutationum varietas. Proponatur Trajectoriarum species enumerare, in quibus corpora moveri possunt, quæ à viribus centripetis in ratione distantiarum triplicata decrecentibus agitantur, quàque de loco dato, cum data velocitate, secundum datam rectam egrediuntur.
Cas. 1. Sit $S$ centrum virium, exeatque corpus de loco $P$ secundum rectam $PQ$ vel $QP$, ea cum velocitate quam acquirere potest ab iisdem viribus, libere cadendo versus centrum $S$ de loco $C$ & cañu suo describendo altitudinem $CP$. In datam rectam $QPT$ demittantur perpendicula $SQ$, $CT$, centroque $S$ & intervallum $\sqrt{SQ^2 + QT^2}$ describatur circulus $RTA$, rectae $SPC$ occurrens in $R$: deinde ad Modulum $\sqrt{SCq - SKq}$ fit arcus $RA$ mensura rationis inter $SR = \sqrt{SRq - SPq} \& SP$, jaceant autem arcus ille $RA$ & punctum $Q$ ad diversas partes rectae $SR$; & punctum $A$ erit Apsis summam Trajectoriae. Exhinc vero Trajectoria dabitur, tumendo $SM$ æqualem ipsi $\sqrt{SCq - SRq}$, deinde in recta $SA$.

capiendo longitudinem quamvis $SD$ quæ fit minor quam $SA$, ad eandem erigendo perpendiculum $DE$ fecans circulum in $E$, & jun- gendo $SE$. Nam si ad utrasque partes puncti $A$ ponatur arcus circularis $AR$, cujus longitudo fit mensura rationis inter $SE + ED$ & $SD$ ad Modulum $SM$, & in semidiametris $SR$ capiantur dis- tanta æ SP æquales ipsi $SD$: erunt puncta $P$ ad Trajectoriam de- scrivendum. Tempus autem quo radius $SP$, à centro ad corpus motum ductus, percurræ areae quamvis $SAP$, erit ut recta $DE$: nam area percurfa æquatur ipsi $DE$ in Modulum dimidiatum $\frac{1}{2}SM$ ductæ. Velocitas vero corporis in loco quovis $P$, erit ad veloc- itatem qua in Circulo, ad eandem distantiam $SP$, cum iisdem viribus revolvi.
revolvi posset, ut \( \sqrt{SCq - SPq} \) ad \( SC \). Ex ipsa constructione patet, hanc Spiralem primam infinitis gyris circa centrum virium contorqui, quin & seiplam infinitis vicibus decusfare, & siti erunt Nodi omnes ad Apsidis lineam \( AS \).

Cas. 2. Recedat punctum \( C \) ad infinitam distantiam à centro \( S \); & corporis de loco \( P \) secundum rectam \( PM \) vel \( MP \) exeuntis ea fit velocitas, quam acquirere posset cadendo libere ad eundem locum \( P \) ab infinita distantia. Ad rectam \( SP \) ducatur normalis \( SM \), que seget \( PM \) in \( M \); deinde centro \( S \) & intervallo \( SP \) describatur circulus, & in ejus circumferentia capiatur arcus \( PX \), cujus longitudo fit mensura rationis inter distantiam quamvis \( SD \) & distantiam datam \( SP \) ad Modulum \( SM \), jaceant autem arcus ille \( PX \) & punctum \( M \) ad diversas partes rectae \( SP \) fi \( SD \) fuerit major quam \( SP \), aliter ad ea- dem, inque semidiametro \( SX \) ponatur \( SZ \) æqualis ipsi \( SD \); & punctum \( Z \) erit ad Trajectoriam describen- dam. Tempus autem quo radius \( SZ \), à centro ad corpus motum ductus, percurret aream quamvis \( S P Z \), erit ut differentia quadratorum \( ex SZ \) & \( SP \); Nam area percursa, est ad illam differentiam, in data ratione Moduli dimidiati \( \frac{1}{2} SM \) ad \( SP \). Velocitas vero corporis in loco quovis \( P \), æqualis erit velocitati qua in Circulo, ad eandem distantiam \( SP \), cum iisdem viribus revolvi posset. Ex constructione patet hanc secundam Spi- ralem esse æquiangulam illam Propositionis sextæ; ea vero migra- bit in Circulum ubi angulus \( SPM \) fit rectus.

Cas. 3. Ut velocitas sit adhuc major, abeat jam punctum \( C \) ad distantiam plusquam infinitam à centro \( S \), vel (quod perinde est) accedat à parte contraria eidem centro, ad finitam distantiam; & corporis de loco \( P \) secundum rectam \( PQ \) vel \( QP \) exeuntis, ea fit velocitas, quam acquirere posset ascendendo libere de loco \( C \) ad in- finitam distantiam, & deinde ab infinita distantia ex altera centri partie
parte descendendo ad locum $P$, viribus centripetis inter ascenden-
dum in aequales vires centrifugas conversis. In datam rectam $PQT$
demittantur perpendiculara $SO$, $CT$; et erit $TQ$ vel major, vel a-
qualis, vel minor quam $SQ$. Si $TQ$ fuerit major quam $SQ$; cen-
tro $S$ & intervallo $\sqrt{TQq - SQq}$ describatur circulus $RBE$ rectæ
$SP$ occurræs in $R$, deinde ad Modulum $\sqrt{SCq - SKq}$ fit arcus
$RB$ mensura rationis inter $SR = \sqrt{SRq - SPq}$ & $SP$, jacent autem arcus ille $RB$ & punctum $Q$ ad partes diversas rectæ $SP$. Ex-
hinc Trajectoria dabitur, sumendo $SM$ aequalem ipsi $\sqrt{SCq - SRq}$,
in rectæ $SB$ capiendo longitudinem quamvis $SD$, ad eandem erigendo
perpendicularum $SE$ circulum fecans in $E$, & jungendo $DE$. Nam si
retro ponatur à puncto $B$ circularis arcus $BR$, cujus longitudo mensura
sit rationis inter $SE = ED$ & $SD$ ad Modulum $SM$, & in semidiameter $SR$
capiatur distantia $SP$ aequalis ipsi $SD$: erit punctum $P$ ad Traje-
goriam describendam. Tempus autem quo radius $SP$, à centro ad
corpus motum ductus, percurret aream quamvis hujus Trajectoría,
erit ut incrementum vel decrementum rectæ $DE$
per tempus illud factum: nam area percursa æquatæ huic incre-
mento vel decremento in Modulum dimidiatum $\frac{1}{2}SM$ ducto. Vel-
ocitas vero corporis in loco quovis $P$, erit ad velocitatem qua in
Circulo, ad eandem distantiam $SP$, cum iisdem viribus revolvi pos-
fet, ut $\sqrt{SCq - SPq}$ ad $SC$. Ex constructione patet, hanc Spira-
lem tertiam infinitis gyris centrum cingere infra punctum datum $P$;
at supra idem punctum vel non undique cinget, si arcus $RB$ minor
fuerit quam circumferentia tota $RBER$; vel toties cinget, quoties arcus ille circumferentiam excedit.

(F)

Cas. 4.
Caf. 4. Reliquis manentibus, fit jam \( TQ \) & \( SQ \) æquales. Centro \( S \) & intervallo \( SP \) describatur circulus \( P \ne X B \), & fit arcus \( PB \) æqualis ipsi \( SC \), jacent autem arcus \( PB \) & punctum \( Q \) ad partes diverzas rectæ \( SP \). Exhinc Trajectoria dabitur, sumendo in recta \( SB \) longitudinem quamvis \( SD \), centroque \( S \) & intervallo \( SD \) describendo circuli arcum \( DZ \) æqualem ipsi \( SC \). Nam si ordine circulari contrario ponantur arcus \( PB \) a puncto \( P \) & arcus \( DZ \) a puncto \( D \); erit punctum \( Z \) ad Trajectoriam describendam. Tempus autem quo radius \( SZ \), a centro ad corpus motum ductus, percurret aream quamvis \( SPZ \), erit ut differentia radiorum \( SZ \) & \( SP \); nam area percurta æquatur huic differentiae duæ in semissim distantiam \( SC \).

Velocitas vero corporis in loco quovis \( P \), erit ad velocitatem quæ in Circulo, ad eandem distantiam \( SP \), cum ipsædem viribus revolvi posset, ut \( \sqrt{SCq} - SPq \) ad \( SC \). Ex constructione patet, hanc Spiralem quartam esse Reciprocam illum, cujus longitudinem supra dimensam dedimus.

Caf. 5. Reliquis adhuc manentibus, fit jam \( TQ \) minor quam \( SQ \). Centro \( S \) & intervallo \( \sqrt{SQq} - TQq \) describatur circulus \( RAE \) rectæ \( SP \) occurrrend in \( R \); deinde fit arcus \( RA \), ad ejusdem circuli arcum cujus fecans est \( SP \), ut \( \sqrt{SCq} + SRq \) ad \( SR \); ponatur autem arcus ille \( RA \) ad eadem partes rectæ \( SP \) cum puncto \( Q \); & \( A \) erit Apsis ima Trajectoria. Exhinc vero Trajectoria dabitur, sumendo \( SM \) æqualem ipsi \( \sqrt{SCq} + SRq \), in recta \( SA \) capiendo longitudinem quamvis \( SD \) quæ fit major quam \( SA \), ducento \( DE \) quæ circulum tangat in \( E \), & jungendo \( SE \). Nam si ad utrasque partes puncti \( A \) ponatur arcus circularis \( AR \), cujus longitudine mensura fit anguli \( DSE \) ad Modulum \( SM \), & in semidiometris \( SR \) capiantur distantiae \( SP \) æquales. ipsi \( SD \); erunt puncta \( P \) ad.
P ad Trajectoriam describendam. Tempus autem quo radius \( SP \), a centro ad corpus motum ductus, percurret aream quamvis \( SAP \), erit ut recta \( DE \); nam area percurta aequatur ipsi \( DE \) in Modulum dimidiatum \( \frac{1}{2} SM \) ductæ. Velocitas vero corporis in loco quovis \( P \), erit ad velocitatem qua in Circulo, ad eandem distantiam \( SP \), cum iisdem viribus revolvi posset, ut \( \sqrt{SCq} - SPq \) ad \( SC \).

Ex constructione patet, hanc quintam Spiralem vel nullum habere Nodium, vel unicum, vel plures, pro varia proportione rectæ \( SM \) ad diametrum circuli \( EAR \): toties enim Trajectoria sese decus-\( \text{fabit, quoties illa recta diametrum excedit, } \& \text{ Nodi omnes siti erunt ad Apidis lineam } AS. \)

Sunt itaque Trajectoriarum quinque Species. Harum primam atque ultimam descriptit olim \textit{Newtonus}, per Hyperbole \& Ellipseos quadraturam.

Geometris integrum erit, ex adduitis haecenus Exemplis de Methodo nostra judicare; quam quidem, si proba fuerit, ulterior excolere peregent \& excolendo latius promovebunt. Patet utique campus amplissimus in quo vires suas experiri poterunt, præfertim si \textit{Logometriae Trigonometriam} insuper adjungant, quibus miram quandam affinitatem in se invicem cunctibus intercedere notabam. Hicce qui-\( \text{dem Principiis haud facile crediderim generaliora dari posse; cum.} \)

\( 39 \)
tota Mathesis vix quicquam in universo suo ambitu complectatur, præter angulorum & rationum Theoriam. Neque fane commodiora sperabit, qui animadverterit Effectiosis facilitatem per amplissimas illas, omnibusque suis numeris absolutas, tum Logarithmorum tum Sinuum & Tangentiun Tabulas, quas antecessorum nostrorum laudatissime folertiae debemus acceptas. Ut vero tanti beneficii uberior nobis exsurget fructus, id nunc exponendum refitut, quibus artibus ad istiusmodi conclusiones rectissima perveniat. In hunc finem Theoremata quædam, tum Logometrica tum Trigonometrica adjectis, quæ parata ad ulum afferveri; ni consultus visum effert, quum absque nimiis ambigibus ea tradi non possint, intaeta potius præterire atque aliis denuo investiganda reliquere. Ceterum isthoc apparatu non semper est opus; nam in Methodo. Fluxionum fape evenit ut ipsa Fluenteris, omillis hujusmodi subsidiis, ad Logometriam satis commode revocentur: id quod uno atque altero Exemplo ostendam.

Egimus in præcedentibus de rectilineo Gravium descensu, per Medii resistentiam continuam retardato, ex Hypothesi quod illa resistentia est in duplicata ratione velocitatis. Ex eadem Hypothesi resistentiam corporis penduli, in Cycloide oscillantis, jam sit propositum invenire. Cycloidis itaque in rectam explicata sit AC dimidium, C punctum insumum, B punctum a quo cadere incipit corpus pendulum, BC, CD arcus descensu ejus & subsequente acenso descripsi. Hilce positis, exquirenda est ratio quam habet resistentia corporis in loco quovis E, ad pondus ejus relativum in Medio resistente. Exponatur pondus illud per AC; & vis ab eodem oriunda, qua pendulum acceleratur ad E, exponetur per CE: quæ si dicatur x, & momentum ejus + x; momentum arcus jam descripi BE erit − x. Exponatur vis resistentiae per x; & vis qua pendulum vere acceleratur, erit ut excessus vis prioris supra resistentiam, hoc est, ut − x − z. Itaque cum resistentia sit ut quadratum velocitatis, resistentiae momentum z erit ut velocitas & velocitatis momentum, hoc est, ut − x & x − z, five ut z x − x x. Nam si tempus in particularis æquales dividatur, erit velocitas ut arcus descripi momentum − x; & velocitatis momentum ut vis acceleratrix x − z quæ momentum illud generat. Quoniam ergo z est ut z x − x x, si capitatur...
piatur quantitas invariabilis $a$, qua sit idonea magnitudinis: erit $a \cdot z = z \cdot x - x \cdot x$.

Ad hanc aequationem construendam, assumatur quantitas $\nu$ qua sit variabilis, & fingatur aequatio $z = p + q \cdot x + r \cdot v$, in qua notae $p, q, r$ deignent alias novas quantitates invariabiles; & erit $z = q \cdot x + r \cdot v$. Hisce porro valoribus ipfum $z$ & $\dot{z}$ substituitis in aequatione prima $a \cdot z = z \cdot x - x \cdot x$, habebitur $aq - p, x + ar \dot{v} = q - r, x \cdot x + r \cdot vx$.

Ut hae aequatio simplicior evadat, ponatur $q - r = 0, & aq - p = 0$; 

five $q = r, & p = a$: & sit $a \frac{\nu}{a} = \dot{x}$, ac praeterea $z = a + x + r \cdot v$.

Jacentibus punctis $D$ & $S$ ad eandem partem puncti $C$, intelligatur $CS$ aequalis ipfi $a$: & erit $z = SE + r \cdot v$, atque $CS \frac{\nu}{c} = \dot{x}$. Sit c valor quantitatis $\nu$, dum incidit punctum $E$ in punctum $C$: & quantum $x$, five $CE$, aequabitur mensura rationis quam habet $\nu$ ad $c$ pro Modulo $CS$, per Propositionem primam: quam aequalitatem sic designare soleo, $CE = CS \frac{\nu}{c}$. Tota ergo Problematis difficultas jam revocatur ad binas illas aequationes $CE = CS \frac{\nu}{c}$, atque $z = SE + r \cdot v$; 

hae vero deduci non poterunt in usum, priusquam determinatae fuerint quantitates $r$ & $CS$. Ad hoc efficiendum, duae restant conditions nondum adimpletae; oportet enim resistentiam esse nullam, atque adeo quantitatem $z$ five $SE + r \cdot v$ evanescere, ubi punctum $E$ in puncta $B$ & $D$ inciderit.

Sint ergo $b$ & $d$ valores ipsius $\nu$, dum incidit punctum $E$ in puncta $B$ & $D$ respective: & in his casibus habebuntur $SB + rb = 0$, $SD + rd = 0$. Unde $r = -\frac{SB}{b}$, $r = -\frac{SD}{d}$, atque $z = SE + r \cdot v = SE - \frac{\nu}{b} SB = SE - \frac{\nu}{d} SD$. Porro erit $SB \frac{\nu}{SD} = \frac{b}{d}$; atque adeo $CS \frac{\nu}{SD} = CS \frac{b}{d} = CS \frac{d}{c} = CB + CD = )BD$: unde dabitur punctum $S$.

Componetur itaque Problema hunc in modum. Producatur $BD$ versus $D$ ad $S$, eo usque, donec $BD$ fuerit mensura rationis inter $SB$ & $SD$ ad Modulum $CS$. Deinde ad arbitrium positae quantitate $c$, ita capiantur quantitates $b$ & $\nu$; ut eodem Modulo $CS$, fiat $CB$ mensura rationis quam habet $b$ ad $c$, fiat quoque $CE$ mensura rationis quam habet $\nu$ ad $c$: & erit vis resistentiae in loco $E$, ad pondus relativum corporis penduli, ut $SE - \frac{\nu}{b} SB$, ad $CA$.

Hujus
Hujus Problematis solution utilitatem habet in Physica non contentendam: quapropter constructionem ejusdem Linearem, ex eadem Analysis deductam, subjungere visum est. Invento uti supra puncto S; ad rectam SA erigantur perpendicula DH, CC, EK, BF, AN, rectae SN uteunque per S ductae occurrence in H, C, K, F, N. Per punctum c ducatur recta da parallela rectae DA, quae illud perpendiculis occurrat in d, c, v, b, a; & ad Asymptomon SA ducatur Logistica HGIF, quae transeat per puncta H & F, secetque perpendicula CC, EK in G & I, ac parallelam da in m: namque his positis, erit pondus relativum corporis penduli, ad vim illam qua pendulum acceleratur ad punctum E in Medio non resistente, ut aN ad eK; erit autem ad vim resistentiae in loco E, ut aN ad KI; atque adeo ad vim qua pendulum acceleratur ad punctum E in Medio resistente, ut aN ad eI. Porro, si per punctum m ducatur ad rectam SMA perpendicularis LmM, quae secet SN in L: erit M locus ubi resistentia sit maxima: atque adeo resistentia illa maxima, erit ad pondus relativum penduli, ut Lm ad Na, hoc est, ut CM ad CA.

Ceterum si ita ducatur recta SN, ut abscindat rectam DH quae fit dupla ipsius SD, centroque C & intervallo CB describatur Circulus BOP, qui occurrat perpendiculo KE in O: erit penduli in Medio resistente oscillantis velocitas in loco E, ad velocitatem penduli ejusdem ad eundem locum E delati per idem pondus relativum in Medio non resistente, ut media proportionalis inter CS & KI, ad EO.

Adhæc
Adhæc sì jungatur CO, & in perpendiculo KE sumatur ER, quæ fit ad CB ut CB ad medium proportionalem inter Ce & Ki; continuoque ductu rectæ ER in basim BE generetur area BQRE; erit tempus quo Cycloidis arcus BE describitur in Medio refletente, ad tempus quo idem arcus describeretur in Medio non refletente, ut area illa BQRE, ad Circuli sectorem BOC. Pergo nunc ad alia.

Denisitatem Aëris invenimus ad quamvis altitudinem, ubi vis Gravitatis vel erat uniformis, vel decrevcebatur in recessu à centro telluris in duplicata ratione distantiae: libet eadem exquirere desnuo, ubi gravitatio vel augetur vel diminuatur in ratione datae cu jusvis dignitatis distantiae. Sit S centrum telluris, A punctum in eju supersicie vel alibi utcunque situm, SAFz recta à centro ad summitatem Atmosphærae producea: & quærenda sit ratio densitatis in loco A, ad densitatem in loco quovis F, ex Hypothesi quod vis gravitatis in F sit ut distantia SF dignitas quæcunque SF", cujus index est n. Pro SF scribatur x, ac designent d & v densitates Aëris ad A & F; & cum densitas sit ubique ut pressura totius Aëris incumbentis, erit densitatis momentum ut momentum pressurae, hoc est; v ut vxx", atque adeo v ut xxn. Sit AC altitudo Atmosphærae, cujus uniformis densitas eadem effet ac densitas loci A; vel sit AC ad altitudinem Hydrargyri barometrici in loco A, ut densitas Hydrargyri ad densitatem Aëris in eodem loco A: & si punctum F accedere intelligatur ad punctum A; erit altitude Hydrargyri barometrici in loco A, ad altitudinem Hydrargyri barometrici in loco F, ut AC ad FC. Aëris ergo in loco A densitas d, est ad Aëris in loco F densitatem v, ut AC ad FC: unde consequitur ut sit d—v fives v, ad d fives v, ut AF fives x, ad AC.

Erit itaque, in hoc casu, \( \frac{AC}{v} = \frac{x}{x^n} \) Quoniam ergo, ubicunque fumeretur punctum F, erat \( \frac{v}{v} \) ut \( x^n \); erit porro \( \frac{AC}{v} = \frac{x}{x^n} \) ubicunque sumatur punctum F.
Jam si gravitatio sit reciprocamente ut distantia a centro, sine ut \( \frac{1}{x} \) vel \( x^{-1} \); erit \( n = -1 \), atque inde

\[ \frac{AC}{v} = SA \frac{x}{x} \],

unde si Fluentes statuantur æquales, mensura rationis inter densitates \( d \) & \( v \) ad Modulum \( AC \), æquabitur mensuraæ rationis inter distantias \( SF \) & \( SA \) ad Modulum \( SA \).

Si gravitationis sit alia quævis Lex: quoniam est \( \frac{AC}{v} = S_A n \); si Fluentes statuantur æquales, erit \( \frac{1}{n+1} \) in \( \frac{SF_n+1}{SA^n} \) – \( SA \) mensura rationis inter densitates \( d \) & \( v \) ad Modulum \( AC \). Itaque si sumanter in progressione Geometrica termini crescentes \( SA \), \( SF \), \( SF_1 \), \( SF_2 \), &c: decrescentes \( SF \), \( SA \), \( SF_2 \), \( SF_3 \), &c: mensura rationis inter densitates Aeris in \( A \) & \( F \) ad Modulum \( AC \), erit \( \frac{1}{2} AF_3 \), si gravitatio sit reciprocamente in triplicata ratione distantiae; erit \( AF_2 \), si gravitatio sit reciprocamente in duplicata ratione distantiae; erit \( AF \), si gravitatio uniformis statuatur; erit \( \frac{1}{3} AF_1 \), si gravitatio sit ut distantia; erit \( \frac{1}{3} AF_2 \), si gravitatio sit in duplicata ratione distantiae. Et sic proceditur in infinitum.

Denique ut plenius constet, Syntheticas etiam demonstrationes ex elementis praemissis levi negotio concinnari possit; sufficit unicum insuper addidisse Exemplum, tædæ utique plura jam proferre. Repetatur itaque divisio illa Nautica Meridiana quam supra attigimus, & videamus etiam abique ope Curve cujusiam Geometricarum, annon simplicior aliquanto sit futura demonstratio ad modum sequentem. Sit \( PXCO \) Telluris axis, \( CO \) semidiameter Equatoris, \( PABQ \) Meridianus; & invenienda sit in planisphere Nautico magnitudo cujusvis arcus \( AB \). Ad arcus illius terminos \( A \) & \( B \) ducantur ab alterutro Polorum \( P \) vel \( Q \) rectæ \( QA \), \( QB \), semidiametro \( CO \) occurrentes in \( D \) & \( E \): Dico magnitudinem Nauticam arcus \( AB \) æqualem esse mensuraæ rationis inter \( EC \) & \( DC \) ad Modulum \( OC \). Nam divisus intelligatur arcus \( AB \) in particulas
culas quam minimas $RS$, & jungantur $QR$, $QS$ quae facent $CO$ in $T$ & $V$; & demiſfo in axem perpendicular $SX$ quod rectæ $QR$ occurrat in $Z$, erit lineola $SZ$ æqualis partículæ $RS$. Itaque magnitudo Nautica nascentis arcus $RS$, erit ad Sphææ semidiametrum $OC$, ut arcus ille $RS$ five lineola $SZ$ ad $SX$, hoc est,

\[
\text{ut } VT \text{ ad } VC. \text{ Unde (per Corol. 2. Prop. 1.) magnitudo illa Nautica æquatur mensuraæ rationis inter $VC$ & $TC$ ad Modulum $OC$: & similes utroſque ſummas colligendo, magnitudo Nautica totius arcus $AB$ æquabitur mensuraæ totius rationis inter $EC$ & $DC$ ad eundem Modulum $OC$.}\]
III. An Extract from the Acta Eruditorum for the Month of March, 1713. Pag. 111.


A Dissertation concerning the dreadful Contagious Distemper, seizing the Black Cattle in the Venetian Territories, and especially about Padua.

It is now (at the Publishing the Discourse) a Year and half, since a dreadful, unexpected and violent Contagion has seiz’d the Black Cattle, which, like an increasing Fire, could neither be extinguish’d nor stop’d by any Human means.

This first began to be observ’d a little in Agro Vincentino, and soon discover’d it self more openly in the Country, spreading it self every way even to the very Suburbs of Padua, with a cruel Destruction of the Cows and Oxen. It has also been taken notice of in Germany, in many Places; nor has it been yet wholly conquer’d, Publick News informing us, that it still remains in the Territories of Milan.

Of this so threatening a Distemper, the famous Dr. Ramazzini, according to his yearly Custom, on November 9. 1711. made a particular Dissertation; in which he inquir’d into the Causes of the Distemper, and shew’d what Remedies might be us’d, to put a stop to its violent Course.

It is sufficiently evident, that this Distemper, in the Cow-kind, was a true Fever, from the coldness, rigor and standing up of the Hair of the Cattle at first, which was soon succeed’d by a violent sharp burning, with a quick Pulse. That this Fever was malignant, mortal and pestilential, its concomitant Symptoms plainly shew’d; such as great uneasiness with difficulty,
facility of breathing, great pantings with a sort of shorting, and at the beginning a kind of Stupor and Drowsiness, a continual Flux of a strong smelling Matter from the Nose and Mouth, a very fetid Dung, sometimes with Blood, all Ruminating ceasing. Pustules breaking out over the whole Body on the 5th or 6th day, like the Small-Pox; they all generally dy'd about the 5th or 7th day, very few of them escaping.

The Author deduces this Distemper from a contagious Original. He tells us, it is certain, that out of a great Drove, such as the Merchants bring yearly into Italy out of Dalmatia and the bordering Countries, one Beast happen'd to straggle from the rest, and be left behind; which a Cowherd finding, brought to a Farm belonging to the Illustrious and Reverend Count Borromeo, Canon of Padua: This Beast infected all the Cows and Oxen of the Place where he was taken in, with the same Distemper he labour'd under; the Beast itself dying in a few days, as did all the rest, except one only, who had a Rowel put into his Neck.

'Tis no strange thing therefore, if from the Effluvia, like an Atmosphere, proceeding from the sick Cattle, from those dead, and from the Cowhouses and Pastures where they were fed, and by that means infected, and chiefly from the Cloaths of the Cowherds themselves, this Infection falling upon a proper Subject, should diffuse itself so largely. When therefore this subtle venomous Exhalation happens to meet with any of the Cow kind, joining itself with the serous Juices and Animal Spirits, whilst it is carry'd all over the Body, disorders the natural consistence of the Blood, and corrupts the Ferments of the Viscera; whence it naturally follows, that the natural Functions of the Viscera are vitiated, and the requisite Secretions stop'd. For

Dr. Ramazzini not only supposes, but asserts, that this Poison is of that kind which rather fixes and coagulates, than dissolves the Blood: For besides the forementioned Symptoms accompanying the Disease, the Eye itself is a Witness; since the dead Carcasses being open'd while they are yet hot, little or no Blood
blood nevertheless runs out; those animals having naturally a thick blood, especially when the fever has continued so many days. Whether therefore this plague came first from the foreign beast, or any other way, it is the same thing, when at last it fell upon some animal in which there was the morbid seminary or ground prepared for it.

In the dead bodies of all the cattle it was particularly observed, that in the omasus, or paunch, there was found a hard compact body, firmly adhering to the coats of the ventricle, of a large bulk, and an intolerable smell: in other parts, as in the brain, lungs, &c. were several hydatides, and large bladders fill'd only with wind, which being open'd gave a deadly stink; there were also ulcers at the root of the tongue, and bladders fill'd with a serum on the sides of it. This hard and compact body, like chalk, in the omasus, the author takes to be the first product of the contagious miasma. He adds a prognostick, believing that from so many attempts and experiments, and the method observ'd in the cure of this venom, at last a true and specific remedy will be found out to extirpate the poisonous malignity wholly: He also expects some mitigation of it, from the approaching winter and north winds. He does not think this contagion can affect human bodies, since even other species of ruminating animals, symbolizing with the cow-kind, are yet untouch'd by it; nor was the infection catch'd from the air, provided due care was taken in the burying the dead bodies.

As for the cure of it: from the chirurgical part, he recommends bleeding, burning on both sides the neck with a broad red hot iron, making holes in the ears with a round iron, and putting the root of hellebore in the hole, a rowel or seton under the chin, in the dew-laps; he also orders the tongue and palate to be often wash'd and rub'd with vinegar and salt.

As for the pharmaceutical part, he recommends alexipharmicks, and specific cordials; and from the vegetable kingdom, three ounces of jessuits bark, infus'd in ten or twelve pints of cordial water or small wine, to be given in 4 or 5 doses; which is
is to be done in the beginning of the Fever, when the Beak begins to be sick. From the Animal, two Drams of Sperma Catuli dissolved in warm Wine. From the Mineral, Aconitum Dha-
thoreticum. Against Worms breeding, an Infusion of Quicksil-
ver, or Petroleum and Milk is to be given. And lastly, as to the Food, Drinks made with Barly or Wheat Flower or Bread, like a Pissare, fresh sweet Hay made in May and macerated in fair Water. In the mean time the Cattle must be kept in a warm Place, and cloath'd, to keep them as much as possible from the cold Air, daily making Fumigations in the Cowhouses with Juniper Berries, Galbanum, and the like. As to Prevention, he enjoyns Care in cleaning the Stalls, and scraping the Crust off from the Walls; Care also is to be taken of their Food, that it be good, the Hay and Straw not spoild by Rain, in the making, and judges their Food ought to be but sparing; Friction, rubbing and currying, not only with the Hand, but with a Currycomb and Bruff, with Setons under their Chin, made with a hot Iron run through the Part, and kept open with a Rope put through it.
IV. A Recipe: On the Ingredients of a Medicine for the spreading mortal Distemper amongst Cows: Lately sent over from Holland, where a like Distemper raged amongst the Black Cattle.


Bleed the Cow, and give her every Morning, for 3 or 4 Mornings, an Ounce of this Powder with a Horn in warm Beer.

If the Cow's Illness continues, after the omission of 2 or 3 Days, repeat the Medicine for 3 or 4 Days again.
PHILOSOPHICAL
TRANSACTIONS.

For the Months of April, May and June, 1714.

The CONTENTS.

I. A New Method for making Logarithms, and vice versa, for finding the Number corresponding to a Logarithm given, by help of a Small Table. Communicated by Mr. John Long, S. Theol. Bacc. and Fellow of Corp. Christi Coll. Oxon.

II. An Extract of a Letter from Mr. Anthony van Leeuwenhoek, F. R. S. containing his Observations on the Small Fibres of the Muscles in several Animals.

III. An Extract from the Journal Literaire, &c. giving an Account of several Observations of the Texture of the Muscles, by Mr. Muys, Professor of Anatomy at Franeker.


A new Method for making Logarithms, and vice versa, for finding the Number corresponding to a Logarithm given, by help of the following Table. Communicated by Mr. John Long, S. Theol. Bacc. and Fellow of Corpus Christi Coll. Oxon.

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This Table is what I sometimes make use of for finding the Logarithm of any Number propos'd, and vice versa. For instance: Suppose I had occasion to find the Logarithm of 2000. I look in the first Class of my Table (the whole Table consists of 8 Classes) for the next less to 2, which is 1.995262315, and against it is 3, which consequently is the first Figure of the Logarithm sought. Again; dividing the Number propos'd 2, by 1.995262315 the Number found in the Table, the Quotient is 1.002374467; which being look'd for in the second Class of the Table, and finding neither its equal, nor a less, I add 0 to the part of the Logarithm before found, and look for the said Quotient 1.002374467 in the third Class, where the next less is 1.002305238, and against it is 1, to be added to the part of the Logarithm already found; and dividing the Quotient 1.002374467, by 1.002305238, last found in the Table, the Quotient is 1.0000690703 which being sought in the fourth Class gives 0, but being sought in the fifth Class gives 2, to be added to the part of the Logarithm already found; and dividing the last Quotient by the Number last found in the Table, viz. 1.000046053, the Quotient is 1.000023015, which being sought in the sixth Class, gives 9 to the part of the Logarithm already found: And dividing the last Quotient by the new Divisor, viz. 1.0000002072, the Quotient is 1.000000249, which being greater than 1.000000115, shews that the Logarithm already found, viz. 3.3010299 is less than the Truth by more
more than half an Unit; wherefore adding 1, you have Briggs's Logarithm of 2000, viz. 3.3010300.

If any Logarithm be given, suppose 3.3010300, throw away the Characteristic, then over against these Figures 3...0...1...0...3...0...0, you have in their respective Classes: 1.995262315...0...1.002305238...0...1.000069080...0...0 which multiplied continually into one another, the Product is 2.000000019966, which by reason the Characteristic is 3, becomes 2000.000019966, &c. that is, 2000, the Natural Number desired. I shall not mention the Method by which this Table is fram'd, because you will easily see that from the use of it.

It is obvious to the intelligent Reader, that these Classes of Numbers are no other than so many Scales of mean Proportionals: In the first Class, between 1 and 10; so that the last Number thereof, viz. 1.258925412 is the tenth Root of 10, and the rest in order ascending are the Powers thereof. So in the second Class, the last Number 1.023292992 is the hundredth Root of 10, and the rest in the same manner are Powers thereof. So 1.002305238 in the third Class, is the tenth Root of the last of the second, and the rest its Powers, &c. Or, which is all one, each Number in the preceding Class, is the tenth Power of the corresponding Number in the next following Class: Whence 'tis plain, that to construct these Tables requires no more than one Extraction of the fifth or surd Root for each Class, the rest of the Work being done by the common Rules of Arithmetick; and for extracting the fifth Root, you will find more than one very compendious Rule in Num. 210 of these Transactions, if any one shall desire to examin the computus of these Tables.

The Process is exactly the Reverse of Mr. Briggs's Doctrine, in Cap. XII. of his Arithmetica Logarithmica of Place's Edition; and had Briggs been appriz'd hereof, it would have greatly eas'd the Labour of deducing the Logarithms of the first prime Numbers, which appear to have cost him so much Pains.
III. An Extract of a Letter from Mr. Anthony van Leeuwenhoek, F.R.S. Dated October the 12th. 1713. Concerning the Fibres of the Muscles, &c.

Gentlemen,

In compliance with your desires, I here send you a Copy of the Observations I communicated to the great Pensionary Monsieur Heinsius, concerning the Membranes with which the Fibrilles of the Muscles appear to be encompassed, both in the Flesh of a Whale, Cod-fish, Salmon, and Smelt; and also in that from an Oxe to a Mouse; in all which the appearance was the same.

The Flesh of the Whale, was a small piece cut off near the Tail of the Fish, which I desired a Sea-Captain to bring me, that I might observe how the Flesh in that part was joined to the Tendons in so large an Animal. Viewing this thro' the Microscope, I judged the Fibres thereof to be four times as large as those I had formerly observed in another piece of Whale's Flesh, taken from another part of the Fish; which made me consider, whether the Fibres of that part might not be, by Nature, furnished with larger Fibres for its greater strength.

Cutting the said Flesh-parts length-wise, and a cross the Fibres, I discovered more plainly than before, that each Particle or Flesh-fibre, was enwrapt in a fine thin Membrane.

To have a better Idea of these Flesh-fibres of a Whale, I cut a thin slice of it a-cross, which I laid on a wetted piece of Glass, that the Flesh which was very dry and shrunk, might, by the moisture be swelled, and thereby distended to the natural size it had when on the Body of the Fish itself. In this state, placed before the Microscope, it appeared as I caused it to be drawn in the Figure. A. B. C. D. in which the Parts were so close together, that their encompassing Membranes, represented by the black Lines, were but just discernable, some whereof appeared larger than others: these, if attentively viewed, seem'd I
plainly to be divided into multitudes of others, cut also transverse, the bigness of which was no larger than a common Sand to the naked Eye. These were so close crowded together, that their Figure was very irregular, as well as their sizes different; for tho' each seem'd encompassed with six others, yet some of them were twice as large as the other.

Having formerly mentioned the slenderness of these Fibrille in the Flesh of a Whale, and judging these, as I said before, to be four times as large, I took a thin slice of the formerly mentioned Whale-flesh (which I had still kept by me) and after having made it thoroughly wet, I viewed it with the fame Microscope as I had done this of the Tail. This appear'd as is represented (Fig. 2.) E. F. G. H. Letting the moisture dry away from these slices, so stuck on to the Glass, the Particles became much smaller; and the Membranes with which each was encompassed, became very visible; that is, those which were not shrunk away; which was a very entertaining Object to the curious; and as often as I made new Cuts, a new Object presented itself.

A small Particle of this Flesh I caused to be drawn, as in (Fig. 3.) I K L M. These Particles seem'd to touch and be joined to others; but now being dried, they shrunk in from the Membranes round about them; for the Membranes could not shrink, because they were all joined to one another.

Along these Flesh-fibres there runs some such thick Membranes, that they equal the thickness of a Hair or more, which are scarce distant the breadth of a Sand from each other; from these larger Membranes other parts are spread, dividing each Fibre into numerous Fibrille; so that it may be said, each flesh-Fibre, no bigger than a Hair, is a little Muscle encompassed in its peculiar Coat or Membrane, as I have said before. Whereas the Designer had not the same apprehension of the size of these Fibres, as I and some other Persons had, I made him draw a little piece as large as it appeared to my apprehension, as in (Fig. 4.) N. O. P. whence appears the difference of one Man's sight from another.
I have also often seen some few of these Fibres, tho' joined to others, yet but one Fourth of their bigness to which they were joined.

When I again moistend those represented in the Third and Fourth Figures, (dryd up and shrunk) they would be again so swelled and distended, as to fill up the spaces between the Membranes, and re-assume the Shape they had before they were dried. Among several little pieces of Flesh placed before another Microscope, and moistend as before, there was one, whose Particles were not separated upon drying, which I supposed to be, from the splitting and tearing of a large Membrane that run thro' the middle of it, as may be seen in (Fig. 5.) Q, R, S, T, V, W where between S, T, and V, the dried Particles remain unseparated; these being cut a little thicker appeared also of a darker hue, and if they had been sliced yet thicker would have appeared of a dark red. By S, W, is represented the thick Membrane dividing this piece, which was about the bigness of a Hair; this at T, sent out a Branch, and near W, is split into two, I apprehend that a great number of Blood-veins are spread over this Membrane, which by their smallnes are not visible; for it is by these the Nourishment is convey'd. Between R, S, and Q, W, the exceeding fine Membranes torn from the great are visible.

Is it not amazing that in such vast Animals as a Whale, such exceeding small Fibrillæ should be found? nay, such they are in small Animals; and that the whole fifth Figure is not so large as a coarse grain of Sand.

This Whale was so large, that the upper part of its Body yielded 60 Quarteels of Blubber or Fat, which allowing 30 Rotterdam Stopes (making each about 3 English Quarts) to one Quarteel, it will nearly amount to 24000 Pound weight; besides, there is a very great deal of Fat about the Entrails.

Then I caused a very little piece, consisting only of five Fibrillæ, to be drawn lengthwise, as they were seen thro' the Microscope, as (in Fig 6.) A, B, C, C, E, F. in which Figure at A, and a little at that place, it is divided into two Fibrillæ. Between C.
and $F.$ are to be seen the little Membranes which incompass the Fibrille, which are here torn asunder.

I have frequently, with pleasure, observed these Flesh-fibres lengthways, to be as it were corrugated or wrinkled, which I imagined to be the Representation of their Rest or unbent Posture; and yet more, when the Part to which they belong is bowed together, or brought nearer; but when the Muscle is extended, and its Antagonist acts, there is not the least wrinkle observable in these Fibrille.

However, all the little Inequalities in these Fibrille must not be taken for those Corrugations, since many of them are only the articles torn off from the Membranes which encompass the Fibrille.

Figure (7) G. H. I. K. represents four little Fibres of a piece of Whales-Flesh I had procured two Years since; this I caused to be drawn to shew the difference. By the two Figures 6 and 7, is visible the Diameters of the Fibres are as big again in one, as in the other, therefore the Fibres must be four times as big in Fig. 6, as in Fig. 7. Now each Flesh-fibre being composed of a great many smaller Fibrille, we may imagine each of these in-lying Fibres do likewise consist of others of the like Nature.

I have a fresh viewed several small Fibres of Ox-Flesh, and observed, that each of the Fibrille in them was encompassed with a thin Membrane. But I cannot shew these Membranes so clearly to other Persons in Cows-Flesh, as in Whales-Flesh, because the Parts of the former are of a much more compact and close Texture than that of the Whale, from whence they do not shrink so much in drying.

I am of Opinion, that what I have said of the Membranes (encompassing the Fibres and Fibrille) of the Flesh in a Whale, will likewise hold true in other kinds of Flesh; yea even down to that of a Rat or a Mouse: Concerning which I shall prosecute my Observations. I conclude, &c.
To these Observations of Mr. Leuenhoek I shall join,

III. An Extract from the Journal Literaire, Publish'd at the Hague, for the Months of January and February, 1714. Pag. 238. Being,

An Account of several Observations concerning the Frame and Texture of the Muscles; By Mr. Muys of Franeker:

The Celebrated Mr. Muys, who always acts for the Honour of the Academy of Franeker, and Advantage of Students in Physick and Anatomy, has made several Discoveries, as to the Mechanism and Texture of the Muscles of Animals; of which these are some.

He has observed, that the fleshly Fibres of the Muscles are composed of other smaller Fibres, which he calls Fibrille: that these Fibrille are of the size of a slender Hair, and that 500 or 600 of them, may be counted in one Fleshly-fibre, whose Diameter is no more than a 24th part of an Inch.

That each of these Fibrille also is made up of more than 300 little transparent Tubuli, but so slender, that if a Blood Globule (which, according to Mr. Leuenhoek, is but the 1,000,000th part of a grain of Sand) were divided into 24 parts, one of these could hardly pass thro' these small Pipes.

He has shewn, that tho' the fleshly Fibres of the Muscles, are joined to the Tendons and tendinous Membrane of a Muscle; yet these tendinous Fibres are not a continuation of the fleshly ones, as most Anatomists suppose: which he proves thus; If by means of a wooden Knife, or only by pulling it, you separate the fleshly Fibres from the Tendon, the end of the Tendon to which they were joined, will remain smooth and even, and not rugged.

Having
Having made several Injections of warm Water into the crural Artery of a Lamb of a Year old, all the fleshly Fibres lost all their redness, and became entirely white. The Fibres being whitened by this Injection, he injected a coloured Liquor by the same Artery; and then not only the small Arteries appeared filled with this tinged Liquor, but he found also that the Liquor past thro’ each Fibre, either in a Serpentine manner, or undulating, or framing several Angles, or joined by a great number of Anastomoses.

He observed also, that many small Branches of the Arteries which before could not be seen, appeared visibly, spread all round the little Fibrillae, and tinged with the same Colour.

Having remarked, that the Parts of the fleshly Fibres which were near the Extremities of the Arteries, appeared tinged with the Liquor, he examined them with a Microscope, and found the little Fibrillae filled and tinged with the same Liquor; and yet there was not least appearance of the Liquor in the Interstices between the Fibrillae.

Having made Injections by the crural Artery, of another coloured Liquor, in the Muscles, whiten’d, as before, with Water, he saw not only the Fibres in some of the Muscles, and the most part of them in the others filled with this matter; but having examined them with a good Microscope, he found the Fibrillae, and even the least Tubuli which compose them, filled and tinged with the same Matter; and nevertheless the small Ramifications of the Nerves appeared perfectly white.

It results from all these Observations,

1st. That the little Tubes, which make a Fibrilla, are really hollow, and that the Extremities of the Capillary Arteries open into them, and empty there a part of their Liquor, which is re-conveyed by the Veins to the Heart.

2d. That the Blood Globules must be divided into an almost infinite degree of smallness, before they can enter and pass these Tubuli. That the Blood-Globules may be so divided, and when so divided pass thro’ the small Tubuli, is evident from
from the redness of the Fibres and *Fibrille* of Animals, which have a red Flesh; which will be no surprize to them who have read Mr. Leeuwenhoeks Letter 42, where he says, that these Globules do divide themselves after this manner, to pass thro' the last Extremities of the Capillary Arteries of the Brain; nor to those who know, that the Globules are extreme soft and easily separable, as *Monseur Muys* has evinced by Arguments grounded on very curious Observations.

*Monseur Muys* has added to his Observations very exact Figures, which contribute very much to the forming a clear and distinct Idea of the Structure of these Fibres of the Muscles, and of the manner of the Arteries passing through them; but I dare not so far depend on my Skill in designing to venture to Copy them.

This knowing Person has also made several Discoveries of the Course and Ramifications of the Nerves in the Muscles: But I wait for an Opportunity of informing my self better of several Particularities, before I communicate them to you.

In my last I wrote to you concerning the Salts which Mr. *Muys* had discover'd in Human Blood; but I had forgot to inform you, that he had found out a way to separate them from the Blood, without any Chymical Analysis, and without making them undergo any change, and to form them into Crystals, visible without a Microscope; as he has shewn to his Students in Physick.

The first Letter directed to Dr. Woodward, is dated at Boston in New England, Nov. 17. 1712. In this the Writer gives an Account of a large Work in Manuscript, in two Volumes in Folio, but does not name the Author. This, according to the account of it, is a large Commentary upon some Passages in the Bible, interspers'd with large Philosophical Remarks, taken out of Natural Historians, and the Observations of himself and others, more particularly as to Matters observ'd in America, whence he entitles the Work, Bibliam Americana. This Work Dr. Mather recommends to the Patronage of some generous Mecenas, to promote the Publication of. As a Specimen of it, he transcribes a Passage out of it, being a Note on that Passage in Gen Chap. 6. v. 4. relating to Giants; and confirms the Opinion of there having been, in the Antediluvian World, Men of very large and prodigious Statures, by the Bones and Teeth of some large Animals, found lately in Albany in New England, which, for some Reasons, he judges to be Human; particularly a Tooth brought from the Place where it was found to New York, 1705. being a very large Grinder, weighing 4 pounds and three quarters, with a Bone, suppos'd to be a Thigh-bone, 17 Foot long. He also mentions another Tooth, broad and flat like a fore-Tooth, four Fingers broad: the Bones crumble to pieces in the Air after they are dug up; they were found near a place call'd Cluverack, about 30 Miles on this side Albany. He then gives the Description of one, which he resembles to the Eye-Tooth of a Man; he says it has four Prongs, or Roots, flat, and something worn on the top;
top it was six inches high, lacking one eighth, as it stood upright on its Root, and almost thirteen inches in circumference; it weigh'd two pounds four ounces Troy weight; there was another near a pound heavier, found under the Bank of Hudson's River, about fifty Leagues from the Sea, a great way below the Surface of the Earth, where the Ground is of a different Colour and Substance from the other Ground, for seventy five Foot long, which they suppose to be from the rotting of the Body, to which these Bones and Teeth did, as he supposes, once belong. It were to be wish'd the Writer had given an exact Figure of these Teeth and Bones.

The second Letter to the same Person, is dated Nov. 18. 1712, from Boston, as all the following are. In this he treats of the Plants of America; and, in the first place, offers a Conjecture of his concerning the Shittim Wood, mentioned in the Sacred Writings to be made use of for the Ark, &c. It is said to be not as most other Woods, subject to rot: He judges that it was the black Acacia; that the Gopher Wood was the Juniperus arbor tetragonophyllos, frequent in the East-Indies, &c. He observes that the Indians often perform very great Cures with their Plants; of which there is a great variety, differing from the European, which he promises a Catalogue and Description of; and, for the present, instances in some. As, a Plant efficacious in curing Inflammations, whence they call it Antierisypelas; it grows plentifully in the Woods: A Chymical Oil extracted from it, taken inwardly, does Wonders in absorbing scorbutick Salts. Another Plant, which goes by the Name of Partridge-berries, excellent in curing the Dropsey; a Decoction of the Leaves being drank several days as a Tea, discharging a vast quantity of Urine, as long as the Disease lasts; after which it may be drank without provoking Urine observably: Gouty Persons drink it with benefit.
The Root call'd the Bleeding Root, curing the Jaun-
dies in five or six days.

Another for Gangrenes, of which he does not give the Name.

Another Specifick for the Bite of the Rattle-Snake, and another for Quinsies, or sore Throats. A Plant, call'd by the Indians, Taututtipang; infallible for the Lues Venerea, the Root being used in a Decoction, and drank half a Pint; a Cataplasm of the same Root, bruised, apply'd to the Ulcers, cures them also.

A Thistle call'd the Boar-Thistle; very short and prickly, with a large and long Root. To this they add a Root, call'd the Cancer Root, and a sort of Devils-Bit: A Decoction of which three Roots is a Cure for the King's-Evil, tho' very far gone; a small quantity being drank every day, and the bruised Roots apply'd to the Scrophulous Tumors.

But of these American Plants he promises a larger Crop.

The third Letter relates chiefly to the Birds of that Country; where, he says, they have many of the same Species with ours in England. He mentions very large wild Turkies, some weighing 50 or 60 pound, but the Flesh is very tough and hard. He takes notice of a very large Eagle with a great Head, soaring very high, as all of that Genus do. As to the Itinerants; he takes notice of vast Flights of Pigeons, coming and departing at certain Seasons: And as to this, he has a particular Fancy of their repairing to some undiscovered Satellite, accompanying the Earth at a near distance.

The next Letter relates to Antipathies, and the Force of the Imagination. As to the first, he says, a Gentlewoman of his Neighbourhood swoons upon the seeing any one cut their Nails with a Knife; which if done with a Pair of Scissors has no Effect upon her. The Wife of a Person, vo-
miting upon seeing her Husband take a Vomit; the Patient that took it being not mov'd, but forc'd to take a fresh Emetic.

Some Histories are here related of the Macula Materne. One particularly, of a Woman longing for Peas, but refusing to gratifie her desire, for fear of a sort of Bug, with which at that time most of their Peas were infest; this Woman's Child, when born, had an Excrecence on the Forehead, resembling one of those Peas, with a black Speck, as the buggy Peas had, which after some time, dry'd away, and shell'd out they fancy'd, as the Bugs are obser'd to leave the Husk of the Pea.

This Letter concludes with an account of a Stone, generated under the Tongue, near the Root.

The fifth Letter gives an account of some monstrous Births, but nothing very observble.

The sixth Letter relates the Stories of some Persons that had Informations of Medicines for the Distempers they lay under, in their Dreams; these Accounts relate little to Natural Philosophy.

The next, and last to Dr. Woodward, relates the Cures of several Wounds in Persons, which were judg'd mortal. In this little of Philosophical Information.

The next Letter, being the first to Mr. Waller, is dated at Boston, Nov. 24. 1712. In this the Writer observes, in the first place, That the Indians have no Division of Time, except by Sleeps, Moons and Winters. Altho' the Indians have not divided or distinguishted the Stars into Constellations, yet it is observble that they call the Stars of Ursa Major, Paukunawaw, that is, the Bear; and this long before they had any Communication with Europeans. He says,
there is a Tradition among them, that in November, 1668, a Star appear'd below the Body of the Moon, within the Horns of it. In the next place he mentions the Evening Glade; first taken notice of by Dr. Childrey, to be constantly observ'd there in February, and a little before and after that Month; adding, that the Cause of that Appearance must be sought for above the Atmosphere. Then he gives a new Method of his own for finding the Julian Period, adding a Table for that purpose; which concludes this Letter.

The next relates the Appearances of several uncommon Rainbows and Mock Suns. On the 2d of January, in a clear Sky, but very cold; the Sun was from Ten o’Clock, for near Three Hours after, attended with four Parhelia, in the midst whereof were two Rainbows.

About six Weeks after this; in a Day much colder than used to be at that time of the Year, the Air a little hazy, a little after One o’Clock, for about half an hour, four Mock-Suns were seen.

He observes, that these Appearances with them are of great varieties; each usually differing in some respect from the other.

The next Letter dated, Nov. 26th. to the same Person, has the Relation of a strange Discovery of the Murder of a Person in England, to his Brother Joseph Beacon, at that time at Boston, in a Dream; the Person wounded appearing with the Wound on his Head; with the Attestations of several Persons, as to the Truth of it. The Information by the Dream was on the 2d of May, 1687, about Five o’Clock in the Morning; on the very same Day his Brother dy’d at London, of the Wounds he had receiv’d in April before: Of which Misfortune his Brother Joseph Beacon neither had, nor could have any notice, till the next Communication by Shipping, towards the latter end of June following; when he had a Letter of his Brother’s Death, and the Cause of it,
it, agreeable to his Dream, but this not directly relating to Natural Philosophy, I omit the Particulars, tho' the Relation seems to be well attested.

The following Letter sent likewise to Mr. Waller, treats chiefly of the Rattle-Snake, hinting at the occasion of its Name, from the Rattles in its Tail, in which he says are sometimes twenty of those loose Rings, tho' he does not come in with the Opinion, that one is added every Year. Next follow these Observations. That the more Northerly they travel, these Snakes are less numerous, as well as less venomous; nor as it is said, are any seen to the North of Merimack River, which is about 40 miles North of Boston. Here he relates a Story, as he says, constantly affirmed by the Indians, that these Snakes frequently lie coiled at the Bottom of a great Tree, with their Eyes fixed on some Squirrel above in the Tree; which tho' seeming by his cries and leaping about, to be in a Fright, yet at last runs down the Tree, and into the Jaws of the Devourer. Then he relates, that the Winter abroad of these Snakes is in the Clefts of inaccessible Rocks, from whence in the Spring, they come forth a Sunning themselves, at first very feeble, which is their chief time of destroying them. At this time the Cystis or Gall bladder in these Snakes is full of an acid azure coloured Juice, which they squeeze out into a Glass, but it is so Spirituous, that if the Glass be not immediately stop'd, it will soon evaporate; this Liquor therefore they mix with a convenient quantity of powder'd Chalk or Indian Meal, and use it as a proper Medicine against the venomous Bite of this Snake; some have named it Trochisci Connecticutians, from the Connecticut Colony. 'Tis observable when the Summer Heats come on, the Snakes have no longer this azure Liquor in their Gall-Bladders, in which there is only found a black thick Sediment, of no known use, at which time they think the forementioned Spirituous Juice is carried to, and lodged in their Gums, and so conveyed or thrown by the hollow of the Teeth into the Wound, when they Bite, having received another Digestion,
and higher Exaltation, by passing thro' several Strainers and Glands before it arrives to the Gums; as an instance of the virulence of this Liquor, he tells us, that a Traveller killing one of these Snakes, suffered the enraged dying Viper, to bite the end of his Switch, with the lashes of which he had disabled him; and a Fly by chance disturbing one of his Temples as he rode on afterwards, he rub'd his Temple with the other end of the Switch, which as he relates it, immediately caused his whole Head to swell to a great excess, the Poison as he supposes permeating the whole length of the Switch. He adds another relation as to the penetrating quality of this Poison, a Person provoking a Rattle-Snake to bite the Edge of a broad Axe he had in his Hand; the colour, of the Steeled part bitten, was immediately changed, and at the first stroke he made with it in using his Ax, the so discoloured part broke out, leaving a gap in his Ax. But to return to the Trochies made of the Gall, he says it is a Cordial sudorifick, and so good an Anodine, that he knows some who take 3 or 4 Grains of it to compose them to rest after Travel. 'Tis good in all Fevers, especially the Malignant. 'Tis an infallible Remedy for Obstructions incident to Women upon catching cold in Childbed. Being taken in a convenient Quantity, 12 Hours before the Fit, it certainly cures a Quartan Ague. The Dose is 1 4 Grains more or less according to the Circumstances of the Patient in any Vehicle.

The next Letter treats of the Effects of Thunder and Lighting very frequent with them, which from its frequent destroying Animals, without any visible Hurt on the external Parts, he compares to the Jewish Punishment of pouring melted Lead down the Throats of the Condemn'd which they call'd combustio animæ. Tho' he likewise observes some have had their Hair singed with Marks on their Skin like those made by small Shot; some have had their Bones made limber like a Gristle. The Captain of their Castle was found Dead in his Bed after a storm of Lighting without any apparent Hurt. Here he relates a Passage of which an Account has sometime since
Since been given in the Philos. Transact. but is here confirm'd. That July 24th, 1681. A Ship whereof one Mr. Lad was Master, about 100 Leagues from New England in Lat. 38. met with a violent Storm of Thunder, which did much Damage to the Ship; at which time, a bituminous Matter fell on the Ship burning with that Violence, as not to be extinguish'd with Water till it was all burnt out, smelling strongly like fired Gunpowder; and when they came to observe the Stars at Night, they found the Polarity or Direction of their Sea-Compasses to be changed; the North-Point being turned to the South, and so continued to do for the rest of the Voyage for a 1000 Leagues. He adds farther, that one of these Compasses continues to do so still, and was upon his Table before him at the time of his writing this present Letter. He makes a Query whether this may be accounted for by Mr. Boyle's Experiment of heating a Loadstone red-hot, and by altering the position in which it was cool'd, he could change its Polarity. Which some may say, might happen to this Needle, supposing it was made red-hot, and turned upon its Center in the Storm.

From Thunder he proceeds to Earth-Quakes, which tho' he says they have not done with them the Mischiefs frequent in Sicily, Italy, &c. yet they have had several very sensible and affrightening. In the Year 1663, they had 6 or 7 violent shakes in the space of 3 Days: a Town lying on the River Connecticut, has had scores of them in a Year, for many Years together. The Indians affirm, that several Rivers have not only been stop'd in their course and diverted, but some wholly swallowed up by Earth-Quakes. He farther adds, a passage out of Josselin who dwelt in the Neighbourhood, that in the Year 1670, at a place called Kenelunch, near the side of the River, a piece of clay Ground was thrown up over the tops of high Oaks, growing between it and the River, which it thereby stop't, and left a Hole in the Place from which it was thrown forty Yards square, &c.

Next as to Storms of Hail, he relates that they have had very extraordinary ones, insomuch that they have lain 3 or 4 Foot thick on the Ground,
some as big as Hens Eggs, others five times as big. He mentions, as an Accident sometimes happening to them in the Winter, that it has rain'd plentifully; and at Night frozen so extremely, that the weight of the Icicles has broken the Limbs of the Trees, and not-unfrequently split their Trunks. Tho' they have not those Hurricanes to which the Caribbe Islands are subject; yet they have had Whirlwinds, or Gulfs, drive along a particular narrow Tract, for divers Miles together, with a violence not to be opposed by anything on Earth; that if their Towns had stood in the way, they must undoubtedly have been destroy'd. Of these, he says, a thick dark, small Cloud has arose, with a Pillar of Light in it, of about 8 or 10 Foot Diameter, and past along the Ground in a Track not wider than a Street, horribly tearing up Trees by the Roots; blowing them up in the Air like Feathers, and throwing up Stones of a great weight to a considerable height in the Air, throwing down all in its passage; the Noise this Cloud made was so great all the while, that the Noise of the Mischiefs done by it, was thereby quite drown'd.

The remainder of this Letter relates to some very ancient Remains, at a Place call'd Amuskeag, a little above the hideous Falls of Merimack River. There is a huge Rock in the midst of the Stream, on the Top of which are a great number of Pits, made exactly round, like Barrels or Hogheads of different Capacities, some so large as to hold several Tuns; the Natives know nothing of the Time they were made; but the Neighbouring Indians have been wont to hide their Provisions in them, in their Wars with the Maqua's; affirming, God had cut them out for that use for them. They seem plainly to be artificial.

In the next place, he gives an account of a strange Inscription found on a Rock, in these Words. At Taunton, by the side of a Tiding River, part in part out of the River, there is a large Rock, on the perpendicular side of which, next to the Stream, are 7 or 8 Lines, about 7 or 8 Foot long, and about a Foot.
Foot wide, each of them ingraven with unaccountable Characters, not like any known Character. He has not yet been able to procure the whole, which he hopes to be Master of before long, and has herewith sent a Copy of two of them, promising the rest; they are as is represented, Fig. 8.

The last Letter of this Collection, dated Nov. 29. 1712, gives a Calculation of the possible Increase of the Descendants of Adam; and from this Introduction proceeds to the Account of some long-liv'd Persons there, as likewise of their Fruitfulness. He says, 'tis no rare thing with them to have an aged Gentlewoman see many more than 100 of her Offspring. He mentions one Woman that had 23 Children, of which 19 liv'd to Man's Estate. Another that had 27: another 26, of which 21 were Sons, one whereof was Sir William Phipps; another 39 Children. Here he gives several Instances of Persons living, with them, to above 100 Years of Age. One Clement Weaver lived to 110, his Wife being upwards of 100. This Man, to the last Year, could carry a Bushel of Wheat to the Mill, above 2 Miles. He relates the Case of an old Man, above 100, that lost the memory of several of the latter Years of his Life, but very well retain'd the Remembrance of what past in his younger days. I do not find, by any of these Relations, that the Persons observ'd any Regularity, or Method, in their manner of Diet, Exercise, or the like.
V. An Account, or History, of the Procuring the SMALL POX by Incision, or Inoculation; as it has for some time been practised at Constantinople:


THE Writer of this ingenious Discourse observes, in the first place, that the Circassians, Georgians, and other Asiatics, have introduced this Practice of procuring the Small-Pox by a sort of Inoculation, for about the space of forty Years, among the Turks and others at Constantinople.

That altho' at first the more prudent were very cautious in the use of this Practice; yet the happy Success it has been found to have in thousands of Subjects for these eight Years past, has now put it out of all suspicion and doubt; since the Operation having been perform'd on Persons of all Ages, Sexes, and different Temperaments, and even in the worst Constitution of the Air, yet none have been found to die of the Small-Pox; when at the same time it was very mortal when it seized the Patient the common way, of which half the affected dy'd. This he attests upon his own Observation.

Next he observes, they that have this Inoculation practised upon them, are subject to very slight Symptoms, some being scarce sensible they are ill or sick; and what
is valued by the Fair, it never leaves any Scars or Pits in the Face.

The Method of the Operation is thus. Choice being made of a proper Contagion, the Matter of the Pustules is to be communicated to the Person proposed to take the Infection; whence it has, metaphorically, the name of Infection or Inoculation. For this purpose they make choice of some Boy, or young Lad, of a sound healthy Temperament, that is seized with the common Small-Pox (of the distinct, not Flux sort) on the twelfth or thirteenth day from the beginning of his Sickness; they with a Needle prick the Tubercles (chiefly those on the Shins and Hams) and press out the Matter coming from them into some convenient Vessel of Glass, or the like, to receive it; it is convenient to wash and clean the Vessel first with warm Water: A convenient quantity of this Matter being thus collected, is to be stop'd close, and kept warm in the Bozom of the Person that carries it, and, as soon as may be, brought to the place of the expecting future Patient.

The Patient therefore being in a warm Chamber, the Operator is to make several little Wounds with a Needle, in one, two or more places of the Skin, till some drops of Blood follow, and immediately drop out some drops of the Matter in the Glass, and mix it well with the Blood issuing out; one drop of the Matter is sufficient for each place prick'd. These Punctures are made indifferently in any of the fleshly Parts, but succeed best in the Muscles of the Arm or Radius. The Needle is to be a three-edg'd Surgeon's Needle; it may likewise be perform'd with a Lancet: The custom is to run the Needle transverse, and rip up the Skin a little, that there may be a convenient dividing of the Part, and the mixing of the Matter with the Blood more easily perform'd; which is done, either with a blunt Stile, or an Ear-picker: The Wound is cover'd with half a Walnut-shell, or the like Concave Vessel, and bound over, that the Matter be not rub'd off by the Garments; which is all
removed in a few Hours. The Patient is to take care of his Diet. In this place the Custom is to abstain wholly from Fleth and Broath for 20 or 25 days.

This Operation is perform'd, either in the beginning of the Winter, or in the Spring.

Some, for caution, order the Matter to be brought from the Sick by a third Person, lest any Infection should be conveyed by the Cloaths of the Operator; but this is not material.

As to the Process of this Matter, in respect of the Idiosyncrasy; the Small-Pox begins to appear sooner in some than in others, in some with greater, in others with lesser Symptoms; but with happy Success in all. In this Place the Efflorescence commonly begins at the end of the seventh day, which seems to favour the Doctrin of Crises.

It was observ'd, in a Year when the common Small-Pox was very mortal, that those by Incision were also attended with greater Symptoms. Of 50 Persons, who had the Incision made upon them almost in the same day, four were found in whom the Eruption was too sudden, the Tubercles more, and Symptoms worse. There was some suspicion, that these four had caught the common Small Pox before the Incision was made. It is enough for our present purpose, that there was not one but recovered after the Incision. In those four the Small-Pox came near the confluent sort. At other times the inoculated are distinct; few and scatter'd; commonly 10 or 20 break out; here and there one has but 2 or 3, few have 100: There are some in whom no Pustule rises, but in the Places where the Incision was made, which swell up into purulent Tubercles; yet these have never had the Small-Pox afterwards in their whole Lives; tho' they have cohabited with Persons having it.

It is to be noted, that a no small quantity of Matter runs for several days, from the place of the Incision.

The Pocks arising from this Operation are dry'd up in a short time, and fall off, partly in thin Skins, and partly con-
contrary to the common sort, vanish by an insensible wall-
ing.

- The Matter is hardly a thick Pus, as in the common, but a thinner kind of Sanies; whence they rarely pit, except at the place of the Incision, where the Cicatrices left are not to be worn out by time, and whose Matter comes near the nature of Pus.

If an Apotheke breaks out in any (which Infants are most subject to) yet there is nothing to be fear'd, for it is safely heal'd by Suppuration. If any other Symptom happens, 'tis easily cur'd by the common Remedies.

Observe, they scarce ever make use of the Matter of the Incisious Pox, for a new Incision. If this Inoculation be made on Persons who have before had the Small-Pox, they find no alteration, and the places prick'd presently dry up; except in an ill Habit of Body, where possibly a slight Inflammation and Ex- ulceration may happen for a few days.

To this time, he says, I have known but one Boy, on whom the Operation was perform'd, and yet he had not the Small-Pox, but without any mischief; and some Months after catching the common sort, he did very well. It is to be observ'd, that the places of the Incision did not swell. I suspeet this Child prevented the insertion of the Matter, for he struggled very much under the Operation, and there wanted help to hold him still. The Matter to be inserted will keep in the Glass very well for 12 Hours. He goes on.

I have never observ'd any mischevious Accident from this Incision hitherto; and altho' such Reports have been sometimes spread among the Vulgar, yet having gone on purpose to the Houses whence such Rumors have arisen, I have found the whole to be absolutely false.

It is now eight Years since I have been an Eye-witness of these Operations; and to give a greater Proof of the Sedulity I have used in this Dilquisition, I shall relate two Histories.
There was, in a certain Family, a Boy of 3 Years old, afflicted with the Falling-Sickness, the King's-Evil, an Hereditary Pox, and a long Marasmus. The Parents were desirous to have the Incision made upon him; the Small-Pox were thrown off with ease; about the 40th day he dy'd of his Marasmus. In another Family, a Girl of 3 Years old, troubled with the like Fits, Strumous, attended with an Hereditary Lues, and labouring under a colliquative Looseness for three Months. The Operation was perform'd on this Child; she came off very well of the Small-Pox, which was all over the 15th day; on the 32d she dy'd of her Looseness, which had never left her the whole time.

But it is true, I never maintain'd the Inoculation as a Panacea, or Cure for all Diseases; nor do I think it proper to be attempted on Persons like to die. Some more quick-sighted, imagin'd these two Children were, as useless Shades, sent to Charon by any means that could be made use of. If I could have collected any more concerning this Matter, I should have imparted it candidly.

The rest of Dr. Timone's Letter contains his Reasons for this Method of Practice; which being the Aetiological Part, is publish'd in his own Words, as follows.

ÆTIOLOGIA.

Contagium Variolarum per puris infusionem propagari hand e quidam mirabilius qui Æsculapii templum vel a primo limine salutavit, et fermentationis doctrinam subodoratus est: Nec obscurior est infestationis modus, quod panificium, aut ars cerevisaria, in quibus ex admixto fermento masse fermentanda surgescunt; conciliato nimium motu intestino minima pulss particularum principii activae systema. Si quis querit interim cur variola periculo sa alioquin & per sepe lethales, ex infestatione sine ullo periculo excluantur. Dico: Variolae communes vel concurrente pravâ aliquâ speciali aeris diathesi suscitantur, vel ab effluviis a variolofo cœ-
corpore emanantibus per contagium propagantur. Primus casus
in paucis individuis accidit, & concurrente quidem vel insigni
caccohymia, vel saltum variolo$ siemini in talibus individuis
latitantis acerrima exaltunge: Secundus casus communissimus est.
In primo casu miasma malignum areum, in secundo virulentum
contagio corpuscula indolis (probabiliter) salino-sulphureae sed spe-
cificam fracedinem seu ranticidatem nacta statim ac per respira-
tionem bauriuntur spiritus ipsos, & labo quidem teterrima insici-
unt; subsequenter autem massa sanguineam & lympham vitiari
manifestum est: Spiritus statim insici rationi consentanetum est,
tum quia in fontes spirituum, cor scilicet & cerebrum, statim
ingressum habent virulentum aпорri#, tum ratione analogismi in-
ter miasmata & effluvia ista ipsosque spiritus, cum utrque spiritu-
to$-aerea texture sint. Deducitur etiam cita & prava spiritu-
um infectio à tot tantisque norvo$ systematis symptomatibusque,
que malas plerumque comitantur variolas, & precipue à convul-
$ionibus epilepticis que infantibus accident quipso momento, quo
variolo$ insiciuntur contagio multo antequam febris illos corri-
piat. Massam autem sanguineam inquinari præter februm puru-
lenta tuberculorum exclusio te$atur. Lymphae vero vitiatae sedem
facient glandularum in faucibus tumor, creatus, & enormis mul-
toties ptyalismus. Inter haec circularis etiam sequitur noxa. Sed
precipue sanguinis particule ab indebita spirituum irradiatione in
plures ataxias & anomalias perducuntur. Duo$us tamen potissi-
num modis in variolis communibus mortem contingere observavi.

Primus est quando paucis erumpentibus variolis, & tarde ad
maturitatem procedentibus, mala alia oborinunt symptomata; sec-
cundus quando nimia tuberculorum cotid cadaverosam putredinem
inacuit. In primo ca$u maligne vulgo dicuntur variolae: causa
autem est vel nimia fusio & dissoluto mass$ sanguineae, vel ejusdem
coagulatio & grumescencia. Si enim impetus spirituum explosio
justo plus augeratur, particula massa sanguineae nimium ad invicem
atteruntur, communicuntur, & tenuissimas nanciscuntur acroitas:
sanguis in hoc statu sollertis naturae mechanismum eludit, cumque
nil fauculentioris in glandulis secretoriis cribrisque deponat, econo-
nomie animalis functionibus requisitas filtrationes & transcolati-
ones
ones celebrari baud patitur: improportionata etenim est figura particularum liquidi ad configurationem pororum in tubulis & colatioris ratione subtillitatis nimis: filtratione enim defecarentur particula sanguinis si naturalem fervarent schematismum & molem: hinc dicitur pepefic fieri per incrassationem. Prater hoc celerritas ipsa transfus sanguinis in causa est ut nihil deponatur in colatioris. Torrens ubi nimmio impetu & precipitit currus fertur aquas turbidas defecari baud patitur; quia vis centripeta gravitatem admixti terrei sequens superatur a fortiorum pulsoria virtute aquorum globulorum rapide ruentium: virtus enim fortis, verbi gratia, ut unum non poterit lineam perpendicularem describere ubi virtus fortis ut duo ad lineam horizontalem protrudit: sic etiam haud pluit vento flante intensissimo; eadem geometrica proportione (probabiliter loquendo) sanguinis particula autro ab effrenibus spiritibus motu, tubulos colatiorios preterfluent nullâ faciâ facum dispositione. Hac probabilia sunt à summa pulsus celeritate, febre intensissima, sudore nullo, & urina cruda. E contra quandoque contingit ut ab acutis, & seidentibus deleterii fermenti particularis frangatur, corrodatur, vel saltem relaxetur elater spirituum: elanguescente igitur spirituum motu torpidiores etiam hebetioresque sanguinis lymphaque particulae: igitur dum in labyrinthis tubulorum anfractibus moram indebitam contrahunt alias turmae in viem complicari, alias autem, congestione faciâ, super alios incidere, & diverso ad invicem superficiem suarum contactu à naturali configuratione disteare, & novas induere angulorum dimensiones necessas est. Sic igitur diversa ab ulla, quam superius narravimus, figurarum ad tubulorum meatus improportione, partem calamitatis eventu dedale naturae machinationes irratas fieri contingit. Hac probabilia sunt à pulsu tardo & raro, ac febris carentia quandoque in summa malignitate observatis, paucis & tardè erumpentibus variolarum pulsulis. Uterius à trepidatoria, su su faltoria ac tumultuosa furentium spirituum irradiatione inaequalis codem tempore in diversis partibus maffe sanguinea, & arteriarum etiam venarumque contingere potest impulsus. Sive igitur fibrilla alicui (ut quidem volunt) referiantur in sanguine, seu chili nondum bene assimilati sint portiones usibus peculiariibus dica-
dictae; probabiliter istarum motum turbari contingit: bas enim
in circulatorio motu secundum longitudinem suam naturaliter mo-
veri necessae est: ab inequali autem pressione dicta rectilineam fi-
guram perdere, & in spiras ac semicirculos crispari coguntur:
bas igitur sic contortas transversaliter postmodum in circulatione
raptari, ad invicem implicatas convolvi, & ramosis schematibus
abortis, racematim adeo conglobari necessae est, ut in majuscules
tanquam gramos coalescant, sive demum fibrilla ille non dentur,
certè cujuscumque figura sint massa sanguinea particulari, illas à na-
turali desciscere situatione ex hac motus inaequalitate contingit:
Confusa igitur particula ista & ad invicem implicatae statim ve-
hiculi sui, serì scilicet globulis per expressionem & suo contubernio
exploso, majorem, ratione motis autque gravitatem nanciscun-
tur, idque impulsæ circulatoriæ facultatis vim superant: Has
igitur hic ilic est retitare ac stagnare necessae est, prout in hoc vel
illo loco prima mutua cohabet forte contigerit: Hinc livida lig-
mata, & simul (quod sepe observavi in variolis cum petechiis
erumpentibus) frequens sequitur miœtas, quo limpidissimum serum
in magna copia exclusitum. En fusio, & coagulatio. Hinc mi-
rum non est car moriantur aliqui in variolis cum petechiis, convul-
sionibus syncope, vigiliis nimiis, emorrhgiis, delirio, vomitibus,
emormibus, dysenteriis, sc. quamvis haud multa pustularum putrilagi-
ne perfundantur: In stygium enim veluti carceris variola-
rum fermentum multoties evexitur, ira ut quamvis haud magnam
crassii puris copiam progignere aptum sit, spiritibus tamen, liqui-
dis & solidis suprarecensita mala modis vel explicatis vel aliis
consimilibus communicare posset, sicque mortem inferre; & hoc
ante undeceum plerumque. Veniamus nunc ad secundum modum.
Diversa enim aliquando contingit pernicies & longè alterius ge-
eris tragœdia: quamvis enim absint illa symptomata, nimia ta-
men puris, materia scilicet cadaverisate, copia corpus obturatur.
Pus autem generali probabile est quando sulphureis oleosifque massa
sanguinea particularis in francede & fusione constituitis acido-sali-
narum particularum coaffuso contingit. Huic assepto facem ac-
cendunt innumera chymica experimenta quibus manifestè edocemur
solutionibus pinguium sulphureorum per alkalia factis acido quali-
M bet
bet coafluso statim massam albicantis coloris emergere. Multoties
igitur miafma seu fermentum variolarum per respirationem ha-
"rum ratione indolis propriae acerrimae & fortassis septicae tales in
massam sanguineam particularum acido-salinarum & oleo-sulphurea-
rum producere potest combinationes, ut non seminia solum vario-
larum, qua omnibus individuis (mole tamen minima) à nativi-
tate indita sunt, agitentur, aeduentur, & in purulentam aceant
putrilaginem, sed massa ipsa sanguinea tota acorem contrahat, &
motu quodam corruptorio putrescat & cadaverifetur. Sic igitur,
incendio veluti cortorto, ulterius furere fermenteentes particulas
contingit, quam variolosus seminis per despumationem eliminandis
opus fit: hic motus non est depuratorius heterogeneis secerndis
inserviens, sed destructivus & corruptorius, fermento nemente mas-
sum totem superante & invertente; fracidis scilicet rebellibusque
particulis victoria potitis, & omnes alias in sua castra migrare
cogentibus. Hoc manifeSté observamus in variis potulentis, in
quibus fermentatione aliquando excitata, motus succedit corrupti-
vus liquores totaliter vitiatis: hinc videmus aliquos quamvis su-
prarecensitis symptomatibus immunes, immenso tamen, ut ita di-
cam, putredinis oceano suffocatos: Et hoc periculum usque ad vi-
ginum secundum protrabitur. Ultimo loco confiderandum solida
etiam & nobiliores partes in hisce caebus pessime affici, & in
spasmos inordinatos fieri: variis horum distortionibus tubulorum
meatus vitiari, at functionum munera depravari necessè est: Ecce
igitur continentia, contenta, & impetum facientia, quorum tri-
umviratu humani corporis respublica regitur, una eademque ruina
ut plurimum involuta: mirabitumne quispiam malorum inde Ilia-
dem in hominis perniciem pullulare? Observandum ulterius mul-
tis, qui peste laboraverint, communibus variolis etiam post annum
corruptis bulones cosdem intumuisse, qui antean in peste eruperant:
nonne hic etiam summam malignitatem testatur. Infectionem mo-
do ad rationis trutinam revocemus. At hercule longe aliter in
hoc contagionis modo rem procedere quis est qui non sateatur:
Primum enim Spiritus nullatenus infici manifestum est: deinde
non lymphae, non sanguini labes illa tertrima inuritur, non soli-
dis viuum aliquod communicatur. Hinc symptomata omnium levia
via, nulla pessima, nulli infantibus epileptici insultus. Contagio-
nis enim bujusce fermentum non spiritale, non aerum & acutum
est, sed humorale, iners, ac pigrum: venenum autem quo subtiliora
eo pejora: Ratione igitur improportionis nulla inter fermentum
hoc & spiritus esse poterit laeta. Pus equidem variolarum
in ipsa substantia sanguini immediate infusum statim in largum
veluti pelagus exceptum diluitur, involvitur, absorbetur, obtun-
ditur: sic illud mitescit, sic in manu ejusdem indolem cicerat
Contagiosa iste particula sanguinem ingressa statim siti congener-
res variolosorum feminis participas sanguinis & nativitate inditas
invenit; igitur improprietatis nulla inter fermentum hoc & spiritus
esse poterit laeta. Pus equidem variolarum
in ipsa substantia sanguini immediate infusum statim in largum
veluti pelagus exceptum diluitur, involvitur, absorbetur, obtun-
ditur: sic illud mitescit, sic in manu ejusdem indolem cicerat.
Contagiosa ista particula sanguinem ingressa statim siti congener-
res variolosi feminis participas sanguinis & nativitate inditas
invenit; igitur improprietatis nulla inter fermentum hoc & spiritus
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veluti pelagus exceptum diluitur, involvitur, absorbetur, obtun-
ditur: sic illud mitescit, sic in manu ejusdem indolem cicerat.
VI. Theoremata quaedam infinitam Materiam Divisibilitatem spectantia, quae ejusdem raritatem & tenuem compositionem demonstrant, quorum ape plurima in Physica tolluntur difficulitates.

Lemma.

Data quavis materiae quantitate, ex eâ, vel ex quavis ejus arte, formari potest sphaera concava, cujus semi-diameter sit datae rectae aequalis.

Sit materiae particula \( a^3 \) & data recta sit \( b \). Ratio peripheriae circuli ad Radium sit \( p \) ad \( r \). dicatur semi-diameter concavitatis \( x \), & crassities, pelliculae concavitatem sphærae ambientis, erit \( b-x \) & Cylindrus sphæræ circumscriptus cujus radius est \( b \) erit \( \frac{p \times b^3}{r} \), unde sphaera cylindro inscripta erit \( \frac{2 \times p^3 b^3}{3 \times r} \). Eadem ratione sphaera cujus radius est \( x \) erit \( \frac{2 \times p^3 x^3}{3 \times r} \) quarum differentia \( \frac{2 \times p^3 b^3}{3 \times r} - x^3 \) ponenda est sphæricæ lamellæ aequalis, seu materiae particulae datae; hoc est erit \( \frac{2 \times p^3 b^3 - x^3}{3 \times r} = x^3 \) seu \( b^3 - x^3 = \frac{3 \times r^3}{2 \times p} \) unde \( x^3 = b^3 - \frac{3 \times r^3}{2 \times p} \) &

\[
 x = \sqrt[3]{b^3 - \frac{3 \times r^3}{2 \times p}}
\]

Eadem ratione fieri possunt ex data materiae quantitate Cubi concavi, Cylindri concavi, vel corpora etiam alterius cujusvis figura concava, quorum latera sunt datae rectae aequalia.

Theorema Primum.

Datâ quavis materiae quantitate quantumvis exigua, & dato spatio quovis finito utcunque ampio; quod v. gr. sit cubus, qui sphæram Saturni circumscriberet: Possibile est ut materia istius Arenulae per totum illud spatium diffundatur, atque
atque ipsum ita adimpleat, ut nullus fit in eo porus cujus
diameter datam superet lineam.

Sit datum spatium Cubus cujus
latus sit recta $AB$, diametro scil. or-
biteæ Saturni æqualis, deturque ma-
terieæ particula cujus quantitas sit $b^3$, 
& data recta (quæ pororum diamet-
tri non majores esse debent) sit $d$
Dividi concipiatur recta $AB$ in par-
tes æquales rectæ $d$, quarum nume-
rus finitus erit, cum nec recta $AB$
ponitur infinitè magna, nec recta $d$
infiniè parva: fit numeros ille $n$, hoc est fit $nd=AB$, adeo-
que erit $n^3d^3$ æqualis cubo rectæ $AB$. Concipiatur item
spatium datum dividi in cubos quorum singulorum latera
sunt æqualia rectæ $d$, eritque cuborum numerus $n^3$, & hi
cubi per spatia $efgh$ in figura reprezententur. Dividi porro
supponatur particula $b^3$ in partes quam numeros sit $n^3$, &
in unquoque spatio cubico ponatur una harum particula-
rum, & hac ratione materia $b^3$ per omne illud spatium dif-
fundetur. Potest præterea unaquæque ipsius $b^3$ particula in
sua quasi cellâ locata in sphæram concavam formari, cujus
diameter sit æqualis data rectæ $d$; unde fiet, ut sphæra qua-
libet proximam quamque tangat, & data materie particula
utcumque exigua $b^3$ spatium datum ita adimplebit, ut null-
lus fiet in eo porus cujus diameter datam rectam $d$ superat.

Q. E. D.

Cor. Hinc dari potest corpus, cujus materia, si in spatium
absolutè plenum redigatur, spatium illud fieri potest prioris
magnitudinis pars quàlibet data.

Theorema Secundum.

Possunt esse duo corpora mole æqualia, quorum materie quan-
titates sint utcumque inaequalis, & datam quamvis ad se
invicem
invicem obtineant rationem, pororum tamen summa, seu spatia vacua inter corpora, ad rationem aequalitatis ferè accedant. Vel in stilo Cartesiano: Spatium emne, quod à materiâ subtili intra unius corporis poros occupatur, posset esse fere æqualis spatium quod à simili materiâ intra alterum corpus tenetur. Licet materia propria unius corporis decies millies vel centies millies superat materiam propria alterius Corporis, & Corpora sint mole æqualia.

Ex. gr. Sit Digitus cubicus Auri & Digitus cubicus Aeris vulgaris non condensati. Certum est quantitatem materiæ in Auro vices millies circiter superare materiam æris, atamen fieri potest, ut spatia in auro vel absolutè vacua, vel materiâ subtili repleta, sint fere æqualia spatii in ære, vel vacuæ, vel materiâ tantum subtili repletis.

Sint A & B corpora duo, magnitudine æqualia: utrumque v. gr. sit cubus uniús digiti. Et corpus A decies millies sit gravius corpore B, unde & corpus A quantitate materiæ decies millies superabit corpus B. Ponamus jam materiæ quantitatem in A redigi in spatium absolutè plenum, quod sit digitæ cubici pars centies millefima; (liqueat enim ex corollis precedentis Theorematis id fieri posse). Unde cum materia in A decies millies superat materiam in B, materia illa in B, si in spatium absolutè plenum compingatur, occupabit tantum digitæ cubici partem \( \frac{1}{100000000} \) seu millies decies centies millefimam; Adeoque partes reliquæ 999999999 vel erunt absolutè vacuae, vel materiâ aliqua subtili, qualis supponitur Cartesiana, tantum repletae. Porro, cum materiæ quantitas in A impleat tantum digitæ partem centies millefimam, erunt in corpore A partes 99999 centies millefimæ, vel vacuae, vel materia subtili repletae, hoc est reducendo fractionem ad denominatorem prioris fractionis, erunt in A partes vacuae 999990000 millies decies centies millefimæ. Adeoque vacuitates in A erunt ad vacuitates in B, ut numerus 999990000 ad numerum 999999999, qui numeri sunt ad se invicem ferè in ratione.
ratione aequalitatis, nam corum differentia, parvam admodum ad ipsos numeros obtinet rationem. Adeoque spatia vacua, vel materiâ subtili tantum repleta, quae sunt in duobus corporibus A & B, eandem cum ipsis numeris, ad se invicem rationem obtinentes, sunt etiam ferè in ratione aequalitatis. Q. E. D.

Corpora autem omnia esse rarissima, hoc est pro mole sua parvam admodum continere materiae quantitatem, ex diaphanorum proprietatibus certissimè constat, nam Radii Lucis intra vitrum, vel aquam non secus ac in aere per rectas lineas diffunduntur; quæcumque luci exposita sit corporis Diaphani facies; Adeoque à minimâ quâvis assignabili Diaphani parte, ad aliam quamvis ejsdem partem, semper extenditur in his corporibus porus rectilineus, per quem transiverit lux, atque hoc fieri non potest nisi Materia Diaphani ad ejus molem, parvam admodum obtineat rationem, nec fortasse materiae quantitas in vitro, ad ejus magnitudinem majorem habet rationem, quam magnitudo unius Arenulae ad totam Terreni orbis molem: Hoc autem non esse impossibile, superius ostenfum est. Unde cum Aurum non sit octuplo densius Vitro; ejus quoque materia, ad propriam molem, exiguam admodum obtinebit rationem.

Hinc ratio reddi potest, cur effluvia magnetica eadem ferè facilitate densum Aurum & tenuem aerem pervadunt.

Ex his etiam propositionibus, & ex maximâ lucis celebritate, ratio reddi potest, cur Lucis radii ex pluribus objectis prodeuntes & per tenue foramen transmissi, se mutuo non impediunt, sed per eandem rectam in motu suo perseverant: Quod per motum seu impulsum fluidi, plenum essentiae, vix explicari potest; corpus enim omne à pluribus potentitis, secundum diversas directiones, simul impulsum, unam tantum & determinatam directionem accipit ex omnibus composition.

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The CONTENTS.


III. Rules for correcting the usual Method of computing Amounts and present Values, by Compound as well as Simple Interest; and of stating Interest Accounts; by Thomas Watkins, Gent.

F. R. S.

STELLA Bayero τ Tauri proxime appellit ad limbus Lunæ, observata per Telescopium duodecim palmarum.

Eadem jam occultata est ab ea parte Lunaris Limbi, quæ media sermæ est inter maculas Ariftarchi & Galilei. Parallelus diurinus à centro Lunæ descriptus apparat Australior quam stella τ partibus Micrometri 7 ½, qualium Lunæ diameter subtendit 37. Stellæ igitur τ declinatio Boreali est declinatione apparente Lunaris centri minutis circulis maximi 5 ½ circiter.

Sirius attingit Meridianum: unde verificata sunt tempora.

Stella τ, quæ aliquot minuta exceederat è limbo Lunæ, in revolutione diurna præcedit limbum occidentalem Lunæ secundis horariis 0 33'', eademque præcedit centrum Lunæ secundis 10 3'' vix 1'. 43''.

Eadem præcedit limbum Lunæ secundis 48'', & centrum 1'. 58''.

Differentia Afcensionis rectæ stellæ & limbi est 1'. 03'', centri vero Lunæ & ejusdem stellæ 2'. 13''.

In limbo Lunæ Penumbra, quæ antea erat dilutior, fœsim fit densior.

Penumbra fit evidentior, sed nondum apparat Umbra vera.

Initium incidentis Lunæ in Umbram veram, ea in parte limbi quæ proxima est macula Schiccardi.

Umbra
<table>
<thead>
<tr>
<th>h°</th>
<th>m'</th>
<th>s&quot;</th>
<th>Observatio</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>5</td>
<td>21</td>
<td>Umbra vera jam obregit partem unam, qualium Lunæ diameter in Micrometro obtinet 37.</td>
</tr>
<tr>
<td>15</td>
<td>7</td>
<td>20</td>
<td>Jam partes duæ obteguntur qualium Lunæ diameter est 37.</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>20</td>
<td>Jam obteguntur Lunaris diametrio.</td>
</tr>
<tr>
<td>15</td>
<td>31</td>
<td>20</td>
<td>Latent Lunaris diametri.</td>
</tr>
<tr>
<td>16</td>
<td>12</td>
<td>0</td>
<td>Jam latent in diametro partes.</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>20</td>
<td>Partes latentes 15, ut antea.</td>
</tr>
<tr>
<td>16</td>
<td>50</td>
<td>20</td>
<td>Jam partes latentes.</td>
</tr>
<tr>
<td>16</td>
<td>54</td>
<td>35</td>
<td>Incipit emergere prior limbus Tychonis.</td>
</tr>
<tr>
<td>16</td>
<td>56</td>
<td>9</td>
<td>Jam totus Tycho emergit.</td>
</tr>
<tr>
<td>17</td>
<td>13</td>
<td>30</td>
<td>Latent Lunaris diametri partes 5 è 37.</td>
</tr>
<tr>
<td>17</td>
<td>27</td>
<td>45</td>
<td>Umbra vera excedit in limbo Lunæ, in loco designato per diametrum duæam inter Aristarchum &amp; Platonem situm inter medio.</td>
</tr>
</tbody>
</table>

N.B. Hec observatio pluris aestimanda, quod occultatio stelle acciderit tam vicina Opposito Solis, ut inde locus Solis inter Fixas rite examinari poterit.

Emergentes Satellitis intimi Jovialium ex umbra Jovis observata Romæ, anno 1713.

dem nocte 7h 46, Primus & Secundus proximi sunt, & 7h 53.
iidem ita sunt vicini ut vix punctulo distinguantur.
Decemb. 9 N.S. vel Novemb. 28. V. S. 5h. 45'. 45' Primus
Satelles incipit emergere ab umbra Jovis.
Decemb. 21. V. S. 5h. 50'. 22, iterum visus est primus Satel-
les incipiens emergere ex umbra.

Ex his observationibus accurato calculo subjectis, manifestum est
aquisitionem secundam, quam a motu Luminis progressivo ortam suppo-
nimus, necessario locum habere. Nam post 57 satellitis intimi revoluti-
ones, quibus Jupiter a Terra plus quam Radio Orbis magni recessit,
novem fere minuti tardius conspecta est Eclipis ultima, quam de-
buit juxta tenorem Observationis prima: quod quidem Hypothesibus
D. Cassini consonum est.

Ex istem etiam confirmatur (quod nos quoque antea anno-
tavimus, nempe) quod motus Intimi Satellitis Jovis paulo celerior
fui quam in Tabulis elaboratissimis D. Cassini, ante viginti annos
cum publico communicatis, & calculi facilitate plurimum se commen-
dantibus. Errorculus autem iste vix excedere videtur duo tempo-
ris minuta in singulis Jovis revolutionibus, sive duodecim annis;
quibus caelum anticipat Cassini calculum. Hac vero adhibita cor-
rectione, satis accuratus habebitur consensus.

Obilitimum est problema Datâ lege Vis centripetæ invenire Curvam quam descriptit Mobile, de loco dato, secundum datam rectam, & cum data velocitate egrediens Concessis figurarum curvilinearum quadraturis, ejus solutionem perfectam olim dedit Dominus Newtonus in Principiis Philosophiae Mathematicis. Hoc ipsum Problema denuo aggressus est vir clarissimus & Geometra celeberrimus Dominus Johannes Bernoulli in Academia Basiliensi Matheos Professor *, qui non paucâ eaque egregia ingenii sui specimina jam pridem edidit, quibus Geometriam reconditioram non parum ditavit. Unde à tantì virî acumine novam pulchramque Problematis solvendi methodum expectabam. Gestiebam itaque solutionem Bernoullianam perlegere, & cum Newtoniana comparare; quibus tandem diligentius perlegi & examinati, hæc quæ sequuntur annotavi.

Dominus Bernoulli eandem praemitterat propositionem quam Newtonus problemati demonstrando prius adhibuit: eaque ea in Principiis XL, non minus pulchra quam demonstratru facilis. Scil.

Si corpus cogente vi quacunque centripeta moveatur utcunque, & corpus aliud recta ascendant vel descendat, sintque eorum velocitales, in aliquo æqualium altitudinum casu, æqualia; velocitales eorum in omnibus æqualibus altitudinibus erunt æqualia.

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Vide Commentarios Physico-mathematicos Parisienses Anno 1710.
Hujus propositionis Demonstrationem Newtonianam ait Bernoullius esse nimis implicatam, & suam, quam simplici-orem vocat, ejus loco substituit. At pace tanti viri liceat mi-lii dicere, si quid discriminis sit inter demonstrationem Ber-

noullianam & Newtonianam, id. in co.situm est, quod haec multo facilior esse, videtur minusque perplesia quam illa. Fig. I.

Nam si centro C describantur circuli DI, EK, quorum inter-
vallum DE est quam minimum, sintque corporum in D & I veloci-tates aequales, & ab N ad IK demittatur perpendicularum NT, fusc ostendit Newtonus vim acceleratricem secundum DE, esse ad vim acceleratricem secundum IK ut IN ad IT. Nimi-
rum si vis secundum DE vel LN exponatur per rectas DE vel LN, vis illa secundum IN resolvitur in duas IT, TN, qua-
rum illa solum qua est ut IT motum secundum directionem
IK accelerat: accelerationes autem seu velocitatum incrementa sunt ut vites & tempora quibus generantur conjunctim. At tempora ob aequales velocitates in D & I, sunt ut viae descriptae DE, IK; quare accelerationes in descurri corpore
per lineas DE & IK, sunt ut DE ad IT & DE ad IK conjunctim; i.e. ut DE quad. quod est IN. quad. ad rectang. IT & IK. adeoque ob IN quad. = IT x IK, incrementa ve-
locitatum sunt aequalia: aequales igitur sunt velocitates in E & K, & eodem argumento semper reperientur aequales in æquali-
bus distantiis. Hæc est summa demonstrationis Newtoni quæ

tam dilucide ab eo exponitur, ut inter propositiones elementa-
res paucas faciores invenies. At non sic procedit Dominus
Bernoullius, sed illi sufficit dicere, Mechanicam ostendere vim
secundum DE esse ad vim secundum IK, ut IK ad DE. Me-
chanicam etiam ostendere incrementa velocitatum esse in ra-
one virium & temporum conjunctim; & initio motus positis
velocitatibus aequalibus tempora sunt ut viae descriptae DE, IK;

& hinc, (argumento prorsus simili ei quo utitur Newtonus),
concludit incrementum velocitatis, quod acquirit corpus dum
describit IK, esse ad incrementum velocitatis dum describitur
DE, ut DE x IK ad IK x DE, & proinde velocitatum incre-

menta ubique in distantiis aequalibus esse æqualia.
At si Tyronibus facilem voluisset tradere demonstrationem; debuisset Propositionem Mechanicam citare, camque ad praefereb-tem casum accommodare. Et quidem pluribus verbis opus est, ut hoc fiat per theorema quod innuere videtur, in quo agitur de descenti Gravium in planis inclinatis: nullum enim est hic Planum datum quod recto corporum descentui obturat; immo taeniam aebet ut corpus a plano cohibeatur, ut e contra a Plano seu Tangente per vim quandam continuo retrahitur. Procudubio igitur manifesta magis foret ejus rationi ii vis, si demissem Mechanicæ propositionibus, rem omnem ex propriis principiis demonstrasset, uti fecit Newtonus. Nam resolvendo triangulo rectang. KN I in duo triangula æquivalenta, est K I ad IN ut I N ad I T, adeoque loco rationis IN ad IT ponere potuisse rationem K I ad IN vel ad DE.

Si de loco quovis $A$ in recta $AC$ cadat corpus, deque loco ejus $E$ erigatur semper perpendicularis $EG$ vis centripetæ proportionalis, sitque $B F G$ linea curva quam punctum $G$ perpetuo tangat; demonstrat Newtonus velocitatem corporis in loco quovis $E$ esse ut Areal curvilineæ $ABGE$ latus quadratum. Adeoque si velocitas dicatur $v$, crít $v^2$ ut Area $ABGE$: & si $P$ sit altitude maxima, ad quam corpus in Trajectoria revolvens, deque quovis ejus puncto eà quam ibi habit velocitate sursum projectum ascendere posti sitque quantitas $A$ distantia corporis à centro, in alio quovis orbitæ puncto; & vis centripeta sit semper ut ipsius $A d i g n i t a s$ quálibet, scil. ut $A^{-1}$, Velocitas corporis in omni altitudine $A$ erit ut $\sqrt{nP^2 - nA^2}$.

Similiter Dominus Bernoullius ostendit, si distantia à centro dicatur $x$, velocitas $v$ & vis centripeta $\varphi$, esse $v = \sqrt{ab - \int \varphi}$ ubi ex Quadraturis constat esse Areal $ABGE = ab - \int \varphi x$. Perinde itaque est sive exprimatur quadratum velocitatis per Areal $ABGE$, sive per quantitatem huiæ æqualem $ab - \int \varphi x$. Et si vis centripeta $\varphi$ sit ut $nA^{-1}$ seu $n x^{-1}$, sit $ab = P^2$ &

\[
f(x) = A^n, \text{ adeoque } ab - f(x) \text{ est ut quantitas } P^n - A^n.
\]

Describat corpus Curvam \( PK \), vi centripeta tendente ad \( C \), deturque circulus \( VXT \), centro \( C \) intervallo quovis \( CV \) descriptus. \( Q \) sit quantitas constans, atque \( \frac{Q}{A} = z \). Sitque \( KI \) elementum Curvae; \( IN \) vel \( DE \) elementum altitudinis, \( XY \) elementum arcus: demonstrat Newtonus Elementum arcus seu \( XY \) exprimi posse per hanc formulam

\[
\frac{Q \times IN \times CX}{AA \sqrt{ABGE} - z^2}
\]

Similiter ex praemissis Dominus Bernoullius, posito Arcu \( UX = z \), & altitudine seu distantia \( -x \), elementum arcus ad hanc reducit formulam scil. \( z = \frac{a^2 c x}{\sqrt{abx^4 - x^4 f(x)} - a^2 c^2 x^2} \). Et primo quidem aspectu videbatur formula Newtoniana quodammodo simplicior Bernoullianæ, eo quod paucioribus constat terminis; at re diligentius explorata, vidi Bernoullianam formulam omnino cum Newtoniana coincidere; nec nisi in notatione quantitatum ab ea dieret.

Nam si pro \( ab - f(x) \) ponatur \( ABGE \), pro \( ac \) ponatur \( Q \), & \( x \) pro \( A \), a pro \( CX \), & \( x \) pro \( IN \), fit

\[
\frac{Q \times CX \times IN}{\sqrt{abx^4 - x^4 f(x)} - a^2 c^2 x^2} = \frac{Q^2 A^4}{A^2} = \frac{Q \times CX \times IN}{AA \sqrt{ABGE} - z^2}
\]

feu ponendo \( z^2 \) loco \( \frac{Q^2}{A^2} \), (quod facit Newtonus commodioris notationis gratia,) Formula Bernoulliana evadit \( \frac{Q \times CX \times IN}{A^2 \sqrt{ABGE} - z^2} \); unde constat formulam illum non magis à Newtoniana discrepant, quam verba Latinis litteris expressa differunt ab ibidem verbis scriptis in Graecis characteribus.

Post traditam generali formam; descendit Dominus Bernoullius ad casum particularem, ubi vis centripeta est reciprocè
procè ut quadratum distantiae; & per varias reductiones &
operationes satis molestas, constructionem ostendit Curvarum
quæ urgentæ cæ vi centripeta describi possunt, easque ad æqua-
tiones reducendo probat esse Sectiones Conicas. Deinde que-
ritur Dominum Newtonum supponere sine demonstratione
Curvas à tali vi descriptas esse Sectiones Conicas.

Impossibile est ut credat nullam Newtono notam fuísse hu-
jus rei demonstrationem; Noverit enim eum primum & lo-
um fuísse qui hanc omnem de vi centripeta doctrinam geome-
tricè tracòavit, quique cam ad tantam perfectionem perduxit,
ut post plures quam viginti annos, parum admodum à præ-
stantissimis Geometris ei additum sit. Noverit etiam Bernoul-
lius Newtonum, præter generalem problematis inversi solutio-
 nem, ostendisse modum quo formari possunt Curvæ, quæ vi
centripeta decrescente in triplicata distantiae ratione descriptu-
tur, adeoque alterum illum casum ignorare non potuisse.
Nec prosequo intelligo qua ratione Bernoullius Newtono ob-
jectat, eum hujus casus demonstrationem prætermississe; cum
ipse non paucæ lœpius proposuit Theoremata, quorum demon-
strationes nusquam dedit; & quidni liceat Newtono ad alia
felicantes hoc idem facere. Interim in nova Principiorum
Editione, facilior multo & magis clara, licet tribus verbis, ex-
tat hujus rei demonstration, quam est Bernoulliana.

Tandem Bernoullius ut necessitatem suæ demonstrationis
inversi Problematis in hoc particulari casu ostendat, ſae addit.
Considerandum est, inquit, quod vis quæ facit ut corpus in
Spirali Logarithmica moveatur, debet esse reciprocè ut cubus
distantiae à centro; at non inde sequitur talibus viribus sem-
per describi debere tales Curvas, cum similes etiam vires facce-
re possunt ut corpus in Spirali Hyperbolica moveatur.

Miror fane quod Vir. Cl. suspicetur Newtonum talem unquam
duxisse consequentiam. Nam præter Spiraalem logarithmicam,
ostendit Newtonus qua ratione alia Curvæ, numero infinitæ
& diversæ, formari possunt, quæ omnes descriptur eadem vi
centripeta qua Spiralis Logarithmica; interque eas reponi de-
bet hæc ipsa Spiralis Hyperbolica, ut in frequentibus ostendamus.

Exinde
Esinde autem concluidit Newtonus Sectiones tantum Conicas necessario, describi debere per vim centripetam quadrato distantia reciproce proportionalem: Nempte quod Curvatura orbitae cujuscunque ex datis velocitate, vi centripeta & positione Tangentis datur; datis autem umbelico, puncto contactus & positione tangenti, semper describi posset Secatio Conica quæ curvaturam illam datam habeat. Hoc à me offensum est in Actis Philosophicis Londinenfibus Anno 1708. In hac igitur Sectione, urgente illa vi corpus movebitur, & in nulla alia; cum corpus de eodem loco, secundum candem directionem, eadem cum velocitate, & urgente eadem vi centripeta exiens, non possit diversas semitas describere.

Liceat jam mihi Dominum Bernoullium imitari, invensum de vi centripeta problema longe diversa methodo resolvere, & ad easum particulararem applicare; ubi scil. vis est reciproce ut cubus distantiae, simulque ostendere demonstrationem Cor. 3. prop. 41. Principiorum Newtoni.

Quod ut fieri, quaedam ex iis quæ in Actis Philosophicis No. 317. exposui hic præmittenda sunt. Fig. II.

Sit VII Curva quævis, quam corpus urgente vi centripeta ad centrum C tendente describit: hanc Curvam in duobus punctis infinitè vicinis I & K tangant rectæ IP, KP, ad quas centro demittantur perpendiculares CP, CF; centro item C descriptur KE, ID, & ducatur CI.

Erit vis centripeta ut Quantitas \[
\frac{P_p}{P C \times I N}
\] Quod Theorema licet in prædicto loco demonstravimus, ecce aliam ejus demonstrationem. Ex K ducantur KM ad CP & KN ad CI paralleæ. Et ob axianguula triangula IC P, IK N, nK m, Item que ob I K m & IP PX axianguula. Erit,

\[IP \text{ vel } IP : IK : : PP : KM\]
\[PC : IP : : KM : mn\]
\[IN : IK : : mn : nK\] unde ex æquo
fiet \[PC \times IN : IK^3 : : PP : nK, \] & erit \[nK = \frac{PP \times IK^2}{PC \times IN}.\]

Præterea tempus quo describitur arcus IK est ut Area
(97)

Area seu triangulum ICX, vel ejus duplum PC × IK; adeoque si tempus detur erit PC × IK quantitas constans. Dato autem tempore, vis centripeta est ut lineola Xn quae sub urgence vi illa describitur, adeoque vis centripeta est ut lineola illa Xn ducta in quantitatem constantem \(\frac{1}{PC^2 × IK^2}\) hoc est, erit vis centripeta ut \(\frac{1}{PC^2 × IK^2} × \frac{PP × IK}{PC × IN}\) seu ut quantitas \(\frac{PP}{PC × IN}\). Quod erat demonstrandum.

Velocitas corporis in quovis loco est ut via in minimo quovis tempore percursa directe & ut tempus illud inversè; adeoque & ut IK × \(\frac{1}{PC × IK}\) hoc est, velocitas erit reciproce ut Perpendicularis è centro in Tangentem.

Si distantia corporis à centro dicatur \(x\), & Perpendicularis in tangentem \(p\), erit IN = \(x\) & PP = \(p\) & vis centripeta exponi potest per quantitatem \(\frac{f^4 \cdot p}{p^3 \cdot x}\), assumendo quantitatem quamlibet profat

Adeoque si cum Domino Bernoullio vim centripetam nominemus \(\varphi\), erit \(\frac{f^4 \cdot p}{p' \cdot x} = \varphi\) & \(\frac{f^4 \cdot p}{p^3} = x \varphi\); & capiendo harum quantitatum fluentes erit \(\frac{f^3}{2p^2} = \) Fluenti quantitatis \(x \varphi\).

At cum velocitas corporis sit reciproce ut perpendicularis \(p\); ejus quadratum exponi potest per \(\frac{f^4}{2p^2}\). Si itaque velocitas \(v\), erit \(v^2 = \frac{f^4}{2p^2} = \) Fluenti quantitatis \(x \varphi\). Quod si A sit locus de quo cadere debet corpus ut acquirat in D vel I veloci-
velocitatem \( v \), de quo loco corporis \( D \) erigatur perpendicularis \( DF = \varphi \) erit rectangulum \( DE \times DF = x \varphi \). Sit jam \( BFG \) linea curva cujus ordinata exponant vires centripetas, seu quantitates \( \varphi \). Fluens quantitatis \( x \varphi \) erit Area curvilinea \( A B F D = v^2 = \frac{f^4}{2p^2} \), adeoque erit \( v \) ut Area \( A B F D \) latus quadratum. Quod si velocitas ea sit quae ab infinita distantia cadendo acquiritur, erit \( v^2 \) seu fluens ipsius \( x \varphi \) æquale area \( O D F O \) indeperite protensa.

Hinc semper dabitur quantitas \( p \) in terminis finitis, quando Area illa curvilinea terminis finitis exponi potest. Sit, verbi gratia, vis centripeta reciprocè ut distantiae dignitatis \( m \), hoc est, sit \( x \varphi = \frac{g}{x^m} \). Si velocitas corporis sit ea quæ acquiritur cadendo ab infinita distantia, erit \( v^2 = \frac{g}{m-1 \times x^{m-1}} = \frac{f^4}{2p^2} \) & in hisce omnibus casibus Area indeperite protensa est quantitas finita. Potest autem corpus in trajectioni revolvi velocitate cujus quadratum vel maiores fieri potest, vel minus quantitate \( \frac{g}{m-1 \times x^{m-1}} \), vel huic æquale. Adeoque erit \( v^2 = \frac{f^4}{2p^2} \) \( \frac{g}{m-1 \times x^{m-1}} \pm \epsilon^2 \).
Sit \( \frac{1}{2} f^2 = a^2 e^2 \) & \( \frac{1}{m-1} \times g = b^2 e^2 \). Et si velocitas corporis sit ea quae ab infinito cadendo acquiritur, erit \( p^2 = \frac{a^2 x^{m-1}}{b^2} \) feu \( p^2 = \frac{a x^{m-1}}{b} \).

At si velocitas major sit aut minor hac velocitate, fieri uti ostensum est \( f = \frac{g}{2b^2} + e^2 = \frac{1}{m-1} g \pm e^2 x^{m-1} \). Unde pro \( \frac{1}{2} f^2 \) & \( \frac{g}{m-1} \) ponendo earum valores \( a^2 e^2 \) & \( b^2 e^2 \), erit:

\[
\frac{a^2 e^2}{p^2} = \frac{b^2 e^2 + e^2 x^{m-1}}{x^{m-1}} \text{ feu } \frac{a^2}{p^2} = \frac{b^2 + x^{m-1}}{x^{m-1}}, \text{ & fier } p^2 = \frac{a^2 x^{m-1}}{b^2 + x^{m-1}}.
\]

Adeoque si Vis centripeta sit reciprocë ut cubus distantiae, hoc est, si sit \( m = 3 \) & \( m - 1 = 2 \). Erit \( p^2 = \frac{a^2 x^2}{b^2} \), vel \( p^2 = \frac{a^2 \cdot x^2}{b^2 + x^2} \), vel denique \( p^2 = \frac{a^2 x^2}{b^2 - x^2} \).

In primo casu constat Curvam esse Spiralem Logarithmicae: nam fit \( p = \frac{a \cdot x}{b} \), & \( b : a :: x : p \). adeoque ob constantem rationem \( b \) ad \( a \), erit angulus \( CIP \) ubique constans.

Fonamus jam esse \( p^2 = \frac{a^2 x^2}{b^2 + x^2} \) & ex hac suppositione tres orientur diversæ Curvarum species, prout \( a \) major est quam \( b \), aut \( a \) equalis, aut minor. Fig. III.

Et primo sit \( a \) major quam \( b \). Centro \( C \) & ad distantiam quamvis datam describatur circulus \( HTX \) cui rectæ \( CK, CI \) productæ occurrent in \( T & X \). Et est \( IN^2 : KN^2 :: IP^2 : PC^2 \) & ita...
& ita $CP = PC' : PC' :: x^2 - p^2 : p^2 :: x^2 = \frac{a^2 x^2}{b^2 + x^2} : \frac{a^2}{b^2 + x^2} :: b^2 + x^2 - a^2 : a^2$. Qua-
ere erit $\sqrt{x^2 + b^2 - a^2} : a :: IN : KN :: x : \frac{a x}{\sqrt{x^2 + b^2 - a^2}}$

$= KN$. Et quoniam est $a$ major quam $b$, erit $b^2 - a^2$ quantitas
negativa. Sit illa $-c^2$, unde fit $KN = \frac{a x}{\sqrt{x^2 - c^2}}$ Dicatur radius
circuli $HT b$, & est $CK : KN :: CT : TX$ hoc est $x$:
$\frac{a x}{\sqrt{x^2 - c^2}} :: b : \frac{b a x}{x \sqrt{x^2 - c^2}} = TX = j$, si arcus $HT$ voce-
cetur $y$.

Sit $x = \frac{c^2}{z}$ unde $x = -\frac{c^2 z}{x} & \frac{x}{z} = -\frac{z}{x}$. Item erit $x^2 - c^2$:
$= \frac{c^4 - c^2 z^2}{z^2} = \frac{c^2 x^2 - z^2}{z^2} \cdot unde \sqrt{x^2 - c^2} = \frac{c x}{z}$
$\sqrt{c^2 - z^2}$: quibus valoribus substitutis, erit $\frac{b a x}{x \sqrt{x^2 - c^2}} =$
$= \frac{-b a z}{c \sqrt{c^2 - z^2}}$ Sit $a : c :: n : 1$. hoc est, sit $a = n c$, & sit $XY$ seu.

$y = -\frac{nb z}{\sqrt{c^2 - z^2}}$. Est vero $\sqrt{c^2 - z^2} ad \frac{c z}{\sqrt{c^2 - z^2}} ut n b ad$
c ; hoc est in ratione data: adeoque eorum fluuentes, si simul in-
cipiantur, erunt in eadem ratione, hoc est erit $HT$ seu $y$ ad flu-
entem quantitatis $\frac{c z}{\sqrt{c^2 - z^2}} ut n b ad c$.

Quod si centro $C$ radio $CV = c$ describatur circulus $VL$, & $CG$
fit = $z$, & $no = z$, sit arcus $mn = \frac{c z}{\sqrt{c^2 - z^2}}$ = fluxioni arcus
$\equiv m$, quando fluxio est quantitas positiva: sed quando est nega-

tiva, ejus fluxus est arcus $V_m$ prioris complementum. Arcus
enim ejusque complementum eandem habent quantitatem
fluxionem denotantem, diversis tantum signis affectam; quia
crescendo uno decrescit alter.

Hinc est $HT$ ad $V_m$ ut $n$ ad $c$: sed est $CV$ ad $CH$ ut $V_e$:

$$HT, 	ext{ hoc est } c : b :: V_e : \frac{b \times V_e}{c} = HT,$$

quare erit $\frac{b \times V_e}{c}$:

$$V_m : n : b : c,$$

unde $V_e : V_m : n : 1$.

Præterea ex natura circuli erit $C$ $G$ : $C V$ : $C V$ : $C T$,

quando $mT$ circulum tangit: hoc est erit $z : c :: c : \frac{c}{z}$

$CT = x$. Hinc si capiatur angulus $VC e$ ad angulum $VC m$ ut $n$
ad $1$, & producatur $C e$ ad $K$ ut sit $CK$ = secanti $CT$, erit
$K$ punctum in Curvâ quæsita.

Hic obiter notandum est, si $n$ sit numerus, hoc est, si sit $a$ ad $c$
vel $a$ ad $\sqrt{a^2 - b^2}$ ut numerus ad numerum, Curva $VI$ fiet
Algebraica: nam in hoc casu relatio $m G$ ad $\sinum$ anguli $VC e$
æquatione definitur, & inde habebitur relatio $\sinus$ anguli $VC e$
ad $CT$ vel $C K$ per æquationem determinatam, & inde demum
dabitur æquatio quam exprimet relationem inter ordinatam &
interceptam à puncto $c$ incipientem. Harum Curvarum ordi-
nes & gradus in Scala æquationum Algebraica diversi erunt
pro magnitudine numeri $n$. In his omnibus Curvis sic de-
scriptis Asympoti positio hac ratione determinatur: Fiat angu-
lus $VC L$ ad rectum angulum ut $n$ ad $1$. In eo angulo dis-
tantia corporis à centro evadit infinita. Jam quad. perpendicu-
laris in Tangentem $PC = \frac{a^2 x^3}{b^3 + x^3}$, ubi $x$ est infinita, sit $PC' =

\frac{a^2 x^3}{x^3}$, seu $PC = a$. Ducatur itaque $CR$ ad $CL$ perpendicular
ris & æqualis rectæ $a$, & si per $R$ ducatur $RS$ rectæ $CL$ parallela,
hæ Curvam tanget ad infinitam distantiam, seu erit Curvæ
Asympotos.

Si corpus in quavis harum Curvarum descendendo, ad Ap-
fidem imam pervenerit; Hinc rursus ascendet in infinitum,
& aliam Curvam priori similem, seu potius ejusdem Curvæ similem portionem, ascendentem describet.

Curvæ hæ possunt pluribus revolutionibus circa centrum torqueri, priusquam ad Asymptoton convergere incipient, & motus angularis rectæ C K erit æqualis totidem rectis quot numerus n constat Unitatibus. v. g. si n sit ico, perficientur viginti quinque integra revolutiones priusquam distantia à centro evadat infinita.

Acuto numero n, cadem manente a, minuitur c: est enim—

\[ a^2 - c = a^2 - b', \] 

unde fiet \[ n^2 - 1 \times a^2 = n^2 b'. \] 

Et proinde fiet a' : b' : : n^2 : n^2 - 1 ; adeoque si b' ad æqualitatem accedat ipsius a', perveniet quoque n^2 - 1 ad rationem æqualitatis cum n, & proinde augebitur n & in cadem ratione minueretur c. Ponatur itaque esse b' fere æquale ipsi a' ; adeo ut cum differentia sit infinite parva, sit n numerus infinite magnus, & radius circuli c sit infinite parvus, seu circulus in suum centrum contrahetur. At sic evanescente c, non pariter evanesceit C T, si angulus V C M sit propemodum rectus : nam in omni circulo, etiam minimo, secans anguli recti est quantitas infinita. Curva itaque hæc, ob n numerum infinitum, infinitis numero revolutionibus centrum ambit, priusquam ad Asymptoton convergere incipiet.

Evanescente autem c fit b = a & \( p = \frac{a x}{\sqrt{x^2 + a^2}} \). Et quoniam in omni calu est \( j = \frac{b a x}{x \sqrt{x^2 + a^2}} \), evanescente c fiet \( y = \frac{b a x}{x^2 + a^2} \), unde capiendo Fluides fiet \( y = \frac{b a}{x} \) feu \( x y = b a = data \) quantitati. Fig. IV.

Hæc Curva est Spiralis Hyperbolica, quæ plures habet notabiles proprietates. Si ducatur radius quilibet C T Curvæ occurrent in I, & peripheræ circuli in T, & ex C ad CI excitetur perpen-
perpendicularis \( CT \), atque \( IT \) tangat Curvam in \( I \), & rectae \( CT \) occurrat in \( T \). erit \( CT \) constans recta, æqualis seu. arcui-\( VE \); qua proprietate Logarithmicam æmulatur, cum \( CT \) Curvæ Subtangens dici potest. Sit enim Radius circuli \( CE = h \), arcus \( VE = a \), dicatur \( CI x & VT \) sit \( y \). Quia est \( b a = x \times y \) erit
\[
\frac{b}{x} = y & \frac{a}{x^2} = \frac{y}{x^2}
\]
Porro est \( CT : CI :: TX : NK \)
hoc est \( b : x :: \frac{a}{x} : NK \): quæ proinde est \( \frac{a}{x} \). Et quoniam

\[ IN : NK : CI : CT \] hoc est \( x : \frac{a}{x} : x : CT \), erit \( CT = a \),

Si centro \( C \), intervallo quovis \( CG \), describatur circuli arcus \( GF \), hic arcus inter rectam \( CV \) & curvam interceptus erit semper æqualis constanti rectæ \( CT \) vel \( a \). Nam quoniam est \( VL \times CF = CV \times VE \), erit \( VL : VE :: CV : CF :: VL : GF \)
unde æquantur \( VE \) & \( GF \). Si ad \( CG \) ex \( C \) excitetur normalis \( CR = VE \) vel \( FG \) vel \( a \), & per \( R \) agatur \( RS \) rectæ \( CV \) parallela, erit \( RS \) Curvæ Asymptotos. Nam est recta \( MS \) æqualis arci \( GF \), & proinde \( FS \) distantia Curvæ ab \( RS \) est semper æqualis excessui quo arcus superat suum finum: at cum distantia crescat in infinitum, excessus ille minuetur in infinitum, & fier tandem data quavis recta minor, & proinde \( RS \) erit Curvæ Asymptotos.

Sit jam \( b \) major quam \( a \); & similiter, ut in priore casu, invenietur \( KN = \frac{a}{x} \): at quoniam \( b \) superat \( a \), erit \( c \)

\[ = b^2 - a^2 \] quantitas positiva, & \( KN = \frac{a}{x} \) & ponendo

radium circuli \( HT = b \); invenietur \( XT = \frac{a}{x} \). Pona-
tur \( x = \frac{c^2}{z} \), & erit \( x = \frac{c^2}{z} \) & \( z = \frac{c^2}{x} \) & \( z = \frac{c^2}{x} \). Erit quoque \( x^2 = \)

\[ = c^2 \] & \( x^2 + c^2 = \frac{c^2}{x^2} + c = \frac{c^2 + c^2}{x^2} = \frac{c^2}{x^2} \times c^2 + x^2 \) unde

\[ \sqrt{x^2 + z^2} \]
\[
\sqrt{x^2 + c^2} = \frac{c}{x} \sqrt{c^2 + z^2}.
\]
His itaque valoribus substitutis fit
\[
\frac{b \alpha x}{x \sqrt{x^2 + c^2}} = - \frac{h a x}{c \sqrt{c^2 + z^2}} = -j.
\]
Nam tale sumi potest initi- tium arcus \(HT\), ut simul cum Flucte quantitatis \(\frac{-h a z}{c \sqrt{c^2 + z^2}}\) crescat & decrecat. Fiat \(nc = a\) & erit \(\frac{nh z}{\sqrt{c^2 + z^2}} = j\), &

\[
\frac{1}{2} nh z \sqrt{c^2 + z^2} = \frac{1}{2} h y = \text{sector} CXT.
\]

Eft autem \(\frac{1}{2} nh z \sqrt{c^2 + z^2} : \frac{1}{2} c^2 z \sqrt{c^2 + z^2} : : nh^2 : c^2\), hoc est in data ratione. Adeoque erit secto \(CXT\) ad \(\frac{1}{2} c^2 z \sqrt{c^2 + z^2}\) sem- per in data ratione. Harum itaque quantitatum fluentes erunt in eadem ratione, cum simul incipere ponantur. Fluo- ens autem securis \(CXT\) est secto \(CVT\), & fluens quantitatis \(\frac{1}{2} c^2 z \sqrt{c^2 + z^2}\) est secto Hyperbolae, quod sic ostenditur. \(\text{Fig. V.}\)

Centro \(C\) semiaxe transverso \(CV = c\) describatur Hyperbola æquilatera, & ex duobus punctis vicinis \(D\) & \(F\) ordinentur ad axem conjugatum rectæ \(DB\), \(EF\); ducantur item \(CD\), \(CF\). Et incrementum seu fluxio trianguli \(BCD\) æquale est \(\frac{1}{2} c^2 z \sqrt{c^2 + z^2}\) unde secto \(DCF\) (qui est Fluxio securis \(CVD\)) æqualis est \(\frac{1}{2} c^2 z \sqrt{c^2 + z^2}\) unde \(BD = \sqrt{c^2 + z^2}\), & \(BE \times BD = z \times \sqrt{c^2 + z^2}\). Triangulum autem \(BCD\) est \(\frac{1}{2} z \times \sqrt{c^2 + z^2}\), cujus fluxio est \(\frac{1}{2} z \times \sqrt{c^2 + z^2} + \frac{1}{2} z \times z^2 \sqrt{c^2 + z^2}\). Subtrahatur hæc quantitas ab \(z \times \sqrt{c^2 + z^2}\), & restabit secto Hyperbolae minimus \(CDF\).
Adeo quae fluens sectoris $C\,DF$ est æqualis fluenti quantitatis \( \frac{z^2}{c^2 + z^2} \). Proinde erit sector $CVD$ fluens quantitatis \( \frac{z^2}{c^2 + z^2} \).

Præterea $DT$ recta tangat Hyperbolam & occurrat Axi conjugato in $T$. Est ex natura Hyperbolæ $BC$:

\[ CV : CV : CT, \text{ hoc est } z : c :: \frac{c^2}{z} = CT = x. \]

Atque hinc oritur constructio quæ sequitur. Fig. VI.

Centro $C$ semiaxe transverso $CV$, describatur Hyperbola æquilatera $Vm$, item circulus $Ve$. Capiatur secto circularis $CVe$ ad sectorem Hyperbolical $C\,Vm$ ut $n$ ad $1$; tangat Hyperbolam in $m$ recta $T\,m$, occurrens Axi conjugato in $T$; producatur $C\,e$ ad $k$ ut sit $C\,k = C\,T$, & quoniam $k$ erit in Curva æquilata. Nemptalis est ea Curva, ut si $C\,k$ dicitur $x$, Perpendicularis à $C$ in tangentem ejus demissa erit semper æqualis \( \frac{a\,x}{\sqrt{b^2 + x}} \).

Quando $x$ est infinita evanescit $b^2$, & perpendicularis fit \( = a \), & tum coincidit $C\,R$ cum $CV$. Si itaque capiatur in axe conjugato $C\,R = a$, & ducatur $RS$ ipsi $CV$ parallela, erit hæ Curvæ Asymptotæs.

Si eo usque augeatur $a$ ut fiat quantitas $b^2 - a^2$ infinita parva, tum evanescet $c^2$, & quantitas \( \frac{b\,a \, x}{\sqrt{x^2 + c^2}} \) fit \( \frac{b\,a \, x}{a^2} = j \). Unde si capiantur harum quantitatum fluentes, habebimus \( x = y, \) & $b\,a = xy$, hoc est rectangulum sub arcu circulari & distantia Curvæ à centro erit semper data quantitas; atque hæ ratione migrabit curva in Spiralem Hyperbolical. Est itaque spiralis Hyperbolical Curva media seu quasi limes, inter eas Curvas quæ constructur per sectores circulares & eas quæ constructur per sectores Hyperbolicos. Itaque spiralis illa Hyperbolica conci-
Ad eum jam devenimus casum ubi velocitas corporis minor est eaque acquisitum cadendo ab infinita distantia, & ubi $p^2 = \frac{a^2 \cdot x^2}{b^2 - x^2}$.

Et hic similis ratiocinio ac in priori case, inveniatur $KN = \frac{a \cdot x}{\sqrt{b^2 - a^2} - x^2}$, ubi necesse est ut sit $b^2$ majus quam $a^2$. Hinc si $b^2 - a^2$ dicatur $c^2$, fit $KN = \frac{a \cdot x}{\sqrt{c^2 - x^2}}$; & proinde

$$XT \text{ seu } j = \frac{h \cdot a \cdot x}{x \cdot \sqrt{c^2 - x^2}}.$$ 

Sit jam $x = \frac{c^2}{z}$, & fieri $\frac{z}{x} = - \frac{z}{x} \text{ seu } \frac{h \cdot a \cdot z}{x} = - \frac{h \cdot a \cdot z}{x}$ &

$c^2 - x^2$ erit $\frac{c^2 \cdot x^2}{z^2} - c^2$, quibus valoribus substitutis fit

$$\frac{h \cdot a \cdot z}{x \cdot \sqrt{c^2 - x^2}} = \frac{h \cdot a \cdot x}{c \cdot \sqrt{z^2} - c^2} = - j.$$ 

Nam tale ponendum est

incipiat: unde erit $\frac{\frac{1}{2} h^2 \cdot a \cdot z}{c \cdot \sqrt{z^2} - c^2} = \frac{1}{2} \cdot h \cdot j = \text{sectori}$

$C \cdot XY = \frac{\frac{1}{2} n \cdot h^2 \cdot z}{\sqrt{z^2} - c^2}$, ponendo $n \cdot c = a$. Eft vero $\frac{\frac{1}{2} n \cdot h^2 \cdot z}{\sqrt{z^2} - c^2}$
ad $\frac{\frac{1}{2} c^2 \cdot z}{\sqrt{z^2} - c^2}$ ut $n \cdot b^2$ ad $c^2$, hoc est in ratione constanti. Qua
det harum quantitatum Fluentes sunt in eadem ratione, hoc est

Fluens quantitatis $\frac{1}{2} \cdot h \cdot j$ seu $\frac{\frac{1}{2} n \cdot h^2 \cdot z}{\sqrt{c^2 - x^2}}$ erit ad fluentem quanti-
tatis $\frac{\frac{1}{2} c^2 \cdot z}{\sqrt{x^2} - c^2}$ ut $n \cdot b^2$ ad $c^2$. Eft autem fluens quantitatis
\[ \frac{1}{2} b y = \sector \cdot CVX, \text{ & Fluens quantitatis} \frac{1}{2} c^2 \frac{z}{\sqrt{z^2 - c^2}} \text{ est sector Hyperbolae, quod sic ostenditur. Fig. VII.} \]

Centro \( C \) femiaxe transverso \( CV = c \) describatur Hyperbola æqualitera, \& ex duobus punctis infinite vicinis \( B \) \& \( D \) ad axem ordinantur duæ rectæ \( B E, DF \); ducantur item \( CB, CD \). Et erit Fluxio seu incrementum trianguli \( CBE = \) triangulo \( CBD + BE \times EF \); unde triangulum \( CBD \), seu sector minimus \( CBD \), erit = incremento trianguli \( CBE - BE \times EF \). Dicatur \( CE z \), \& erit \( BE = \sqrt{z^2 - c^2} \), \& \( BE \times EF = z \times \sqrt{z^2 - c^2} \). Est quoque triangulum \( CBE = \frac{1}{2} z \times \sqrt{z^2 - c^2} \), cujus Fluxio est \( \frac{1}{2} z \times \sqrt{z^2 - c^2} + \frac{1}{2} z \times \sqrt{z^2 - c^2} \); \& \( CE z \times \sqrt{z^2 - c^2} \); unde constat sectorem \( CBE \) esse fluentem quantitatis \( \frac{1}{2} c^2 \frac{z}{\sqrt{z^2 - c^2}} \). Præterea si \( BT \) tangens Hyperbolam Axi transverso occurrat in \( T \), \& ex natura Hyperbolæ fit \( CE : CV :: CV : CT \), hoc est \( z : c :: c : \frac{c^2}{z} = c T = x \). Fig. VIII.

Hinc deducimus sequentem constructionem. Centro \( c \)

Semiaxe transverso \( CV = c \), describatur Hyperbola æqualitera \( VB \), \& circulus \( C e G \) ex centro \( C \). Ad hyperbolam ducatur recta \( CB \), \& hyperbolæ Tangens \( BT \) axi transverso occurrat in \( T \). Capiatur circuli sector \( CV e \), qui fit ad sectorem Hyperbolicum \( CVB \) but \( n \) ad \( r \). In \( C e \) capiatur \( CK = C T \), \& erit \( K \) punctum in Curva quaestita, cujus perpendicularum est centro \( C \) ad Tangentem in \( K \) demissum, si \( CK \) dicatur \( x \), est æquale

\[ \frac{a x}{\sqrt{b^2 - x^2}} \]

Et
Et in hac Curva, urgent vi centripeta quæ fit reciproce ut cubus distantiae, movebitur corpus, si secundum directionem Tangentis cum juulta velocitate exeat. Qualis autem debet esse velocitas quæ faciat ut corpus harum Curvarum quamvis describat, sic invenitur.

Cum velocitas qua corpus in trajectoria quacunque movetur fit reciproce ut quantitas \( p \), assumendo constantem quamvis \( a \), ea semper exponi potest per \( \frac{a}{p} \). Et si ad Axem \( CV \) ordinentur rectæ quæ sint reciproce ut cubi distantiarum à centro, seu ut vires centripetæ, & hac ratione formetur Figura curvilinea, ejus Area indefinitely extensa semper exponi potest per \( \frac{b^2}{x^2} \), ut ex Quadraturis constat. At Area illa est ut quadratum velocitatis quæ acquiritur ab infinita distantia cadendo, adeoque velocitas hoc casu acquisita erit ut \( \frac{x}{b} \). Hinc si velocitas illa dicatur \( y \), et velocitas qua corpus in Trajectoria movetur dicatur \( v \), talesque assumantur quantitates \( a \) & \( b \), ut in una aliqua à centro distantia fit \( y : v :: \frac{b}{p} : \frac{a}{x} \), erit ubique in omnibus distantias \( y : v :: \frac{b}{x} : \frac{a}{p} : \frac{a}{b} \). Unde si \( y = v \), erit \( p = \frac{ax}{b} \), & Curva hac velocitate descripta erit Spiralis Nautica; vel Circulus existente \( p = x \) & \( a = b \).

Si \( y \) sit major quam \( v \), tunc \( p \) major erit quam \( \frac{ax}{b} \); eritque illa, ut ex precedentibus constat, \( = \frac{ax}{\sqrt{b^2 - x^2}} \). Curva autem constructur per sectorem Hyperbolicum, ut in ultimo casu ostensum fuit, ubi distantia corporis à centro per concursum Tangentis Hyperbolæ cum Axe transverso determinatur. Si
Si $y$ fit minor quam $v$, at in tantilla ratione ut maneat $b$ major quam $a$, Curva formabitur per eundem sectorem hyperbolicum. At distantia corporis à centro dehemitur ex concurfu Tangentis cum Axe conjugato.

Si fit $y : v : p : x$, erit in eo cafu $a = b$, & Curva evadit Spiralis Hyperbolica, ubi est $p = \frac{a \times \sqrt{a^2 + x^2}}{a^2 + x^2}$. Hinc fi de loco quovis projiciatur corpus secundum datam rectam, cum ea velocitate quæ fit ad velocitatem ab infinito cadendo acquisita, ut distantia corporis à centro ad perpendiculararem è centro ad lineam directionis demissam, movebitur illud corpus in Spirali Hyperbolica. Si denique sit $v$ tantò major quam $y$, ut fit etiam $a$ major quam $b$, Curva constructur per Sectores Circulares. Atque hac ratione datæ velocitatis semper determinata possit relatio quantitatum $a$ & $b$, ac proinde Curva descriptur in qua corpus cum illa velocitate movebitur: & viceversa data Curva, seu datis quantitatibus $a$ & $b$, inventur velocitas quam Curva illa describitur.

Omnium Curvarum Area (si cireulum excipias) quæ urgen te hac vi centripetæ describi possunt, sunt perfecte quadrables.

Nam primo, in spirali Logarithmica, quia est $p = \frac{a \times}{b}$, erit $KN = \frac{a \times}{\sqrt{b^2 - a^2} \times} = \frac{a \times}{c}$, ponendo $b^2 - a^2 = c^2$: *vid. Fig. II.* adeoque erit triangulum $C \times I = \frac{1}{2} \frac{a \times \times}{c}$ cujus Fluens est $\frac{a \times x^2}{4 c} = \text{Areæ Curvæ}$. 

Si $p$ fit $\frac{a \times}{\sqrt{b^2 + x^2}}$, & $a$ major quam $b$, ostensum est esse $KN = \frac{a \times}{\sqrt{x^2 - c^2}}$, unde $KN \times \frac{1}{2} CI = \frac{1}{2} \frac{a \times \times}{\sqrt{x^2 - c^2}}$ cujus Fluens est $\frac{1}{2} \frac{a \times \sqrt{x^2 - c^2}}{x^2} = \text{Areae Curvæ}$. At si $a$ minor sit quam $b$, sit $KN = \frac{a \times}{Q_a}$.
HO ScKNxi Cl = —£==, cujus Fluens est [a.

\[ \sqrt{x^2 + c^2} - Q = \text{Area Curvæ.} \]

Ponatur \( x = 0 \), & fier \( \frac{1}{2} ac - Q = 0 \), unde \( Q = \frac{1}{2} ac \), & Area Curvæ fit \( \frac{1}{2} a \sqrt{x^2 + c^2} = \frac{1}{2} a \).

In Spirali Hyperbolica evanescit quantitas \( c \), & Area Curvæ fit \( \frac{1}{2} a x \).

Si \( p \) fit \( \frac{a x}{\sqrt{b^2 - x^2}} \), ostendit esse \( KN = \frac{a x}{\sqrt{c^2 - x^2}} \), unde \( CI \times KN = \frac{a x}{\sqrt{c^2 - x^2}} \), cujus fluens est \( \frac{1}{2} a \sqrt{c^2 - x^2} \).

\( = \text{Area}. \) Fiat \( x = 0 \), & erit \( Q = \frac{1}{2} ac = 0 \), feu \( Q = \frac{1}{2} ac \); unde erit Area Curvæ femper æqualis \( \frac{1}{2} ac = \frac{1}{2} a \sqrt{c^2 - x^2} \). Fiat \( c^2 - x^2 = 0 \) feu \( c = x \), & Area curvæ fit \( \frac{1}{2} ac \). Unde (vid. Fig. VII.) erit Area semper æqualis \( \frac{1}{2} a \sqrt{c^2 - x^2} \).

De Areis quas describunt corpora radiis ad centrum ductis, urgente vi centripeta quæ fit reciproce ut distantiarum cubi, sequentia adnotavit Collega meus perissimus Geometriae Professor. Halleius. Nempe si corpora diversos circulos vel diversas Spirales Hyperbolicas hac legem describunt; erunt areæ sectorum, tam in Circulis quam in Spiralibus illis omnibus; æqualibus temporibus descriptæ, semper æquales: Nam velocitates corporum in circulis motorum secundum hanc legem, debent esse radiis seu distantis reciproce proportionales, adeoque arcus simul percurserunt quoque in eadem radiorum reciproca ratione, unde statim patebit sectores simul descriptos esse æquales.

In reliquis omnibus Curvis cum fit velocitas ad velocitatem corporis in eadem distantia in circulo moti ut

\[ \frac{a}{b} \times x \text{ ad } p, \]

(vid. Fig. III.) feu ut \( \frac{6}{x} \times KN \); interea dum corpus in Traiectoria.
III. Rules for correcting the usual Methods of computing Amounts and present Values, by Compound as well as Simple Interest; and of stating Interest Accounts. Offer'd to Consideration, by Thomas Watkins, Gent. F. R. S.

I. Of Compound Interest.

The Supposition whereon the Method of computing by Compound Interest is founded; viz. That all Interest Money, Rents, &c. are or may be constantly receiv'd, and put out again at Interest, the Moment they become due, without any Charge, or Trouble, being impracticable; therefore all Computations by this Method (except of Fee-Simples or other Perpetuities) must needs be erroneous. Thus for Instance, the Amount of a Sum of Money, or Annuity, for want of Deductions out of the Profits, for the unavoidable Trouble, Charge, and Delay in the Management, will be too great: and for the same reason, the present value of a Sum of Money payable in any time to come, will be too little; also the present value of an Annuity (being only the Amount of the difference between the Annuity, and Interest of the said present value) will be too much. But in long terms of Years, as that difference becomes less so does the Error, as the term is...
greater; and in Fee-Simples it vanishes; the contrary to which happens in Amounts of Sums of Money, and Annuities.

All which is propos'd to be rectify'd, only by a just reduction of the Rate, and Annuity; (which is done by deducting so much per Cent. thereof, as the whole Trouble, and Charge of Management is supposed to amount to, and reducing the Remainder, by a Discount equivalent to the supposed loss of time) and then by working with the Rate so reduced, for Sums of Money, and with the Rate and Annuity reduced in the like proportion for Annuitys, according to the common Method of Compound Interest; as follows. Put \( r \) for the rate of Interest of \( l.c \) for the Charge and Trouble of the Management of \( l. \) Then is \( r - cr = \) the Rate after deducting the said Charge, = (putting \( d \) for \( i - c \) ) \( dr \). And for the Discount put \( t \) for the time lost, that is for such part of the Period of time in which the Payments are made (whether Yearly, \( \frac{1}{4} \) Yearly, Quarterly, or otherwise) as is supposed to be spent in receiving and putting them out again at Interest. Then, \( dir \), being = the Interest of \( l. \) for that time; lay, as \( 1 + dir : 1 : : dr : \frac{r}{x + dir} = \) (putting \( e \) for \( 1 + dir \) )

\[
\frac{dr}{e}, \text{ which is equal to the reduced Rate, near enough for practice, for which put } t. \text{ But if the utmost accuracy be required, the Discount itself must be made with regard to the like loss of time, which is done by a Series of Discounts rais'd thus; } e(=1+tdr): tdr : : dr : \frac{t^2x{-}r^2}{e} : : \frac{t^3x{+}d^3r^3}{e} : \frac{t^4x{+}d^4r^4}{e} \&c.
\]

Whence \( dr = \frac{t^2x{-}r^2}{e} + \frac{t^3x{+}d^3r^3}{e} - \frac{t^4x{+}d^4r^4}{e} \&c. = \) (putting \( q \) for \( tdr \)) \( \frac{dr}{e}t - q + q^2 - q^3 \&c. \times dr = \frac{dr}{e}(=1-qxdr) + q^2 - q^3 \&c. \times dr \), is \( = t \), = the true Rate reduced. Put \( s = 1 + t \), \( n = \) the time, \( p = \) the present Sum or Value, \( m = \) the Amount.

Then
Then will \( r + \frac{dr}{e} + q - q' \) &c. \( \times dr \) \( \times p = r + \frac{dr}{e} \times p = p s^3 \),

be exactly \( m \) : But \( r + \frac{dr}{e} \times p = m \) is sufficient for practice.

And for the Amounts and present values of Annuities.

Put \( A = \text{Annuity per annum} \).

\( a = \text{Annuity yearly, quarterly, &c.} \)

\( R = \text{Yearly rate of Interest of} \ i \).

\( r = \text{Rate yearly, quarterly, &c.} \)

\( n = \text{Number of Years,} \ i \text{ years, quarters, &c.} \)

Then will \( \frac{r}{R} = \frac{a}{r} \) be \( \text{reduc'd Annuity taken yearly} \)

\( \frac{r}{R} = \text{yearly, quarterly, or otherwise} \); and by Compound Interest

\( \frac{r}{R} = \text{will be} \frac{1 + r^n}{r} \times \frac{r}{R} = \frac{1 + r^n}{r} A = \frac{s^3_r}{R} A = m \), and \( \frac{s^3_r}{R} A (\frac{m}{s^n}) = p \). Whence the

Theorems for solving all the other Cases are easily deduced.

And if the Rate be requir'd, when 'tis for a Sum of Money, the Solution is obvious : when for the Amount or Value of

an Annuity, since \( r + r^n = \frac{m r + a}{a - pr} = \frac{a}{a - pr} \) are the Equations whence Theorems for the Rate are usually deriv'd, which

by this Correction become \( r + r^n = \frac{m r + a}{a - pr} = \frac{a}{a - pr} \). That the same \( r \) may be had on both sides the Equation, put \( m r + a \)

for \( r \), and 'twill be \( m r + a = \frac{a}{a - pr} \) : then, by the Rate assum'd as near the truth as may be, find the value

of \( a (\frac{m}{a} + r = \frac{1}{a - pr}) \) and in any Theorem for the
Rate, putting $m$ for $m$, and $pu$ for $p$, the result will be the
Rate reduced nearly: and by repeated Operations correcting $t$ and thereby $n$, the true $t$, and thence $R$ the whole Rate, may
be found.

The only difficulty that remains, is the right assuming the
Quantities $c$ and $t$, the impossibility of doing which with
perfect Exactness, I suppose to be the reason why neither this,
nor any Method of Correction to the like purpose, has yet
been taken notice of by the Writers on this Subject; and what
may therefore be very likely to be objected to this. But the
same Objection I take to be of equal force against the Estimates
of any other Uncertainties whatsoever, as Estates for Lives,
Insurances &c.

First then, for the Quantity $c$, which is put for the Trouble
and Charge of Management, viz. of collecting and placing
out the Money on good Security, together with all
Contingencies attending the same, as travelling Charges, Ex-
pences, Attorney's Bills, &c. of all which, the principal Arti-
cle is the Charge of Collection or Receiver's Fees, which is
commonly a fix'd Rate, customarily allow'd in the Place, or
upon the Estate itself out of which the Purchase is made, if
it be a Rent; and for Interest Money, or any other Annuity,
the like Estimate is to be made, whether the Proprietor acts
for himself, or by another. Then for the Charge of placing
out the Money at Interest when receiv'd; though this be for
the most part defray'd by the Borrower, yet because it high-
ly concerns the Lender to see it be securely done, there are
usual Allowances made to Agents and Scriveners, to encourage
their Care and Fidelity therein; besides the Time, Expense
and Trouble of the Proprietor himself, in finishing Contrac-
ts, inspecting Securities &c. and whatever is sav'd in this Arti-
cle, we must suppose to be fully made up by an Equivalent
Degree of Risque in the Security.

In the next place, for the Loss of time; though 'tis also im-
possible for this to be exactly ascertain'd, nor perhaps so
nearly as the former, since it depends very much on the Dilig-
ence of the Manager: Yet if the usual times of Payment of
of the particular Rent or Annuity to be purchased, with a moderate
degree of diligence in the Manager, and the usual indulgence
practis'd by Men of Business in this Case towards one another
be observ'd, a reasonable Estimate may be also made of the loss
of time; in which 'tis to be noted, that Interest Money being
usually paid in small Sums, when any Sum of Money to be
made up of several such Payments, is intended to be put out
at Interest, the whole must lie dead till the last Payment be
made; also that the Principal lies dead sometimes as well as
the Interest; and that on the other hand, to save time, Bor-
rowers may be found out, and treated with during the time of
Collection; but this Advantage is in a great Measure lost by
the difficulty of fixing the time or Quantum of a Loan, till the
whole be paid in. Note also, that if the Charge of Collect-
on, or Loss of time, on the Rent, or Annuity, of any par-
ticular Estate or Place, be found to differ from that of the In-
terest of the Purchase Money, and so much exactness be re-
quired, as that the Computation be made with regard to such
difference: It must be done, either by assuming a proper
Medium for both, or more accurately thus: For the reduc'd
Rate of the \( \frac{\text{Annuity}}{\text{Interest-Mony}} \) put \( \frac{2}{r} \). Then \( r : q : : a : a \frac{2}{r} \);
and \( r : \frac{1 + q^n - 1}{a} : \frac{1 + q^n - 1}{a} \times \frac{2}{r} = m = \frac{1 + q^n}{1 + q^n \times q} \).

To give a Specimen of this Method in Numbers, first the
Quantities \( c \) and \( q \) are to be assum'd, which are not here to be
accommodated to any particular Place or Estate, but to be ta-
ten in general: And first for \( c \) the Charge of Management;
the usual Rates of Collectors Fees in these Kingdoms, as I am
inform'd, are 6d. 12d. and 18d. per Pound, but the most usual
12d. which is 5 per Cent. However to be within compass, I
shall take 4 per Cent. for the Medium, including what further
 trou-
trouble and charge may attend the Receipt of the Mony, besides Receivers Fees; and 2 per Cent. for all the other Charges before mention'd, in placing it out at Interest, both which make 6 per Cent. so that e is = 0.06, and d (= 1 - e) = 0.94. Next for t the Loss of Time; since few Annuities are paid yearly, and a Discount being given for the Loss of Time, we are to lose no more than is discounted for; therefore I choose Half-yearly Payments for the Examples, being the most usual, which with little Alteration may serve for quarterly; and considering the before-mention'd Circumstances relating to the Time, I look upon two Months the least, and seven or eight Months the most, that can well be suppos'd to be spent, one Time with another, in receiving and putting out the Mony, upon a moderate Management; between which the Medium is about four Months and half, which being 1/2 of a Year, gives

\[ \frac{d r}{e} = t \] and if \( \frac{d}{e} \) be = \( r \), the yearly Rates of 4, 5, 6, 8 and 10 per Cent. will produce for Half-yearly Rates reduce'd of 1 l.

0.018539, 0.023093, 0.027616, 0.036569 and 0.0454 each = \( \frac{d r}{e} \). But if \( \frac{d}{e} \) be \( \frac{1}{2} + q - q \) &c. \( \times d r \), it will be, 0.01854219,

0.0230999155, 0.027627775, 0.036596289 and 0.04545235, each = \( r \), (so that each Rate loses by this Estimate about \( \frac{1}{2} \) Part.). Whence the following Amounts, and present values of 1 l. per Annum computed Half-yearly, are produced, and compared with those of the usual Method computed yearly, to agree with the common Tables.
## Notes

**Notes:**

*Note: That this Correction is also applicable to the Valuation of Estates for Lives, in which the first Step being to find an Equivalent in Years of Continuance, brings them to the Case of Estates for Years.*

### Table 1: Amounts of 1 l. per An. at 5 per Cent. by Comp. Int. Co. Int. cor.

<table>
<thead>
<tr>
<th></th>
<th>Amounts of 1 l. per An. at 5 per Cent. by</th>
<th>Differences</th>
<th>Amounts of 1 l. per An. at 6 per Cent. by</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Comp.</td>
<td>Int.</td>
<td>Co. Int. cor.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5,52563</td>
<td>5,15104</td>
<td></td>
<td>39459</td>
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<tr>
<td>10</td>
<td>12,57789</td>
<td>11,57846</td>
<td></td>
<td>99943</td>
</tr>
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<td>20</td>
<td>33,66595</td>
<td>29,85996</td>
<td></td>
<td>3,20599</td>
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<td>66,43855</td>
<td>58,72546</td>
<td></td>
<td>7,71339</td>
</tr>
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<td>104,30079</td>
<td></td>
<td>16,49898</td>
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### Table 2: PROFITS.

<table>
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<tr>
<th></th>
<th>Prof. Values of 1 l. per An. at 5 per Cent. by</th>
<th>Differences</th>
<th>Prof. Values of 1 l. per An. at 6 per Cent. by</th>
<th>Differences</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Comp.</td>
<td>Int.</td>
<td>Co. Int. cor.</td>
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<td>33860</td>
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<td>10,37966</td>
<td>9,91935</td>
<td></td>
<td>46031</td>
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<td>20</td>
<td>12,46221</td>
<td>11,97753</td>
<td></td>
<td>48468</td>
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<td>16,78200</td>
<td></td>
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<td>50</td>
<td>18,55593</td>
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<tr>
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<td>19,18247</td>
<td></td>
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<tr>
<td>F.S.</td>
<td>20,00000</td>
<td>20,00000</td>
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</table>
II. Of Simple-Interest.

The absurdity of the Supposition, on which the usual Method of computing present Values by Simple-Interest, is founded, viz. That the Rent or Annuity is constantly received, and put out again at Interest, as it becomes due; but that the Interest of the Purchase Money lies dead during the whole Term, is so apparent, and the Errors arising from it so gross, that the Writers who have laid down this Method, have at the same time caution'd against the Use of it for any more than 6 or 7 Years, the Error for that time being not considerable.

The same Supposition does also occasion the miscomputation of Amounts, or rather the misapplication of them to their proper Cases. Wherefore, since the Simple-Interest of Money is of equal Value pro rata, and of the same regard with Rents or Annuities, being each the Original Profits issuing alike from a principal Stock, Estate or Value, and equally improveable; This general Rule may serve for a just Correction of this Method, viz. That supposing in any Case an Interest ought to be, or not to be allowed to either of those Profits, the same be done in the like Case to the other. Thus, in the Case of Debts, or Amounts of Sums of Money, Rents, or Annuities for the time past; its usual in Practice to allow no Interest to either: For though the Law, to curb the exorbitant Avarice of Usurers, and for other Reasons, does more expressly disallow Interest upon Interest for a Debt; our Courts both of Law and Equity, as I am inform'd, will be as far from allowing the Charge of Interest against a Tenant for Rent in Arrear; except on a nomine pente (which is now become almost obsolete) so that in this Case (putting \( a \) and \( r \) for the Annuity and Rate yearly, Half-yearly, or otherwise) as \( prn + p \) is the Amount of a Sum of Money, so is \( an \) the Amount of an Annuity or Rent in Arrear, and not \( \frac{n-1}{2} r + 1 \times \text{ar} \) as Arithmeticians commonly make it. But in the Computation of present Values, or Amounts for the time to come,
come, the same being made on the Expectation of a constant regular Income of the Profits, without any extraordinary Interruption, an Interest ought to be allow'd to both, especially in present Values, which are found by setting the Amounts of both against each other: so that in these Cases, putting \( x = n - 1 \), if \( \frac{1}{2} xr + 1 \times a n \), be made the Amount of a Rent or Annuity; then \( \frac{1}{2} xr + 1 \times prn + p \), will be the proper Amount of a Sum of Money, and not \( prn + p \): and consequently \( \frac{1}{2} xr + 1 \times an \) will be the present Value of a Rent or Annuity, the subsequent Interest being remitted on both sides in lieu of the Loss of Time and Charge of Management; which such as are apt to depreciate long Futurities, may think the properest Method of approaching the true Value; but I rather look upon the former Method of Compound Interest corrected as more exact, as well as more general, the Interest remitted in this, being in short Terms less, and in long Terms more than an Equivalent for the Trouble, Charge and Delay in the Management. But it is however the most exact of any of the Methods, that have yet been deduc'd from Simple-Interest. The reduc'd Rate may also in some Cases be properly made use of for Amounts, but not for present Values, except for short Terms, and then, since \( r' = a' = \frac{1}{2} xr + 1 \times t n \):
\[
\frac{1}{2} xr + 1 \times an \quad \frac{1}{r} \quad \text{twill be} \quad \frac{1}{2} xr + 1 \times an \frac{1}{r} = m = \frac{1}{2} xr + 1 \times \frac{p \times \text{tn} + p}{r} \quad \text{and} \quad \frac{1}{2} xr + 1 \times \frac{1}{n} \times \frac{1}{r} = p.
\]
Examples of this Method compar'd with those of the former, will stand as follows; in which all is computed Half-yearly, except the last Column of Compound Interest.

Amounts of 1 l. at 5 per Cent. computed 6 several Ways.

<table>
<thead>
<tr>
<th>Years</th>
<th>Simple Int.</th>
<th>Id. for Bonds</th>
<th>Sim. Int. cor.</th>
<th>Co. Int. cor.</th>
<th>Comp. Int.</th>
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<tbody>
<tr>
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<td>1.25</td>
<td>1,25813</td>
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<tr>
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<td>2.2</td>
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<td>2,34021</td>
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<td>2.5</td>
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<td>6.6</td>
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<td>31,19026</td>
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</tbody>
</table>

Amounts of 1 l. at 6 per Cent. by the same Theorems.

<table>
<thead>
<tr>
<th>Years</th>
<th>Simple Int.</th>
<th>Id. for Bonds</th>
<th>Sim. Int. cor.</th>
<th>Co. Int. cor.</th>
<th>Comp. Int.</th>
</tr>
</thead>
<tbody>
<tr>
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<td>7.7</td>
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</tbody>
</table>

Amounts of 1 l. at 10 per Cent. by the same Theorems.

<table>
<thead>
<tr>
<th>Years</th>
<th>Simple Int.</th>
<th>Id. for Bonds</th>
<th>Sim. Int. cor.</th>
<th>Co. Int. cor.</th>
<th>Comp. Int.</th>
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<td>9.9</td>
<td>49,8000</td>
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<td>100</td>
<td>11.1</td>
<td>11.1</td>
<td>60,7500</td>
<td>51,20222</td>
<td>72,585398</td>
</tr>
</tbody>
</table>

Years
The Theorems to the preceding Columns of Amounts (of which the fourth and fifth are infinitely variable in the Result, by assuming \( c \) and \( t \) in the reduc'd Rates at Pleasure) may serve to answer all the simple Cases of Amounts that occur in Business: To instance in some,

1. The first Column contains the Amounts of such Debts, or Sums of Money as carry a simple Interest till the Principal be paid.

2. The second Column answers the common Case of Debts due by Bond, that by Law are allow'd not to exceed the Penalty, which is generally double the principal Debt.

3. The third Column answers the Case of a Security or joint Obligor, that has duly and constantly paid the Interest, and at last the principal Sum of a Debt, from which he has a Counter-Bond from the principal Debtor to save him harmless, against whom he may make his Charge from this Column.

4. In case the Parties shall agree that the Debt shall lie for any time certain, or uncertain; and for the much greater Case, advantage and satisfaction of both of them, no Interest to be call'd for, till the Principal itself is paid; but to carry Interest as it becomes due, the Lender allowing for the time and charge he must have been at, in receiving and putting out his Interest, the fourth Column will fit this Case, or else the fifth.
as a greater or less degree of lenity is agreed upon in favour of the Borrower.

5. The fifth Column is also proper in the following Case, viz. if it be demanded, what Estate in reversion, after a certain number of Years, any Sum in Hand will Purchase; the first step being to find the Amount of that Sum to the time the Reversion commences, its had in this Column.

6. The last Column gives the Amount of a Sum of Money, according to the common Method of Compound Interest, but being computed with that extraordinary rigor as has been said, (except some small allowance for the loss of time, by being done Yearly) 'tis hardly suitable to any Case.

Other Cases might be enumerated, to which the foregoing Theorems might be equitably apply'd; besides such extraordinary ones, wherein it may appear to Arbitrators, or a Court of Equity, that either Partie deserves Favour, either by way of Compensation for Injuries suffer'd from the other, by means of any fraudulent or opprobrious Practices, or otherwise, for which no other redress is provided.

III. Of Interest Accounts.

The Inequality of the usual Method of stating Interest Accounts, as practis'd in our Courts of Equity, will best appear by an Example, for which I shall take the common general Case of an Interest Account to be stated on a Mortgage, viz. suppose 1000 l. to be let out at 6 per Cent. on a Mortgage of 120 l. per annum, payable Half Yearly, and the Mortgagee after five Years end, to have Possession till the Arrear of Interest, accruing Interest and Principal be discharged: Quære, How long that will be? supposing also, for the sake of brevity in the Account, the Payments to be equally, and punctually made as they become due. By the Chancery Method, the Rent is first apply'd to discharge the Arrear of Interest; and then the remainder of every Half-Year's Rent, after deducting the same Half-Year's Interest, is apply'd towards the Discharge of
of the Principal, and thereby the Principal and Interest constantly lessens, till the whole be paid off. Now by this means the Mortgagee, after the Arrear is discharged, pays Comp. Interest, with the utmost rigour, for so much per annum of the Rent, as exceeds the Interest of the whole Principal Money, and receives but Simple Interest for his Debt; which, however strange it may seem, is easily prov'd, by applying the proper Theorems of Simple and Compound Interest to this Case, in which the Annuity, Principal Money, Rate and Arrear of Interest are given, and the time require'd; the result being the same with that of the Chancery Method, except a very small difference only when any part of the time is express'd by a Fraction: viz. putting \( L \) for Logarithm, \( \alpha = a - p r = 30, s = 1 + r = 1.03, \)
\[ t = \text{time of contracting the Arrear} = 10 \text{ Half-years}, n = \text{any number of \( \frac{1}{2} \) Years spent in discharging the whole or any part}, \]
\[ N = \text{number of Years required}; \]
the Equation for the Arrear will be \( ptr + prn = au \); and for the Principal and accruing Interest \( prn + p = prn + \frac{s^n - 1}{r} \times \alpha. \)

Whence
\[ ptr + \frac{La - L \alpha}{2 \alpha} + \frac{s^n - 1}{2Ls} = N = 16,7249 \text{ Years} = \text{the time demanded}; \]

i.e. \[ \frac{ptr}{2 \alpha} = 5 \text{ Years} = \text{the time of discharging the Arrear, and}; \]
\[ \frac{La - L \alpha}{2Ls} = 11,7249 \text{ years, = the time in which the Principal and accruing Interest is discharged}; \]
during which it is evident, the Mortgagee pays full Compound Interest for 60 l. per annum of the Rent. For the Correction of which inequality, in the first Place, to the end that neither Branch may exceed, or be depriv'd of its due Profits; This general Rule is propos'd as necessary to be always observ'd, viz. That Amounts of the Produce on each side be stated separately, and set against each other in the Account, in order to a Balance. And in the common Cases of Mortgages, Government, and Stock-
Stock Securities, &c. where the Debt is paid off by a Rent, Annuity, Pension, Dividend, or other Payments issuing in the same manner, and with the like trouble, charge, &c. as Interest Money does; I presume this Rule will also be easily admitted, viz. That the same equitable Advantage be impartially allow'd on both sides; for which the Method of Simple Interest, as corrected under the foregoing Head, seems truly adapted; whereby the Original Profits on each side are supposed to be deem'd, either as Interest, or else as Principal Money; and since the Amounts both of an Annuity and Sum of Money, for the time past, as there stated, on the first of these Suppositions (t being there = o) are likewise vouch'd by our Laws, and the practice of our Courts, to be good when separately us'd; I think it's very evident, that the Account ought to be stated by setting those Amounts against each other thus,

\[ p = p \cdot r \cdot n + p = a \cdot n \]  (whence \( \frac{1 + tr}{2} = p = N, = 21,6666 \) Years) and that this Method is most proper for general Use, in the Cases mention'd; Unless it should be thought fit, in consideration of the various Ways found out for the ready improvement of Money, to allow a further Advantage on both sides, by charging the Original Profits as Principal Money, and giving a Simple Interest there to, which still falls short of the Advantage allow'd to Rents by the Chancery Method. And this is to be done two ways, viz. either by applying an Amount of Rent to pay off the Arrear first, and afterwards another Amount of Rent to discharge the Principal, and accruing Interest; or else by letting the Profits with all Arrears and other Charges run on at Simple Interest on each side, till the end of the Term: viz. putting \( x = n - 1, y = t - 1, a = a - p r, f = 2 - r \times a, g = f - 2 p \cdot r^2, \mu = yr + 1 \times p r t = \text{Arrear of Interest} ; \) by the first of these 'twill be, for the Arrear, \( \mu + p \cdot r^2 n = \frac{1}{2} x r + 1 \times a n, \) and for the Principal and accruing Interest, \( \frac{1}{2} x r + 1 \times a n = p ; \) Whence
\[ \frac{8\mu ar + g^2}{4 ar} - \frac{\sqrt{8par + f^2 - f + g}}{4ar} = N = 18,7653 \text{ Years.} \]

By the other way it will be, for the whole, \( p + \frac{\mu + p \times 8ar + g^2 - g}{4ar} = N = 18,1648 \text{ Years.} \) The Lender will have to allege for the first of these two Ways, that as the Rent is not hindered by any other parallel Charge from making the utmost produce it can, so for that reason ought his Principal Money to have the Advantage of the Arrears being first discharged, which also agrees with the Chancery Method in this particular.

Lastly, another way of stating this Account, may be taken from that Notion of Simple-Interest, whereby the Annuity only is charged as Principal Money, and then it will be, for the Arrear, \( pr + prn = an \), and for the Principal and accruing Interest, \( \frac{1}{2} xr + 1 \times an = p; \) Whence \( \frac{\sqrt{8par + f^2 - g}}{4ar} \)(= \( \frac{p + \frac{\mu + p \times 8ar + g^2 - g}{4ar}}{2a} = N = 17,3072 \text{ Years; which appears to be same with the Chancery Method, only that the Compound Interest in that, is turned into Simple in this; and as it still retains part of the same inequality, to the Advantage of the Borrower, it seems only fit to be observed in such Cases wherein the Borrower may be thought to merit favour, as when the Debt is paid out of the Profits of Trade, arising by extraordinary Risque or Industry. But since such a Rule of distinction is hardly possible to be reduced to general practice, the use of this Theorem seems restrained to such Cases only, wherein the Parties themselves, or a Court of Equity shall think it reasonable.

For a further illustration of these Rules, the following Specimen is added, which shews at first, how the Results of the several Methods differ, as the Rent, Arrear; or Rate of Interest, is greater or less, and consequently of how much more
or less concern it is to the Parties, as well as to the due Administration of Justice, to have regard thereto.

The time requir'd in

To discharge a Mort. of 1000 l. by a Rent of 12 l. per annum.

<table>
<thead>
<tr>
<th>Years (computed Half-yearly)</th>
<th>5 per C.</th>
<th>6 per C.</th>
<th>90 l. per annum</th>
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<td>No Arrear of Interest.</td>
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</tr>
<tr>
<td>By the Chancery Method</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>By the same turning the Comp. Int. into Sim.</td>
<td></td>
<td></td>
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<tr>
<td>By Simp.Int.cor.</td>
<td>Princ. and J cont. at Int.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>the Original Pro.</td>
<td>the Arrear</td>
<td>first discharged</td>
<td></td>
</tr>
<tr>
<td>fits charged as</td>
<td>Interest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Years Arrear of Interest.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>By the Chancery Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>By the same turning the Comp. Int. into Sim.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>By Simp.Int.cor.</td>
<td>Princ. and J cont. at Int.</td>
<td></td>
<td></td>
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<tr>
<td>the Original Pro.</td>
<td>the Arrear</td>
<td>first discharged</td>
<td></td>
</tr>
<tr>
<td>fits charged as</td>
<td>Interest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Years Arrear of Interest.</td>
<td></td>
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<td></td>
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<tr>
<td>By the Chancery Method</td>
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<td></td>
</tr>
<tr>
<td>By the same turning the Comp. Int. into Sim.</td>
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<tr>
<td>By Simp.Int.cor.</td>
<td>Princ. and J cont. at Int.</td>
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<tr>
<td>the Original Pro.</td>
<td>the Arrear</td>
<td>first discharged</td>
<td></td>
</tr>
<tr>
<td>fits charged as</td>
<td>Interest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 Years Arrear of Interest.</td>
<td></td>
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<td></td>
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</table>

I must also observe for the sake of such as are unacquainted with Specious Arithmetic, that though for brevity's sake, the foregoing Theorems and Examples are laid down, and wrought in Algebraic Terms: Yet the same Accounts may be stated after the Chancery manner it self, according to the several Principles before deliver'd, and with the same Results, with this only caution, that (instead of a continual deduction of the Rent or Annuity out of the Principal and Interest of the Debt, which occasions the Error before mention'd) the preceding General Rule of stating separate Amounts be observ'd, which may be done by continually adding the profits together on each side, in the same manner, as if the Parties were
make a separate Charge against each other, which is the rather to be noted, as being the only Course that can be taken, in case the Sums or Times of payment should differ, but the respective Results will notwithstanding be analogous to the above Examples.

All which is submitted to the consideration of more discerning Judgments, especially the Applications of the Rules to particular Cases, for exemplifying the Theorems. But if any of those Rules or Theorems should be objected against, merely because they tend to introduce some Alterations in the present practice; I shall for answer only add, with submission, to what is before said, that in former Ages, when our Laws relating to these matters had their rise, (the Profits of England arising chiefly from Husbandry and Tillage, and little from Trade,) the Cash of the Kingdom was but low, the Rates of Interest very high to the Advantage of Usurers, and those ways for the ready Improvement of Money accommodated to all Peoples use, not known; (much like to which we are told was the State of the Jewish Affairs, when they were forbidden to take Usury of any but Strangers.) But latter Ages have produc'd vast Alterations in all these Respects, which having happen'd by insensible degrees, may be one reason why neither our Legislature, nor Courts of Judicature have yet taken such notice thereof, as time and leisure, with the Tender of proper and practical Methods of Computation, may hereafter induce them to do.

FINIS.
<table>
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<tr>
<td>$n + 1 + p$</td>
<td>Y</td>
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<tr>
<td>$n + 1 + 2p$</td>
<td>X</td>
</tr>
<tr>
<td>$n + 1 + 3p$</td>
<td>V</td>
</tr>
</tbody>
</table>

**N.² 2**

\[
Z = \frac{1}{2} x H - p + \frac{1}{2} x H - p + hp + \frac{1}{8} x H - p + 2hp + \frac{1}{16} x H - p + 3hp + \ldots
\]

\[
Y = \frac{1}{2} x K - \frac{1}{2} x H - p + hp + \frac{1}{8} x H - p + 2hp + \frac{1}{16} x H - p + 3hp + \ldots
\]

\[
X = \frac{1}{2} x L - \frac{1}{2} x H - p + 3hp + \frac{1}{8} x H - p + 4hp + \frac{1}{16} x H - p + 5hp + \ldots
\]

\[
V = \frac{1}{2} x y - \frac{1}{2} x H - p + 6hp + \frac{1}{8} x H - p + 7hp + \frac{1}{16} x H - p + 8hp + \ldots
\]

**N.³ 3**

\[
H = \frac{1}{2} x n - c - ncp - cp + \frac{1}{2} x D + ndp - dp + \frac{1}{2} x F + ncp + dp - 3fp - 3cp + 2n - 4 \times F + nfp - 4fp + \ldots
\]

**N.⁴ 4**

\[
X - Z = \frac{1}{2} x K - \frac{1}{2} x H + \frac{1}{2} hp + \frac{1}{8} hp + \ldots
\]

\[
X - Y = \frac{1}{2} x L - \frac{1}{2} x K + \frac{1}{2} hp + \frac{1}{8} hp + \ldots
\]

\[
V - X = \frac{1}{2} x M - \frac{1}{2} x L + \frac{1}{2} hp + \frac{1}{8} hp + \ldots
\]

**N.⁵ 5**

\[
K - H = \frac{1}{2} x n - c - ncp - cp + \frac{1}{2} x D + ndp - dp + \frac{1}{2} x F + ncp + dp - 3fp - 3cp + 2n - 4 \times F + nfp - 4fp + \ldots
\]

**N.⁶ 6**

\[
C - A = Y - Z = \frac{1}{2} x n - c - ncp + dp + \frac{1}{2} x D + ndp + \frac{1}{2} x F + ncp + dp - 3fp - 3cp + 2n - 4 \times F + nfp - 4fp + \ldots
\]

**N.⁷ 7**

\[
L - K = \frac{1}{2} x n - c - ncp + dp + \frac{1}{2} x D + ndp + \frac{1}{2} x F + ncp + dp - 3fp - 3cp + 2n - 4 \times F + nfp - 4fp + \ldots
\]

**N.⁸ 8**

\[
M - L = \frac{1}{2} x n - c - ncp + dp + \frac{1}{2} x D + ndp + \frac{1}{2} x F + ncp + dp - 3fp - 3cp + 2n - 4 \times F + nfp - 4fp + \ldots
\]

**N.⁹ 9**

\[
Z = A + ap x 2^n - nc.
\]

\[
Y = C + dp x 2^n - nc.
\]

\[
X = 2D - C - ap.
\]

\[
V = 4E - 2D - C - 2dp - 2cp.
\]

**N.¹ 10**

\[
C - A = Y - Z = \frac{1}{2} x n - c - ncp + dp + \frac{1}{2} x D + ndp + \frac{1}{2} x F + ncp + dp - 3fp - 3cp + 2n - 4 \times F + nfp - 4fp + \ldots
\]

**N.¹ 11**

\[
C = A \times 2^n + ap \times 2^n - nc.
\]

\[
D = C \times 2^n + cp \times 2^n - 2n - 1 - ndp.
\]

\[
E = D \times 2^n + dp \times 2^n - 2n - 2 + ap \times 2^n - 2n - 2 - nc.
\]

**N.¹ 12**

\[
D = A + ap x 2^n - nc.
\]

\[
E = D + dp x 2^n - nc.
\]

I. Senex sculp.
PHILOSOPHICAL
TRANSACTIONS.

For the Months of October, November and December, 1714.

The CONTENTS.

I. An Account of the Rain which fell every Year at Upminster in Essex, for the last 18 Years: With Remarks upon that of the last Year, 1714. By Mr. W. Derham, Rector of Upminster, and Fellow of the Royal Society. Also a Comparison of what has been observed of the same kind at Paris, by M. De la Hire.


IV. An Account of several extraordinary Meteors or Lights seen in the Sky. By Dr. Edmund Halley, Professor of Geometry at Oxon, and Secretary to the Royal Society.

V. Some Remarks on the Variations of the Magnetical Compasses, published in the Memoirs of the Royal Academy of Sciences, with regard to the General Chart of those Variations made by Dr. Halley. As also concerning the true Geographical Longitude of the Magellan Straights. By the same.
I. An Account of the Rain which fell every Year at Upminister in Essex, the last Eighteen Years, with Remarks upon that of the Year 1714. By W. Derham, F. R. S. Also a Comparison of what has been observed of that kind at Paris, by M. De la Hire.

A S T Year having been so remarkably Dry, that Fonds hereabouts are for the most part dry, and the Springs generally either very low or quite failing, I had the Curiosity to make an Extract (out of my Registers of the Weather, &c.) of the Quantity of Rain which fell at Upminister the last 18 Years. The Particulars of which, every Year, may be seen in the following Table. In one Column of which, the Weight of the Rain in Pounds Troy and Centesimals of Pounds, may be seen; in the other, the Depth of it in Inches and Centesimals of Inches, or what Height it would have been, had it not been imbibed by the Earth, or lessened by Exhalations, but been suffered to have stagnated on the Ground.

Among the Dry Years, 1704 was complained of for one; which I remember the News-Papers reported to have been so considerable at Venice, that they were forced to fetch their Water in Barks five Leagues off, as far as the Brenta; so that publick Prayers were put up for Rain. Yet we may observe that several other Years were drier than that with us at Upminister. But among them all, none comparable to the last Year 1714. In which the whole Quantity of Rain was no more than 55 l 95 Hundredths, or 11 Inches 19 Hundredths; whereas the least Quantity of any of the preceding 18 Years, exceeded 15 Inches in Depth.

What Effects this Drought hath had in the Bodies of Animals, I leave others to judge. It is well known how contagious and fatal a Distemper hath raged among, not only
only our own Black Cattle, but in many other Parts of Europe. And I observed the Itch was epidemic among the poorer sort, at the beginning of the Year; that the Measles were very common, some Parts of the Year; and that Pleurises and Malignant Fevers infested a great many, especially in the Summer Months. But how far these Distempers might be owing to the Dry Season, I leave to the Judgment of our learned Physicians.

With greatest Duty and Respect I am the Society's

Most humble Servant, &c.

---

A TABLE of Rain which fell at Upminster, from the Year 1697, to the Year 1714.

<table>
<thead>
<tr>
<th>Year</th>
<th>Weight</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>l. Cent.</td>
<td>inch. Cent.</td>
</tr>
<tr>
<td>1697</td>
<td>77 60</td>
<td>15 52</td>
</tr>
<tr>
<td>1698</td>
<td>122 32</td>
<td>24 46</td>
</tr>
<tr>
<td>1699</td>
<td>75 54</td>
<td>15 11</td>
</tr>
<tr>
<td>1700</td>
<td>95 13</td>
<td>19 03</td>
</tr>
<tr>
<td>1701</td>
<td>93 45</td>
<td>18 69</td>
</tr>
<tr>
<td>1702</td>
<td>101 89</td>
<td>20 38</td>
</tr>
<tr>
<td>1703</td>
<td>119 94</td>
<td>23 99</td>
</tr>
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<td>79 02</td>
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<tr>
<td>1705</td>
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</tr>
<tr>
<td>1706</td>
<td>121 43</td>
<td>24 29</td>
</tr>
<tr>
<td>1707</td>
<td>81 55</td>
<td>16 31</td>
</tr>
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<tr>
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To compare with these, we have collected out of the **Memoirs** of the **Royal Academy of Sciences**, the **Quantity of Rain** and **Dissolved Snow** which has fallen at the **Observatory at Paris** for 23 Years together; according to the accurate Observation of M. De la Hire. And that the Comparison might be made more justly, we have reduced the French Measure to our own. But it is to be observed that the Diversity of Stile makes the Years not exactly the same, though, as to this Matter, the difference may seem very inconsiderable. We have forborne to make any Remarks upon this Comparison, leaving it to the Consideration of the curious Reader.

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II. *Solutio*

CUM Methodus synthetica, quâ usus est D. de Moivre ad inveniendam cujusque Collusoris forum, in usum verti nequeat tune quando plures quam tres sunt collusores, obt in perspicientiam legem progressionis serierum quae se offerunt; ostendam hic quo modo Analysis in ejusmodi Problematibus, ubi depositum continuo augetur, adhiceri quae: cumque in finem demonstrationem dabo analyticam triam Theorematum, quæ inveni, & quidem diu ante visum D. Moyvrei libellum de Mensura Sortis, occasione triplicis questionis mihi ab amico circa ludum hunc, quem Galli vocant le Jeu de la Poule, propostæ, pro inveniendis scil. probabilitate vincendi, lucro item vel damno cujusque Collusoris, & duratione certaminis.

THEOREMA 1.

Si Collusores aliquot A, B, C, D, E, &c. quorum numerus est n + 1 & dexteriates sunt aequalis, deponant singuli 1, & illis conditionibus certent. 1°. Ut illorum duo A & B ludum incipiant. 2°. Ut vicimus locum suum tertio C cedat, ita ut ille tertius C jam cum victore contendat, quique ex hoc certamine victor evaserit cum quarto D ludat, & ita deinceps. 3°. Ut illé depositum totum obtineat, qui omnes collusores successively vicerit. Dico probabilitates vincendi duorum quorumlibet collusorum se æ immediate in ordine ludendi sequentium esse in ratione $1 + 2^n$ ad $2^n$, adeoque expectationes lusorum A (B), C, D, E, &c. esse in progressione Geometrica.

U

Demon
Demonstratio.

Ponantur expectationes vincendi ipsius $A$ vel $B = a$, ipsius $C = c$, ipsius $D = d$, ipsius $E = e$, &c. Porro cum accidere possit, ut collusor aliquis prima vice in ludum intrans inveniat adversarium qui vel nondum, vel semel, vel bis, vel ter, &c. jam successive victor extitit, vocetur expectatio lusoris illius primo cafu $= z$, secundo $= y$, tertio $= x$, quarto $= u$, quinto $= t$, &c. Item cum collusor aliquis vinci possit ab adversario qui antea jam vel nullum, vel unum, vel duos, vel tres, &c. collusores successive vicit, ita ut exiens è ludo relinquat adversarium qui vel semel, vel bis, vel ter, vel quater &c. victor extitit, vocetur expectatio seu probabilitas vincendi ejus qui exit è ludo primo cafu $= b$, secundo $= k$, tertio $= l$, quarto $= m$, &c. Hisce omnibus positis habebuntur sequentes novem series aequationum signatae N°. 1, N°. 2, N°. 3, &c. usque ad N°. 9. Tab. 1. Ratio eas inveniendi breviter hæc est.

Inter aequationes N°. 1°. reperitur ex. gr. $f = \frac{1}{8} t + \frac{1}{8} u + \frac{1}{4} x$ + $\frac{1}{2} y$. Nam collusor $F$ certabit vel cum collusore $A$, vel $B$, vel $C$, vel $D$, vel $E$: ut primum vel secundum contingat, oportet ut vel $A$ vel $B$ quater successive victor exsitat, cujus eventus probabilitas est $\frac{2}{16} = \frac{1}{8}$: Ut tertium contingat oportet ut $C$ ter victor exsitat, cujus eventus probabilitas est etiam $\frac{1}{8}$: Ut quartum contingat oportet ut $D$ bis successive vincat, quod probabilitatem habet $\frac{1}{4}$: Ut quintum contingat, oportet ut $E$ semel vincat, cujus eventus probabilitas est $\frac{1}{2}$; ergo
ergo lusoris $F$ probabilitas vincendi est \[ \frac{1}{8} f + \frac{1}{8} u + \frac{1}{4} x \]
\[ + \frac{1}{2} y. \] Sic inter æquationes No. 2. est, ex. gr. \[ x = \frac{1}{2} \]
\[ + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n} \times h + \frac{1}{2} \times l. \] Collusor enim qui certat cum adversario qui jam bis successe victor extitit, vincere potest vel omnes collusores, vel aliquos, vel nullum. Prioris eventus probabilitas est \[ \frac{1}{2^n}, \] secundi \[ \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n} \times h + \frac{1}{2} \times l. \]
\[ \ldots \frac{1}{2^n}, \] & tertii \[ \frac{1}{2}; \] si primus eventus contingat, probabilitas vincendi evadit certitudo integra seu \[ 1; \] si secundus, exit è ludo relinquens collusorem qui semel vicit; si tertius, exit è ludo relinquens collusorem qui ter successive vicit; adeoque
\[ \text{fors ejus totalis est } \frac{1}{2} \times l + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n} \times h + \frac{1}{2} \times l. \]

Simili ratiocinio inveniuntur æquationes No. 3. Collusor enim qui victus ab adversario exit è ludo, relinquens ex. gr. collusorem unius tantum ludi victorem, acquirit fortém vel ipsius $C$, vel ipsius $D$, vel ipsius $E$, vel ipsius $F$, &c. prout adversarius à quo victus est vincit vel omnes collusores prater unum, vel omnes prater duos, vel omnes prater tres, &c.
unde sequitur quod 
\[ h = \frac{1}{2^n-1} \times c + \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f + \&c. \]
Æquationes No. 4. inveniuntur subtrahendo æquationes No. 2. ab invicem: & æquationes No. 5. subtrahendo æquationes No. 3. ab invicem. Æquationes No. 6. inveniuntur substituendo in æquationibus No. 4. valores inventos per æquationes No. 5. Æquationes No. 7. inveniuntur quandam valorem ipsarum $z, y, x, u, \&c.$ per æquationes No. 1. Et his valóribus substitutis in æquationibus No. 4. habebuntur æquationes No. 8, quae comparatae cum æquationibus No. 6: 2 Ú 2 dant
Corollarium.

Hinc facile inveniuntur probabilitates vincendi singulorum Collusorum, quas habent tum ante ludum inceptum, tum in quolibet statu in quem ludum prosequendo pervenire possint. Si sint, ex gr. tres collusores A, B, C, erit $n = 2$, & $1 + 2^n : 2^n :: 5.4 :: a : c ::$ id est, probabilitates vincendi ipsorum A, B, C, antequam A vicerit B, vel B vicerit A, se habent ut numeri $5.5.4$. Adeoque ipsae probabilitates sunt $\frac{5}{14}, \frac{5}{14}, \frac{4}{14}$; omnes enim simul sumpta facere debent seu certitudinem integrum. Postquam A vicerit B, probabilitates vincendi ipsorum B, C, A, erunt $b, y$ seu $c,$ & (quia A aequalis habet expectationem ad victoriam, & ad sortem ipsius B obtinendam) $\frac{1 + b}{2}$ respective,

hoc est, quia per æq. I. No. 3, $b = \frac{1}{2^n - 1} \times c = \frac{1}{2} c$, & $c = \frac{4}{14} = \frac{4}{7}$ ut modo inventum, hæ probabilitates erunt $\frac{1}{7}, \frac{2}{7}, \frac{4}{7}$, ut.

D. de Moivre invenit Coroll. 1. Prop. 15. pag. 242.

Si sint quatuor collusores A, B, C, D, erit $n = 3$, & $1 + 2^n$. $2^n :: 9, 8$, adeoque probabilitates collusorum ab initio ludi erunt ut $9, 9, 8, \frac{8}{9}$, five ut $81, 81, 72, 64$, hoc est, ipsae $a, 4, c, d$, erunt $\frac{81}{298}, \frac{81}{298}, \frac{72}{298} \& \frac{64}{298}$. Postquam A vicerit B, probabilitates ipsorum B, D, C, A, erunt $b, d, c, \frac{1 + 3 b}{4}$, est autem per æq. I. No. 3, $b = \frac{1}{2^n - 1} \times c + \frac{1}{2^n - 2} \times d = \frac{1}{4} c + \frac{1}{4} d$, &
\begin{align*}
\xi &= \frac{72}{298} = \frac{36}{149}, \quad \eta = \frac{64}{298} = \frac{32}{149}, \text{ut modo inventum:}
\end{align*}

\text{ergo hæ probabilitates erunt } \frac{25}{298}, \frac{32}{149}, \frac{36}{149}, \frac{56}{149} \text{ respective.}

Postquam } A \text{ vicerit } B \& C, \text{probabilitates vincendi ipsorum } C, B, D, A, \text{ erunt } k, \frac{c}{2}, \frac{x}{2}, \text{feu (quia per æq. 2. N°. 3:}
\begin{align*}
k &= \frac{2}{2^n} x d = \frac{1}{2} d, \quad \text{& per æq. 3. N°. 7. } x = 2 d - c) \frac{16}{149}, \frac{18}{149}, \\
\frac{28}{149}, \frac{87}{149}. \text{Et nota quod calculi bonitas confirmetur ex eo, quod summæ harum probabilitatum, hoc est, } \frac{1}{7} + \frac{2}{7} + \frac{4}{7} \text{ in priori exemplo, & } \frac{25}{149} + \frac{32}{149} + \frac{36}{149} + \frac{56}{149}, \text{nec non } \frac{16}{149} + \frac{18}{149} + \frac{28}{149} + \frac{87}{149} \text{ in posteriori exemplo, singulae sunt } = 1 \text{ feu certitudini integrae.}

\text{THEOREMA II.}

\text{Positis quæ prius & infuper hac conditione, ut victus semper multetur summa } p, \text{ quæ deposito augendo intersiavat; quod depositum sic gradatim autem illi soli cedat, qui omnium successivum collorum victor extiterit; denotatis etiam ut antea per literas minusculas } a, c, d, e, \&c. \text{probabilitatibus vincendi ipsorum } A (\text{vel } B), C, D, E, \&c. \text{respectively: per eadem vero literas majusculas } A, C, D, E, \&c. \text{ipsorum } A (\text{vel } B), C, D, E, \&c. \text{expectationibus, hoc est, portionibus depositi quas singuli expectant: Dico, fore semper } C = \frac{A + a p \times 2^n - ncp}{1 + 2^n} \text{&c.}
\[
D = \frac{C + c \cdot p \times 2^n - n \cdot d \cdot p}{1 + 2^n}, \quad E = \frac{D + d \cdot p \times 2^n - n \cdot e \cdot p}{1 + 2^n}, \quad \text{&c.}
\]

Demonstratio.

Denotetur ut prius per literas minusculas \(z, y, x, u, t, &c.\) probabilitas vincendi ludentis cum adversario, qui jam vel nullum, vel unum, vel duos, &c. collusores successivi vicit, per easdem vero literas majusculas \(Z, Y, X, U, T, &c.\) ejus expectatio, quam scil. habet diversis illis caulisbus, deposito existente \(n + 1, n + 1 + p, n + 1 + 2p, n + 1 + 3p, &c.\) respective. Sic etiam per literas minusculas \(b, k, l, m, &c.\) denotetur probabilitas vincendi lusoris victi ab adversario, qui anteae vel nullum, vel unum, vel duos, &c. collusores successivi vicerat; quemadmodum per literas majusculas \(H, K, L, M, &c.\) ejusdem expectatio diversis illis caulisbus, deposito existente \(n + 1 + p, n + 1 + 2p, n + 1 + 3p, &c.\) respective. Hic positis iisdem quibus anteae ratiociniiis inveniuntur sequentes duodecim equationum series in Tab. II. signatæ No. 1. No. 2. No. 3, &c. Inter equationes No. 1. ex. gr. est \(E = - \frac{U}{4} + \frac{X + x \cdot p}{4} + \frac{T + 2y \cdot p}{2}\).

Lusor enim \(E\) ludet vel cum lusore \(A\), vel lusore \(B\), vel \(C\), vel \(D\). Si ludit cum \(A\) vel \(B\), expectatio ejus erit \(= U\), quia ludit cum adversario qui jam tres adversarios vicit, deposito existente \(n + 1 + 3p\). Si ludit cum lusore \(C\), expectatio ejus erit \(= X + x \cdot p\), ludit enim cum adversario qui jam duos collusores vicit, adeoque si depositum esset \(n + 1 + 2p\) ejus expectatio esset \(= X\): verum quia ludente \(E\) depositum esset \(= n + 1 + 3p\), ob tres collusores victos & summa \(p\) multatam; addenda est expectationi \(X\) portio illa multæ unius \(p\), quam lusor \(E\) sperare potest: est autem hac portio (quia probabilitas ejus vincendi esset \(x\)) = \(x \cdot p\), ejus igitur expectatio totalis tunc erit \(= X + x \cdot p\). Sic si ludit cum lusore \(D\), expectatio ejus erit \(= T + 2y \cdot p\): additur ad \(T\) (quæ esset ejus expectatio deposito existente \(n + 1 + p\)) portio \(2y \cdot p\), quæ ipsi debetur de duabus multatis.

Corollarium.

Hinc quoque facile inveniuntur singulorum Collusorum fortes seu expectationes, ipsorumque adeo lucra vel damna. Sint ex gr. tres collusores \(A, B, C\): erit \(C = \frac{A + \frac{4}{5}p}{1 + \frac{2}{5}}\) per coroll. Theor. 1.) \(\frac{4A + \frac{4}{5}p}{5} = \frac{2}{7}\). Unde cum omnium trium expectationes simul sumpta, id est, \(A + A + C\) æquare debent id quod ab initio depositum fuit, id est 3, erit \(2A + \frac{4A + \frac{4}{5}p}{5} = \frac{14A + \frac{6}{5}p}{5} = 3, \) & \(14A = 15 - \frac{6}{5}p, \) &c.

\(A = \frac{15}{14} - \frac{3}{49}p = \text{expectationi luforis } A \text{ vel } B: \text{ proinde } C\)

\(\text{expectatio luforis tertii } C = \frac{4A + \frac{6}{5}p}{7} = \frac{6}{49}p. \) A qui-
bus expectationibus si subtrahatur \( r \), id quod ab initio singuli deposuerunt, remanebit \( \frac{1}{14} - \frac{3}{49} \) \( p \), hic \( \frac{6}{49} \) \( p = \frac{1}{7} \); quæmadmodum D. de Moivre invent. Exempl. 2. Sint collusores \( 4 \). \( A, B, C, D \), etit \( C = \frac{A + ap \times 2^n - nc \ p}{1 + 2^n} = (ob n = 3) \)

\[ 8A + 8ap - 3cp = (ob a = \frac{81}{298} & c = \frac{36}{149} \text{, per coroll.} \]

Theor. 1.) \[ \frac{8A + \frac{315}{149} p}{9} \]; item \( D = \frac{C + cp \times 2^n - ndp}{1 + 2^n} = \)

\[ 8C + 8cp - 3dp = (ob d = \frac{32}{149} \text{per id. corr.}) \frac{8C + \frac{108}{149} p}{9} = \]

\[ \frac{64A + \frac{315}{149} p}{81} \text{: unde habebitur æquatio } 2A + C + D = 2A + \]

\[ \frac{8A + \frac{315}{149} p}{9} + \frac{64A + \frac{315}{149} p}{81} = \frac{298A + \frac{1400}{149}}{81} = 4, \text{ sive } 149A \]

\[ + \frac{2700}{149} p = 162, & A = \frac{162}{149} - \frac{2700}{149} p. \text{ Hinc } C = \]

\[ \frac{8A + \frac{315}{149} p}{9} = \frac{144}{149} + \frac{1176}{22201} p, \text{ & } D = \frac{64A + \frac{108}{149} p}{81} = \frac{128}{149} \]

\[ + \frac{4224}{22201} p. \text{ Subtra} \alpha \text{ta autem unitate } 1, \text{ quam singuli ab initio ludi deposuerunt, remanebit } \frac{13}{149} - \frac{2700}{22201} p \text{ pro lusore } A \text{ vel } B, \frac{1176}{22201} p = \frac{5}{149} \text{ pro } C, \text{ & } \frac{4224}{22201} p = \frac{21}{149} \text{ pro } D \]; quæ singula indigitaabunt lucrum vel damnum, prout pars affirmata præpollet negatæ, vel contra. Simili ratione habebuntur etiam fortes quas acqurunt in quolibet statu in quem ludum prosequendo pervenire possunt.
THEOREMA 3.

Positis quae prius, si addint spectatores \( Q, R, S, T, U \), &c. quorum numerus sit \( n \) unitate minor quam numerus collusorum, quorumque prior \( Q \) affirmet certamen finitum iri post \( n+p \) ludos peractos, \( R \) post \( n+p-1 \), \( S \) post \( n+p-2 \), \( T \) post \( n+p-3 \), \( U \) post \( n+p-4 \), &c præcise, non anrea; sintque \( q, r, s, t, u, \) &c. fortis ipsisorum \( Q, R, S, T, U, \) &c. Di-
ço fore \( q = \frac{1}{2} r + \frac{1}{4} s + \frac{1}{8} t + \frac{1}{16} u + \) &c.

Demonstratio.

Vocetur \( A \) collusor ille, qui post \( n+p \) ludos vincere supponitur: hic intrare debebit in ludum post \( p \) ludos peractos, & tum ludet contra adversarium, qui jam vel unum vel duos, vel tres, &c. collusores successive victor. Jam cum, ut primus casus contingat, & ut collusor \( A \) omnes suos collusores præter unum, id est, \( n-1 \) collusores successive vincat, æquæ probabile sit quam ut adversarius ejus vincat \( n-1 \) collusores, id est, (quia jam unius collusoris victor fuit) ut certamen finiat post \( n+p-1 \) ludos peractos; hujusque eventus probabilitas sit \( r \): erit probabilitas ut collusor \( A \) unum adhuc collusorem vincat, id est, certamen finiat post \( n+p \) ludos = \( \frac{1}{2} r \). Sic, ut se-
cundus; casus existat, & ut \( A \) omnes collusores præter duos vincat, æquæ probable sit quam ut certamen finiatur post \( n+p-2 \) ludos, adeoque ut tunc \( A \) vincat adhuc duos collusores, id est, ut certamen finiat post \( n+p \) ludos, probabilitas est \( \frac{1}{4} s \).

Eodem modo ut, tertio casu existente, \( A \) vincat omnes collusores, probabilitas est \( \frac{1}{8} t \); ut quarto \( \frac{1}{16} u \), &c. Quare ut

\[ Y \quad \text{indiffe-} \]
indifferentem certamen finiatur post \( n + p \) ludos, probabilitas est
\[
\frac{1}{r} + \frac{1}{s} + \frac{1}{t} + \frac{1}{u} + \cdots = q. \quad \text{Q.E.D.}
\]

**Corollarium 1.**

Facile hinc inventur quænam sit probabilitas ut certamen finiatur intra datum quemvis ludorum numerum. Series enim fractionum incipientium à fractione \( \frac{1}{2^n-1} \), quarum denominatores crescent in continua proportione dupla, numerator autem cujusque fractionis sit summa numerorum tot fractionum immediate præcedentium quot sunt unitates in \( n-1 \), dabit omnes successive probabilitates, ut certamen finiatur peraquis præcisè \( n \), \( n+1 \), \( n+2 \), \( n+3 \) &c. ludis: & per consequens si addantur tot termini hujus seriei quot sunt unitates in \( p+1 \), summa ipsorum exprimet probabilitatem ut certamen finiatur ad minimum ludis \( n + p \) peraquis. Ex.gr. Si sint collusores 4, adeoque \( n = 3 \), habebitur hæc serie \( 1, \frac{1}{4}, \frac{2}{8}, \frac{3}{16}, \frac{5}{32}, \frac{8}{64}, \frac{18}{128} \), &c. Æqua si fiat alia \( 1, \frac{1}{2}, \frac{3}{4}, \frac{8}{8}, \frac{19}{16}, \frac{43}{32}, \frac{94}{64}, \frac{201}{128}, \frac{256}{256} \), &c. cujus termini sint summæ terminorum præcedentis seriei, denotabunt idem termini qualis sit probabilitas ut certamen finiatur ad minimum 3, 4, 5, 6, &c. ludis.

**Corollarium 2.**

Poteft terminus quicunque prioris seriei (excepto primo termino,) ut & summa omnium terminorum, id est, terminus quicunque posterioris seriei, per formulam generalem exprimi hoc modo. Si \( n + 1 \) sit numerus collusorum, & \( p \) sit numerus terminorum, erit ultimus terminus prioris seriei
\[
\begin{align*}
\text{Tabula I.} \\
\text{Intrat} & \quad \text{Exit.} & \text{N°. r} \\
\text{Sors} & \quad 0 & a = z \\
& \quad 1 & b = \gamma \\
& \quad 2 & d = \frac{x + y}{2} \\
& \quad 3 & e = \frac{u + x + y}{4} \\
& \quad 4 & f = \frac{t + u + x + y}{8}
\end{align*}
\]

\[
\begin{align*}
z &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots + \frac{1}{2^n} \\
y &= \frac{1}{2} \times b + \frac{1}{2^n} \\
x &= \frac{1}{2} \times b + \frac{1}{2^n} \\
u &= \frac{1}{2} \times b + \frac{1}{2^n} \\
\text{N°. 3:}
\end{align*}
\]
\[b = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f + \ldots\]

\[k = \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f + \ldots\]

\[l = \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f + \ldots\]

\[m = \frac{1}{2^{n-4}} \times f + \ldots\]

\[\begin{array}{c}
N^\circ 4.
\end{array}\]

\[x - y = \frac{1}{2} b - \frac{1}{2} k = \frac{1}{2^n} \times e = a - c\]

\[y - x = \frac{1}{2} k - \frac{1}{2} l = \frac{1}{2^{n-1}} \times d = 2c - 2d\]

\[x - u = \frac{1}{2} l - \frac{1}{2} m = \frac{1}{2^{n-2}} \times e = 4d - 4e\]

\[\begin{array}{c}
N^\circ 5.
\end{array}\]

\[\begin{array}{c|c|c|c}
N^\circ 6. & N^\circ 7. & N^\circ 8. \\
& & \\
| & | & \\
\hline
b = k = \frac{1}{2^{n-1}} \times c & z = a & e = a \times \frac{2^n}{1 + 2^n} \\
& y = c & \text{Corollarium 3.} \\
k - l = \frac{1}{2^{n-2}} \times d & x = 2d - y = 2d - c & d = c \times \frac{2^n}{1 + 2^n} \\
l - m = \frac{1}{2^{n-3}} \times e & u = 4e - x - 2y = 4e - 2d - e & e = d \times \frac{2^n}{1 + 2^n}
\end{array}\]

Potest quis priusquam ludus inchoetur in se suscipere, ut sum-
mam \(n + 1\) de qua collusores contendunt, & multas omnes
pendat, si sibi initio in manus datum sit \(n + 1 + 2^n - 1 \times p\).

Demonstrationem duorum præcedentium corollariorum cu-
riosis indagandam relinquo.

Designationes.

Si B & C collusores duo simul certent, ad designandum B victorem esse, C victum, scribatur BC; atque vice simili ad designandum C victorem esse, B victum; scribatur CB: & sic de ceteris.

Ponatur 1° B vincere A, certamenque concludi tribus ludis

\[
\begin{align*}
&BA \\
&BC \\
&BD
\end{align*}
\]

Sic patet B victorem necessario evadere.

Ponatur 2° B vincere A, certamenque concludi quatuor ludis

\[
\begin{align*}
&BA \\
&CB \\
&CD \\
&CA
\end{align*}
\]

Sic patet C victorem necessario evadere.

Ponatur 3° B vincere A, certamenque concludi quinque ludis

\[
\begin{align*}
&BA & BA \\
&CB & BC \\
&DC & DB \\
&DA & DA \\
&DB & DC
\end{align*}
\]

Sic patet D victorem necessario evadere, idque duplici modo.

Ponatur 4° B prima vice vincere A, certamenque concludi sex ludis.

\[
Z \quad B \quad A
\]
\[
\begin{array}{cccc}
B A & B A & B A \\
C B & C B & B C \\
D C & D C & D B \\
A D & A C & A C \\
A B & A B & A D \\
A C & A D & A B \\
\end{array}
\]
Sic patet \( A \) victorem necessario evadere, idque triplici modo.

\[
\begin{array}{cccc}
B A & B A & B A & B A \\
C B & C B & C B & B C \\
D C & D C & D B & D B \\
A D & A D & A D & A D \\
B A & B D & B A & C A \\
B C & B C & B D & C B \\
B D & B A & B C & C D \\
\end{array}
\]
Sic patet \( B \) vel \( C \) necessario victores evadere, \( B \) triplici modo, \( C \) duplici.

\[
\begin{array}{cccc}
B A & B A & B A & B A \\
C B & C B & C B & C B \\
D C & D C & D C & D C \\
A D & A D & A D & A D \\
B A & B D & B A & C A \\
B C & C A & B D & D A \\
C D & C D & C A & D B \\
C A & C B & C D & D A \\
\end{array}
\]
Sic patet \( C \) victorem evadere triplici, \( D \) duplici, \( B \) triplici modo, \&c.

Nunc ordine scribuntur literae quibus victores designantur:

1, \( B \)  
1, \( C \)  
2, \( D \)  
3, \( A \)  
3, \( B + 2 C \)  
3, \( C + 2 D + 3 B \)  
3, \( D + 2 A + 3 C + 3 D \)  
3, \( A + 2 B + 3 D \)  
\&c.

Per\*
Perspecta illarum formatione, patebit 1° literam B in ordine aliquo semper toties reperiri, quoties A in ordine ultimo & penultimo reperitur: 2° C in ordine aliquo toties reperiri quoties B in ordine ultimo & D in penultimo reperiuntur: 3° D in ordine aliquo toties reperiri quoties C in ultimo & B in penultimo: 4° A in ordine aliquo semper toties reperiri quoties D in ordine ultimo & C in penultimo reperiuntur.

Sed numerus variationum dato cuilibet ludorum numero competens, duplue est numeri variationum omnium dato ludorum numero unitate diminuto competentis: adeoque Probabilitas quam habet Collusor B ut vincat dato ludorum numero, est subduplica probabilitatis quam habebat A ut vinceret dato ludorum numero minus uno; atque etiam subquadrupla probabilitatis quam habebat idem A, ut vinceret dato ludorum numero minus duobus: & sic de ceteris.

Probabilitas quam habet C, ut vincat dato ludorum numero, est subduplica probabilitatis quam habebat B, ut vinceret dato ludorum numero minus uno; atque etiam subquadrupla probabilitatis quam habebat D, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet D ut vincat dato ludorum numero, est subduplica probabilitatis quam habebat C, ut vinceret dato ludorum numero minus uno; atque etiam subquadrupla probabilitatis quam habebat B, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet A ut vincat dato ludorum numero, est subduplica probabilitatis quam habebat D, ut vinceret dato ludorum numero minus uno; et subquadrupla probabilitatis quam habebat C ut vinceret dato ludorum numero minus duobus.

Ex jam observatis facile est componere Tabulam Probabilitatum, quas B, C, D, A habent ut victores evadant dato ludorum numero, atque etiam illorum fortium seu expectationum.
Tabula Probabilitatum, &c.

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{4} \times 4 + 3p)</td>
<td>(\frac{1}{8} \times 4 + 4p)</td>
<td>(\frac{2}{16} \times 4 + 5p)</td>
<td>(\frac{3}{32} \times 4 + 6p)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{3}{64} \times 4 + 7p)</td>
<td>(\frac{2}{64} \times 4 + 7p)</td>
<td>(\frac{1}{128} \times 4 + 8p)</td>
<td>(\frac{3}{128} \times 4 + 8p)</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{256}{64} \times 4 + 9p)</td>
<td>(\frac{6}{256} \times 4 + 9p)</td>
<td>(\frac{2}{128} \times 4 + 9p)</td>
<td>(\frac{9}{256} \times 4 + 9p)</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{1024}{512} \times 4 + 10p)</td>
<td>(\frac{1024}{512} \times 4 + 10p)</td>
<td>(\frac{512}{1024} \times 4 + 10p)</td>
<td>(\frac{9}{1024} \times 4 + 10p)</td>
</tr>
<tr>
<td>6</td>
<td>(\frac{2048}{1024} \times 4 + 12p)</td>
<td>(\frac{2048}{1024} \times 4 + 12p)</td>
<td>(\frac{1024}{2048} \times 4 + 12p)</td>
<td>(\frac{4}{2048} \times 4 + 12p)</td>
</tr>
</tbody>
</table>

Jam vero Series istae sunt convergentes, adeoque singulae summari possunt per vulgarem Arithmeticae; & obtinebuntur vel summari accurate si possint, vel sitem approximatae, si non liceat, terminos multos adhibere.

Inveni-
Invenire summas Probabilitatum ad infinitum usque pergentiam, quas Collusores habent ut victores evadant.

Sint Probabilitates omnes ipsius $B$ ad infinitum, nempe

$B' + B'' + B''' + B''' + B'''' + B''''' + B'''''' + B''''''' + \&c. = \gamma$

Probabilitates ipsius $C$

$C' + C'' + C''' + C''' + C'''' + C''''' + C''''' + \&c. = z$

Probabilitates ipsius $D$

$D' + D'' + D''' + D''' + D'''' + D''''' + D'''''' + D''''''' + \&c. = \nu$

Probabilitates ipsius $A$

$A' + A'' + A''' + A''' + A'''' + A''''' + A'''''' + A''''''' + \&c. = \alpha$

Scribantur autem in Scala perpendiculariter descendente, ad hunc modum.

$B' = B'$
$B'' = B''$
$B''' = \frac{1}{3} A'' + \frac{1}{3} A'$
$B'''' = \frac{1}{3} A'''' + \frac{1}{3} A''$
$B''''' = \frac{1}{3} A''''' + \frac{1}{3} A''''$

Proinde $\gamma = \frac{1}{3} + \frac{1}{3} \alpha$.

Ergo $\gamma = \frac{1}{3} + \frac{1}{3} \alpha + \frac{1}{3} \alpha$.

Demonstratio.

Etenim prima columna perpendicularis $= \gamma$, ex Hypothesi

Est vero $A' + A'' + A''' + A''' + A'''' + A''''' + \&c. = \alpha$, ex hypothesi

Ergo $\frac{1}{3} A' + \frac{1}{3} A'' + \frac{1}{3} A''' + \frac{1}{3} A''' + \frac{1}{3} A'''' + \frac{1}{3} A''''' + \&c. = \frac{1}{3} \alpha$.

Proinde $\frac{1}{3} A'' + \frac{1}{3} A''' + \frac{1}{3} A''' + \frac{1}{3} A'''' + \&c. = \frac{1}{3} \alpha - \frac{1}{3} A'$.

Et $B' + B'' + \frac{1}{3} A' + \frac{1}{3} A'' + \frac{1}{3} A''' + \&c. = \frac{1}{3} \alpha - \frac{1}{3} A' + B'' + B'''$. 

Sed
Sed \( \frac{1}{2} A' = 0, B'' = 0 \) & \( B' = \frac{1}{4} \), ut patet ex Tabula.

Ergo secunda columna perpendicularis = \( \frac{1}{4} + \frac{1}{2} \times \).

Sed tertia columna perpendicularis = \( \frac{1}{4} \times \).

\[ \text{erit igitur } y = \frac{1}{4} + \frac{3}{4} \times. \]

Simili modo scribantur

\[ C' = C' \]
\[ C'' = C'' \]
\[ C''' = \frac{1}{3} B'' + \frac{1}{4} D' \]
\[ C'''' = \frac{1}{3} B'' + \frac{1}{4} D'' \text{ hoc est } z = \frac{1}{3} y + \frac{1}{3} x. \]
\[ C'''' = \frac{1}{3} B' + \frac{1}{4} D'' \]
\[ C'''' = \frac{1}{3} B' + \frac{1}{4} D'' \]

\[ \text{Ergo } z = \frac{1}{3} y + \frac{1}{3} x. \]

Scribantur etiam

\[ D' = D' \]
\[ D'' = D'' \]
\[ D''' = \frac{1}{3} C' + \frac{1}{4} B' \]
\[ D'''' = \frac{1}{3} C'' + \frac{1}{4} B'' \text{ & pari Argumento patebit } \]
\[ D'''' = \frac{1}{3} C'' + \frac{1}{4} B'' \]
\[ D'''' = \frac{1}{3} C' + \frac{1}{4} B' \]

\[ \text{&c.} \]

Scribantur denique

\[ A' = A' \]
\[ A'' = A'' \]
\[ A''' = \frac{1}{3} D'' + \frac{1}{4} C' \]
\[ A'''' = \frac{1}{3} D'' + \frac{1}{4} C'' \]
\[ A''' = \frac{1}{3} D'' + \frac{1}{4} C'' \text{ Unde concludetur } x = \frac{1}{3} v + \frac{1}{3} z \]
\[ A'''' = \frac{1}{3} D' + \frac{1}{4} C''' \]
\[ \text{&c.} \]
Refutatis autem quatuor ipsis equationibus, reperietur

\[ B' + B'' + B''' + B'''' \&c. = y = \frac{56}{149} \]

\[ C' + C'' + C''' + C'''' \&c. = z = \frac{36}{149} \]

\[ D' + D'' + D''' + D'''' \&c. = v = \frac{32}{149} \]

\[ A' + A'' + A''' + A'''' \&c. = x = \frac{25}{149} \]

Valoris ipsis inventis, ponatur jam \( \frac{56}{149} = b, \frac{36}{149} = c, \frac{32}{149} = d, \frac{25}{149} = a. \)

Iterum sit.

\[ 3 B' + 4 B'' + 5 B''' + 6 B'''' \&c. = p y. \]
\[ 3 C' + 4 C'' + 5 C''' + 6 C'''' \&c. = p z. \]
\[ 3 D' + 4 D'' + 5 D''' + 6 D'''' \&c. = p v. \]
\[ 3 A' + 4 A'' + 5 A''' + 6 A'''' \&c. = p x. \]

\[ 3 B' = 3 B' \]
\[ 4 B'' = 4 B'' \]
\[ 5 B''' = \frac{1}{2} A'' + \frac{1}{2} A' \]
\[ 6 B'''' = \frac{1}{3} A''' + \frac{2}{3} A'' \]
\[ 7 B''' = \frac{1}{2} A'''' + \frac{1}{2} A''' \]
\[ 8 B'''' = A'''' + \frac{1}{2} A''' \]

\[ \text{Ergo } y = \frac{2}{3} + \frac{1}{4} x + a \]

Etenim prima Columna perpendicularis = \( y \), ex Hypothesi:

\[ 3 B' + 4 B'' = \frac{3}{4}; \text{ Nam est } B' = \frac{1}{4}, \& B'' = \frac{1}{2}. \]
\[ 3 A' + 4 A'' + 5 A''' \&c. = x \text{ ex Hypothesi.} \]
\[ A' + A'' + A''' \&c. = a, \text{ ut repertum est.} \]

Eft igitur \( 4 A' + 5 A'' + 6 A''' + 7 A'''' \&c. = x + a \)

\[ \text{Et } \frac{4}{5} A + \frac{4}{5} A' + \frac{5}{6} A'' + \frac{2}{3} A''' + \frac{1}{2} A'''' \&c. = \frac{1}{2} x + \frac{1}{2} a. \]

\[ A a 2 \]

Sed
Sed $A' = 0$
Ergo secunda Columna perpendicularis $= \frac{1}{2} + \frac{1}{2} x + \frac{1}{2} a$.

\[ 3 A' + 4 A'' + 5 A''' + 6 A'''' + \text{c.} = x \]
\[ 2 A' + 2 A'' + 2 A''' + 2 A'''' + \text{c.} = 2a \]

Eft igitur $5 A' + 6 A'' + 7 A''' + 8 A'''' + \text{c.} = x + 2a$.

Et $\frac{1}{4} A' + \frac{1}{4} A'' + \frac{7}{4} A''' + \frac{3}{4} A'''' + \text{c.} = \frac{1}{4} x + \frac{1}{4} a$.

Eft igitur tertia Columna perpendicularis $= \frac{1}{2} x + \frac{1}{2} a$.

Erit igitur $y = \frac{1}{3} + \frac{1}{3} x + \frac{1}{3} a$.

five $y = \frac{1}{3} + \frac{1}{3} x + a$, quod erat probandum.

\[ 3 C' = 3 C' \]
\[ 4 C'' = 4 C'' \]
\[ 5 C''' = \frac{1}{3} B'' + \frac{1}{3} D' \]
\[ 6 C''' = \frac{1}{3} B'' + \frac{1}{3} D'' \]
\[ 7 C'''' = \frac{1}{3} B''' + \frac{1}{3} D''' \]
\[ 8 C'''' = \frac{1}{3} B'''' + \frac{1}{3} D'''' + \text{c.} \]

Ergo $z = \frac{1}{3} y + \frac{1}{3} b + \frac{1}{3} v + \frac{1}{3} d$.
Etenim prima Columna perpendicularis $= z$, ex Hypothesi.

\[ 3 C' + 4 C'' = \frac{1}{3} \]
\[ 3 B' + 4 B'' + 5 B''' + 6 B'''' + \text{c.} = y \]
\[ B' + 4 B'' + 5 B''' + 6 B'''' + \text{c.} = b. \]

Eft igitur $4 B' + 5 B'' + 6 B''' + 7 B'''' + \text{c.} = y + b$.
Sed $4 B' = r$.
Ergo $5 B'' + 6 B''' + 7 B'''' + \text{c.} = y + b - r$.
\[ \frac{5}{3} B' + \frac{2}{3} B'' + \frac{1}{3} B''' + \frac{1}{3} B'''' + \text{c.} = \frac{1}{3} y + \frac{1}{3} b - \frac{1}{3} \]

Ergo secunda Columna perpendicularis $= \frac{1}{3} y + \frac{1}{3} b$.

Iterum, $3 D' + 4 D'' + 5 D''' + 6 D'''' + \text{c.} = v$
\[ 2 D' + 2 D'' + 2 D''' + 2 D'''' + \text{c.} = 2d \]

Eft igitur $5 D' + 6 D'' + 7 D''' + 8 D'''' + \text{c.} = v + 2d$.

Et $\frac{1}{4} D' + \frac{1}{4} D'' + \frac{1}{4} D''' + \frac{1}{4} D'''' + \text{c.} = \frac{1}{4} v + \frac{1}{4} d$.
Ergo tertia Columna perpendicularis $= \frac{1}{4} v + \frac{1}{4} d$
Eft igitur $z = \frac{1}{3} y + \frac{1}{3} b + \frac{1}{3} v + \frac{1}{3} d$, quod erat probandum.
Eodem prorius ordine scribantur.

3 \( D' \) = 3 \( D' \)  
4 \( D'' \) = 4 \( D'' \)  
5 \( D''' \) = \( C'' + \frac{1}{3} B' \)  
6 \( D'''' \) = \( C'''+ B'' \)  
7 \( D'''' \) = \( C'''' + B'''' \)  
8 \( D'''' \) = \( C'''' + D'''' \)  

\& c.

Unde \( v = \frac{1}{3} x + \frac{1}{3} c + \frac{1}{4} y + \frac{1}{4} b \).  

Et \( x = \frac{1}{3} v + \frac{1}{3} d + \frac{1}{3} z + \frac{1}{3} c \).

Quae quidem Conclusiones eodem modo demonstrantur ac superiores.

Solutis autem quatuor istis equationibus, elicietur

\[
y = \frac{45536}{149^2}, \quad z = \frac{38724}{149^2}, \quad v = \frac{37600}{149^2}, \quad x = \frac{33547}{149^2} = \frac{33547}{22201}
\]

Ergo, si velint \( B, C, D, A \) vendere Spectatori cuidam \( R \) fummas quas singuli obtinere sperant, \( \varepsilon q u u m \ e r i t \ u t \ \text{emptor} \ \text{pendat} \).

\[
\text{ipfi } B \quad 4 \times \frac{56}{149} + \frac{45536}{22201} p, \quad \text{ipfi } C \quad 4 \times \frac{36}{149} + \frac{38724}{22201} p.
\]

\[
\text{ipfi } D \quad 4 \times \frac{32}{149} + \frac{37600}{22201} p, \quad \text{ipfi } A \quad 4 \times \frac{25}{149} + \frac{33547}{22201} p.
\]

Invenire Probabilitates quas habent \( B, C, D, A \), ut muletentur, dato ludorum numero:

Si Ludi duo tantum sint, erunt hoc modo:

\[
\begin{align*}
B & A \\
\hline
C & B
\end{align*}
\]

Unde patet \( B \) vel \( C \) necessario muletari.

Si Ludi tres fuerint, hoc modo se res habet:

\[
\begin{align*}
B & A \\
\hline
C & B
\end{align*}
\]

Hinc patet \( C \), vel \( D \) vel \( B \) necessario muletari.

Si
Si vero quatuor Ludi fuerint:

\[
\begin{array}{cccc}
B & A & B & A \\
B & A & B & A \\
C & B & C & B \\
D & C & D & C \\
D & A & C & A \\
\end{array}
\]

Debet igitur \( A \) triplici modo, \( D \) duplici, \( C \) simplici, \( B \) multari.

Et sic de cæteris. Ex quibus manifesta est Compositio Tabulæ subjunctæ Probabilitatum quas \( B, C, D, A \) habent ut multifarentur, dato ludorum numero.

<table>
<thead>
<tr>
<th>Num Ld.</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<tr>
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<td>3</td>
</tr>
<tr>
<td>7</td>
<td>64</td>
<td>64</td>
<td>4</td>
<td>64</td>
</tr>
</tbody>
</table>

Sint autem \( y, z, \nu, \kappa \) summx omnium Probabilitatum quas \( B, C, D, A \) habent respective ut multarentur.

Scribantur eodem ordine ac in praecedentibus.

\[
\begin{align*}
B' &= B' \\
B'' &= B'' \\
B''' &= \frac{1}{2} A'' + \frac{1}{4} A' \\
B'''' &= \frac{1}{2} A'''' + \frac{1}{4} A'' \\
B''' &= \frac{1}{2} A''' + \frac{1}{4} A'' \\
B'' &= \frac{1}{2} A'' + \frac{1}{4} A' \\
\end{align*}
\]

\&c.

\[
\begin{align*}
C' &= C' \\
C'' &= C'' \\
C''' &= \frac{1}{2} B''' + \frac{1}{4} D' \\
C'''' &= \frac{1}{2} B'''' + \frac{1}{4} D'' \\
C''' &= \frac{1}{2} B''' + \frac{1}{4} D'' \\
C'' &= \frac{1}{2} B'' + \frac{1}{4} D' \\
\end{align*}
\]

\&c.

\[
\begin{align*}
\text{Ergo } y &= \frac{3}{4} + \frac{1}{2} \kappa + \frac{1}{4} \kappa \\
\text{Ergo } z &= \frac{3}{2} + \frac{1}{2} y + \frac{1}{4} \nu \\
\end{align*}
\]
(155)

Scribantur deinde

\[
\begin{align*}
D' &= D' \\
D'' &= D'' \\
D''' &= \frac{1}{2} C'' + \frac{1}{3} B'' \\
D'''' &= \frac{1}{2} C'' + \frac{1}{3} B'' \\
D''' &= \frac{1}{2} C'' + \frac{1}{3} B'' \\
A' &= A' \\
A'' &= A'' \\
A''' &= \frac{1}{2} D''' + \frac{1}{2} C'' \\
A'''' &= \frac{1}{2} D''' + \frac{1}{2} C''
\end{align*}
\]

\[\&c.\]

Ergo \( v = \frac{1}{4} + \frac{1}{4} z + \frac{1}{4} y \) \hspace{1cm} \text{Ergo } x = \frac{1}{4} v + \frac{1}{4} z.

Resolutis autem quatuor his aequationibus, invenietur

\[\begin{align*}
y &= \frac{243}{249} \hspace{1cm} z &= \frac{252}{149} \hspace{1cm} v &= \frac{224}{149} \\
&\& x &= \frac{175}{149}
\end{align*}\]

Ergo si velit Spectator aliquis \( S \) mulctas omnes sustinere, aequum est ut ipse \( S \)

\[B \text{ tradat } \frac{243}{149} p \quad C \frac{252p}{149} \quad D \frac{224p}{149} \quad \& \quad A \frac{175p}{149}.\]

Sublatis itaque summis probabilitatum quas singuli Collusores habent ut mulctentur, è summis expectationum quas habent iisdem si victores abeant, restabunt fortes eorum respectice: nempe

\[B \text{ recipit } ab R \frac{4 \times 56}{149} + \frac{45536}{22201} p.\]

\[B \text{ tradit ipse } S \frac{243}{149} p.\]

Ergo ipse \( B \) superest \( \frac{224}{149} + \frac{9329}{22201} p.\)

Sed \( B \) depoluerat \( 1 \) priusquam ludus inciperetur.

Ergo \( B \) lucratur \( \frac{75}{149} + \frac{9329}{22201} p.\)
C recipit ab $R \frac{4 \times 36}{149} + \frac{38724}{22201} p$

C tradit ipfi $S \frac{252}{149} p$

Ergo ipfi $C$ supereft $\frac{144}{149} + \frac{1176}{22201} p$

Sed $C$ deposuerat $r$

Ergo $C$ lucratur $\frac{5}{149} + \frac{1176}{22201} p$

$D$ recipit ab $R \frac{4 \times 32}{149} + \frac{37600}{22201} p$

$D$ tradit ipfi $S \frac{224}{149} p$

Ergo ipfi $D$ supereft $\frac{128}{149} + \frac{4224}{22201} p$

Sed $D$ deposuerat $r$

Ergo $D$ lucratur $\frac{21}{149} + \frac{4224}{22201} p$

$A$ recipit ab $R \frac{4 \times 25}{149} + \frac{33547}{22201} p$

$A$ tradit ipfi $S \frac{175}{149} p$

Ergo ipfi $A$ supereft $\frac{100}{149} + \frac{7472}{22201} p$

Sed $A$ deposuerat $r + p$, nempe $r$ priusquam ludus inchoaretur, $& p$ postquam semel victus fuerat a $B$

Ergo $A$ lucratur $\frac{49}{149} = \frac{14729}{22201} p$

Lucrum
Lucrum iplius $B = \frac{75}{149} + \frac{9329}{22201}$

iplius $C = -\frac{5}{149} + \frac{1176}{22201}$

iplius $D = -\frac{21}{149} + \frac{4224}{22201}$

iplius $A = -\frac{49}{149} - \frac{14729}{22201}$

Summa Lucrorum =

Summa autem lucrorum iplorum $B & A = \frac{26}{149} - \frac{5400}{22201}$

sed posueramus $B$ vicisse iplum $A$ semel, priusquam Collufores pacta inirent cum $R & S$. Priusquam vero ludus inchoaretur, $A$ poterat æqua sorte expectare ut vinceret iplum $B$, adeoque summa lucrorum \( \frac{26}{149} - \frac{5400}{22201} \) in duas partes æquales est dividenda, ita ut utriusque lucrum cenfendum sit \( \frac{13}{2700} \).

Ponatur \( \frac{13}{149} - \frac{2700}{22201} \) \( p = 0 \), & erit \( p = \frac{1937}{2700} \).

Ergo si sit mulēta \( p \) ad summam quam singuli deponunt ut 1937 ad 2700, \( A & B \) nihil lucrantur, nihil perdunt. Verum hoc in Casu $C$ lucatur \( \frac{1}{225} \), quam $D$ perdit.

Coroll. 1. Spectator $R$, priusquam ludus inchoetur, id suscipere in se poterit, ut summam 4 de qua Collufores contendunt, & mulētas omnes pendat, si sibi initio in manus darentur 4 + 7 \( p \).

Coroll. 2. Si dexteritates Colluforum sint in ratione data, fortes Colluforum eadem ratiocinatione determinabuntur.
Coroll. 3. Si Series aliqua ita sit constituata, ut continuo decrecat, et terminus quivis ad præcedentes quolibet habeat rationes datas, five easdem five diversas, Series ista accurate summabitur. Infuper si termini omnes hujus Seriei multiplicentur per terminos progressionis Arithmeticæ, singuli per singulos, Series nova resultans accurate summabitur.

Coroll 4. Si sint Series plures collaterales, ita relatae ut terminus quilibet ejusque Seriei ad præcedentes quolibet aliarum Serierum habeat rationes datas, five easdem five diversas, ita ut Series istæ collaterales se decussent data qualibet legem constanti, Series istæ accurate summabuntur. Infuper si termini omnes harum Serierum multiplicantur ordinatim per terminos Progressionis Arithmeticæ, singuli per singulos, Series novæ ex hac multiplicatione resultantes etiamnum accurate summabuntur.

Clavis ad Problema generale.

Si sint Collufores quotunque v. g. Sex, B, C, D, E, F, A & Probabilitates quas habent ut victores evadant, five ut multentur, dato Ludorum numero, denotentur respecte B, C, D, E, F & A; & Probabilitates dato Ludorum numero his proximo & minori competentes, per B,, C,, D,, E,, F,, A;; & Probabilitates dato Ludorum numero his itidem novissimis proximo & minori competentes, per B,,, C,,, D,,, E,,, F,,, A,,, & sic deinceps; erit semper,

\[
\begin{align*}
B &= \frac{1}{2} A'' + \frac{1}{3} A' + \frac{1}{4} A + \frac{1}{5} A'' + \frac{1}{6} A''
C &= \frac{1}{2} B'' + \frac{1}{3} F' + \frac{1}{4} E'' + \frac{1}{5} D''
D &= \frac{1}{2} C'' + \frac{1}{3} B' + \frac{1}{4} F'' + \frac{1}{5} E''
E &= \frac{1}{2} D'' + \frac{1}{3} C' + \frac{1}{4} B' + \frac{1}{5} F'
F &= \frac{1}{2} E'' + \frac{1}{3} D' + \frac{1}{4} C' + \frac{1}{5} B
A &= \frac{1}{2} F'' + \frac{1}{3} E' + \frac{1}{4} D' + \frac{1}{5} C
\end{align*}
\]

Et fiat semper retrogressus ordinatim ad tot literas minus duobus quot sunt Collufores, omittaturque semper litera A, prima æquatione excepta, ubi litera A terminos omnes præter primam occupat.
IV. An Account of several extraordinary Meteors or Lights in the Sky. By Dr. Edmund Halley, Savilian Professor of Geometry at Oxon, and Secretary to the Royal-Society.

The Theory of the Air seemeth, at present, to be perfectly well understood, and the differing Densities thereof at all Altitudes, both by Reason and Experiment are sufficiently defined; for supposing the same Air to occupy Spaces reciprocally proportional to the Quantity of the superior or incumbent Air, I have elsewhere proved that at 40 Miles high the Air is rarer than at the Surface of the Earth about 3000 times; and that the utmost Height of the Atmosphere, which reflects Light in the Crepusculum, is not fully 45 Miles Notwithstanding which, 'tis still manifest that some sort of Vapours, and those in no small Quantity, arise nearly to that Height. An Instance of this may be given in the great Light the Society had an account of (vid. Transact. Sept. 1676) from Dr. Wallis, which was seen in very distant Counties almost over all the South Part of England. Of which though the Doctor could not get so particular a Relation, as was requisite to determine the Height thereof, yet from the distant Places it was seen in, it could not but be very many Miles high.

So likewise that Meteor which was seen in 1708. on the 31st of July, between Nine and Ten a Clock at Night, was evidently between 40 and 50 Miles perpendicularly high, and as near as I can gather, over Shereness and the Buoy on the Nore. For it was seen at London moving horizontally from E. by N. to E. by S. at least 50 Degrees high, and at Redgrave in Suffolk, on the Yarmouth Road, about 20 Miles from the East Coast of England, and at least 40 Miles to the Eastward of London, it appeared a little to the Westwards of the South, suppose S. by W. and was seen about 30 Degrees high, sliding obliquely downwards. I was shown in both Places the Situation thereof, which was as described, but could wish some Person skill'd in Astronomical Matters had seen it,
that we might pronounce concerning its Height with more certainty: Yet, as it is, we may securely conclude, that it was not many Miles more Westerly than Redgrave, which, as I said before, is above 40 Miles more Easterly than London. Suppose it therefore, where perpendicular, to have been 35 Miles East from London, and by the Altitude it appear'd at in London, viz. 50 Degrees, its Tangent will be 42 Miles, for the Height of the Meteor above the Surface of the Earth; which also is rather of the least, because the Altitude of the Place shewn me, is rather more than less than 50 Degrees: and the like may be concluded from the Altitude it appear'd in at Redgrave, near 70 Miles distant. Though at this great Distance, it appear'd to move with an incredible Velocity, darting, in a very few Seconds of Time, for about 12 Degrees of a great Circle from North to South, being very bright at its first Appearance; and it died away at the End of its Course, leaving for some time a pale Whiteness in the Place, with some Remains of it in the Track where it had gone; but no hissing Sound as it pass'd, or Bounce of an Explosion were heard.

It may deserve the Honourable Society's Thoughts, how so great a Quantity of Vapour should be raised to the very Top of the Atmosphere, and there collected, so as upon its Accension or otherwise Illumination, to give a Light to a Circle of above 100 Miles Diameter, not much inferior to the Light of the Moon; so as one might see to take a Pin from the Ground in the otherwise dark Night. 'Tis hard to conceive what sort of Exhalations should rise from the Earth, either by the Action of the Sun or Subterranean Heat, so as to surmount the extreme Cold and Rariness of the Air in those upper Regions: But the Fact is indisputable, and therefore requires a Solution.

Like to this, but much more considerable, was that famous Meteor which was seen to pass over Italy on the 21st of March 0. S. Anno 1676. about an Hour and Three Quarters after Sun set, which happen'd to be observed and well consider'd by the famous Professor of Mathematicks in Bononia.
Gemini Montanari, as may be seen in his Italian Treatise about it, soon after published at Bononia. He observes that at Bononia, its greatest Altitude in the S. S. E. was 38 Degrees, and at Siena 58 to the N. N. W.; that its Course by the Concurrency of all the Observers was from E. N. E. to W. S. W., that it came over the Adriatick Sea as from Dalmatia; that it crost over all Italy, being nearly vertical to Rimini and Savigniano on the one Side, and to Leghorn on the other: that its perpendicular Altitude was at least 38 Miles; that in all Places near this Course, it was heard to make a hissning Noise as it passed, Ronzare, Far fi sacrificato comme un fusco artificiale, Fischiare per aria comme un Raggio di polvere; that having past over Leghorn it went off to Sea towards Corsica, and lastly that at Leghorn it was heard to give a very great Blow, Tuono di maggior rumore di grossa Cannonata; immediately after which another sort of Sound was heard like the rattling of a great Cart running over Stones, which continued about the time of a Credo.

He concludes from the apparent Velocity it went on with at Bononia, at above 50 Miles Distance, that it could not be less swift than 160 Miles in a Minute of Time, which is above Ten times as swift as the diurnal Rotation of the Earth under the Equinoctial, and not many times less than that wherewith the annual Motion of the Earth about the Sun is performed. To this he adds the Magnitude thereof, which appeared at Bononia bigger than the Moon in one Diameter, and above half as big again in the other; which with the given Distance of the Eye, makes its real lesser Diameter above half a Mile, and the other in proportion. This supposed, it cannot be wondered that so great a Body moving with such an incredible Velocity through the Air, though so much rarified as it is in its upper Regions, should occasion so great a hissning Noise, as to be heard at such a Distance as it seems this was. But 'twill be much harder to conceive, how such an impetus could be impressed on the Body thereof, which by many Degrees exceeds that of any Cannon Ball; and how this
this impetus should be determined in a direction so nearly parallel to the horizon; and what sort of substance it must be, that could be so impelled and ignited at the same time: there being no Vulcano or other Spiraculum of subterraneous fire in the N. E. parts of the world, that we ever yet heard of, from whence it might be projected.

I have much considered this appearance, and think it one of the hardest things to account for, that I have yet met with in the Phenomena of Meteors, and am induced to think that it must be some collection of matter form'd in the æther, as it were by some fortuitous concourse of atoms, and that the earth met with it as it past along in its orb, then but newly formed, and before it had conceived any great impetus of descent towards the sun. For the direction of it was exactly opposite to that of the earth, which made an angle with the meridian at that time (the sun being in about 11 degrees of aries) of 67 gr. that is, its course was from W. S. W. to E. N. E. wherefore the meteor seem'd to move the contrary way: and besides falling into the power of the earth's gravity, and losing its motion from the opposition of the medium, it seems that it descended towards the earth, and was extinguish'd in the tyrrhene sea, to the W. S. W. of Leghorn. The great blow being heard upon its first immersion into the water, and the rattling like the driving a cart over stones being what succeeded upon its quenching; something like which is always observed upon quenching a very hot iron in water. These facts being past dispute, I would be glad to have the opinion of the learned thereon, and what objection can be reasonably made against the above-said hypothesis, which I humbly submit to their censure.

P. S. Since this was written, there has fallen into my hands an account of much such another appearance, seen in Germany, in the Year 1686, at Leipzig, by the late Mr. Gottfried Kirch, who was for many years a very diligent observer of the heavens.
Heavens, and perfectly well instructed in Astronomical Matters. He in an Appendix to his Ephemerides for the Year 1688, gives us this remarkable Relation in the following Words.

Die 9 Jul. st. vct. Hora 1 ½ matutina, Globus ardens cauda preditus in 8 ½ Gr. Aquarii & 4 Gr. Sept. apparuit, quia per semi-quadrantem Hora immotus perstitit, cujus diameter semidiametrum Lune circiter aquabat. Primo lux tanta erat, ut eurus ope fine candelis legere potuissimus: postea pedetentim in loco suo evanescebat. Phenomenon istud dico tempore multis aliis in locis pariter visum est, præsertim Schlaizii, oppido undecim milliaribus Germanicis abhinc (i. e. a Lipsia) versus Meridiem distante, altitudine circiter 60 Gr. ab Horizonte meridiano.

At the time of this Appearance the Sun was in 26 ½ Gr. of ©, and by the given Place of the Meteor, 'tis plain, it was seen about ¾ of an Hour past the Meridian, or in S. by W. and by its Declination it could not be above 24 Degrees high at Leipsic, though the same, at Schlaize was about 60 Gr. high: The Angle therefore at the Meteor was about 36 Gr. Whence by an easy calculus it will be found, that the same was not less than 16 German Miles distant in a right Line from Leipsic, and above 6 ½ such Miles perpendicular above the Horizon, that is at least 30 English Miles high in the Air. And though the Observer says of it, immotus perstitit per semi-quadrantem hore; 'tis not to be understood that it kept its Place like a Fixt Star, all the time of its Appearance; but that it had no very remarkable progressive Motion. For himself has at the End of the said Ephemerides given a Figure of it, which he has marked Fig. D, whereby it appears that it darted downwards obliquely to the Right-Hand, and where it ended, left two Globules or Nodes, not visible but by an Optick Tube.

The same Mr. Gottfried Kirch in the beginning of a German Treatise of his, concerning the great Comet which appeared in the Year 1680, intituled Neue Himmels Zeitung, D d 2
printed at Nurenbürg anno 1681, (of which perhaps we shall have further occasion to make mention) gives us a Relation of such another luminous Meteor seen likewise at Leipsick on the 22d of May 1680. about three in the Morning: which though himself saw not, was yet there observ'd by divers Persons who made various Reports of it, but the more intelligent agreed that it was seen descending in the North, and left behind it a long white Streak where it had past. At the same time at Haarburgh the like Appearance was seen in N.E. or rather N N.E.; as also at Hamburg, Lubeck and Stralsund, all which are about 40 German Miles from Leipsick: but in all these Places, by Persons unacquainted with the manner of properly describing things of this kind. So that all we can conclude from it is, that this Meteor was exceeding high above the Earth, as well as the former.

All the Circumstances of these Phænomena agree with what was seen in England in 1708, but it commonly so happens that these contingent Appearances escape the Eyes of those that are best qualified to give a good Account of them. 'Tis plain however that this sort of luminous Vapour is not exceedingly seldom thus collected; and when the like shall again happen, the Curious are entreated to take more Notice of them than has been hitherto done, that we may be enabled thereby better to account for the surprizing Appearances of this sort of Meteor.
V. Some Remarks on the Variations of the Magnetical Compas published in the Memoirs of the Royal Academy of Sciences, with regard to the General Chart of those Variations made by E. Halley; as also concerning the true Longitude of the Magellan Streights.

It must be acknowledged that the Gentlemen of the Royal Academy of Sciences in France, have, for some Years past, apply'd themselves with much Candour, as well as Diligence, to examine the Chart I publish'd in the Year 1701, for shewing at one View the Variations of the Magnetical Compas, in all those Seas with which the English Navigators are acquainted: And, to my no small Satisfaction, I find that what I did so long ago, has been since abundantly verified by the concurrent Reports of the French Pilots, who of late have had frequent Opportunities of enquiring into the Truth thereof. So that I am in hopes I have laid a sure Foundation for the future Discovery of an Invention, that will be of wonderful Use to Mankind when perfected; I mean that of the Law or Rule by which the said Variations change, in Appearance regularly, all the World over. Of this I have adventured long since to give my Thoughts in No. 148 and No. 195 of these Transactions, and as yet I see no Cause to retract what I there offer for a Reason of this Change; but of this we might be more certain, had we a good Collection of Observations made in that Ocean which divides Asia and America, and occupies about two fifths of the whole Circumference of the Globe. This, we hope, from the natural Curiosity of the French (who want no Means of performing it) may be effectually supply'd by such of that Nation who may return from Peru by the East-Indies.
In the mean time I cannot omit to take Notice of two Particulars, seeming to call in Question the Truth of my aforesaid Map, which I have lately observed in the Memoirs of the Royal Academy of Sciences.

The one is in the Memoirs of the Year 1700, concerning the Variation observed at Paraíba in Brašile, about 25 Leagues to the Northwards of Pernambouc, by M. Couplet le fils, whose Words are these,

Le 20 Mai, 1698. ayant auparavant tracé seigneusement une ligne Meridienne, dont je m’étois servi pour les Observations Astronomiques, observai la declinaison de l'aiguille aimantée de 5° 35' Nordouest. And the same Observer tells us, that he found the Latitude of the Town of Paraíba 6° 38' 18''.

Now it so fell out, that my self was in the River of Paraíba, in the Month of March, 1699. and there fitted and cleaned my Ship, so that I had full Opportunity to observe the Variation both on Board and on Shore, and found it constantly to be above 4 Gr. North-East; so that I am willing to believe this to be an Error of the Press, putting N. W. for N. E; or rather of the Memory of M. Couplet, who, it seems, lost all his Papers by Shipwreck in his Return. The like may be said of the Latitude of Paraíba, which, though I did not observe my self, yet at the Fort of Cabo Dello, at the Mouth of the River, and which is about 3 Leagues more Northerly than the Town, I found the Latitude not less than 6° 55' South, and by Consequence that of the Town more than 7 Degrees.

The other is in a Discourse of M. de Lisle, in the Memoirs of 1710; where he compares the Variations observed in some late Voyages, with my Map of the Variations. Among other Things, 'tis there said, that on the East-side of the Island St. Thomas, under the Equinoctial Line, M. Bigot de la Canté, second Lieutenant of the King's Ship la Sphere, had, in the beginning of the Year 1708, found the Variation 11; Gr. whereas my Chart makes it but 5; Gr. 'Tis true, that I never observed my self in those Parts; and 'tis from
from the Accounts of others, and the Analogy of the whole; that in such Cases I was forc'd to supply what was wanting; and 'tis possible that there may be more Variation on that Coast than I have allowed. But consulting my Chart (which was fitted to the Year 1700) I find then make the Variation at the Isle of St. Thomas full 7½ Gr. and not 5½ Gr. the which, by the Year 1708, might well arise to nearly r. So that the Difference will become very tolerable; whereas an Error of 6 Degrees, such as is here represented, would render the Credit of my Chart justly suspected, and the same by consequence wholly useless, as not to be confided in.

But a further Thing I might complain of is, that in the same Memoire of M. de Lisle, the Geography of my Chart is called in Question; and we are told that I have placed the Entrance of the Magellan Straights at least 10 Degrees more Westerly than I ought to have done: for that the Ship St. Louis, in the Year 1708, failing from the Mouth of Rio Gallega, in about the Latitude of 52 Gr. South, and not far from Cape Virgin, directly for Cape Bonne Esperance (which Course perhaps was never run before) had found the Distance between the two Lands not more than 1350 Leagues, which, he concludes, is much less than my Chart of the Variation makes it. I know not from what Computation M. de Lisle has deduced this Consequence, but I find by my Chart that I have made the Longitude of Rio Gallega 75 Gr. West from London, and that of Cape Bonne Esperance 16½ East from it; that is in all 91½ Gr. difference of Longitude. This, with the two Latitudes, gives the Distance, according to the Rhumb-line 1364 Leagues, but according to the Arch of a great Circle, no more than 1287 Leagues; so that instead of invalidating what I have there laid down, it does absolutely confirm it, as far as the Authority of one single Ship's Journals can do it.
I do not pretend that I have had Observations made with all the Precision requisite, to lay down incontestably the Magellan Straights in their true Geographical Site; but yet it has not been without good Grounds that I have placed them as I have done. For when Sir John Narborough, in the Year 1670, wintered in Port St. Julian, on the Coast of Patagonia, Capt. John Wood, then his Lieutenant, and an approved Artist in Sea Affairs, did observe the beginning of an Eclipse of the Moon, Sept. 18° Stil. vet. at just 8° of Night: And the same beginning was observ'd by M. Huygens at Dantzick at 14° 22'; whence Port St. Julian is more Westerly than Dantzick 6° 22', or than London 5° 6', that is 76° 5 Gr. Besides, I have had in my Custody a very curious Journal of one Capt. Strong, who went into the South Seas in quest of a rich Plate-wreck, and who discover'd the two Islands he called Falkland's Isles, lying about 120 Leagues to the Eastwards of the Patagon Coast, about the Lat. of 51°. This Capt. Strong had a quick Passage from the Island of Trinadada (in 20° South) to the Magellan Straights; and in this Journal, which was very well kept, I found that Cape Virgin was, by his Account, 45 Degrees of Longitude more Westerly than that Island, whose Longitude I know to be just 30 Degrees from London: that is in all 75 Gr.

From these concurrent Testimonies, wanting better, I ventured to fix the Longitude of this Coast as I have done; and I can by no means grant an Error of 10 Degrees to be possible in it, though perhaps it may need some smaller Correction. I will however readily grant, that those that go thither from Europe, shall find the Land more Easterly than is here express'd, by reason of a constant Current setting to the Westward near the Equator, where Ships are many times long detained by Calms, whilst the Stream carries them along with it; which thing befalls all Ships bound to any Part of the East Coast of the South America.

Printed for William Innys, at the Prince's-Arms in St. Paul's Church-yard, MDCCXV.
Curulis Cometes Annii 1680 Observations primae Coburgo Saxoniae ad D. no Gottfried Kirch habitae.

Philosoph. Transact. N° 542

Fig. I.
PHILOSOPHICAL
TRANSACTIONS.

For the Months of January and February, 1717.

The CONTENTS.

I. Observationes quaedam accuratæ insignis Cometes sub finem anni 1680 visti, Coburgi Saxoniæ a Domino Gottfried Kirch habitæ; decimo tertio die ante quam a quoquam alio observatus sit.

II. An Account of the Book entitled Commercium Epistolicum Collinii & aliorum, De Analyse promota; published by order of the Royal-Society, in relation to the Dispute between Mr. Leibnitz and Dr. Keill, about the Right of Invention of the Method of Fluxions, by some call'd the Differential Method.
I. Observationes quaedam accuratæ insignis Cometeæ sub finem anni 1680 visit, Coburgi Saxonœ a Domino Gottfried Kirch habiæ; decimo terto die antequam a quoquam alio observatus sit.

COMETA ille qui anno exeunte 1680 visus est, plurimis de causis praecipuus merito habendus est; tamen ob Cursum ejus quadrimestrem, quo novem integra Signa percurrunt; quam ob immensam Caudæ magnitudinem & claritatem: maxime vero ob insignem Orbitæ Curvitatæm, cujus ope tandem patefacta est Cometæ Theoria. Nam dum Astronomi omni adhibita diligentia motum ejus observationibus deśiniendo desiderarunt, conatus eorum secundavit D. Newtoni in Geometricis vis pene divina, qui primus mortaliæ Cometas Orbes Parabolicis maxime affines describere pro- bavit, ac datis tribus locis accurato observatis eisdem consu- mere docuit, remque hujus Cometeæ exemplo illustravit: id quod sub finem Lib. III. Principiorum Philosophiæ videre est.

Accidit autem, nequeo quo facto, ut Cometa hic (quem vespertinum tantopere prosecuti sunt Coelispices) antequam Solæm attigerat matutinum, nec Parisis neque Germani ne semel quidem observatus sit: quique cum viderunt & observarunt, incongrua & inter se pugnantiæ, ac pro rei subtilitate parum idonea prodideræ: neque ante Novemb. 17. mane a quopiam Observationibus visus est. Unde factum ut Orbitæ pars illa, qua ad Solæm descendit Cometa, paulo incertius definiri potuit. Bona autem fortuna nuper incidimus in Librum meritissimi Astronomi Domini Gottfried Kirch, Germani, anno 1681 Noribergæ impressi, cui titulus NEWE HIMMELS ZEITUNG, hoc est NOVUS NUNCIUS COELISTIS; ubi auctorum diligentissimos nobis exponit, quo eam ductus Cometa hunc non- tum adnata Cauda obscurum, ac vix nudis oculis conspicuum detec-
detexerit; dum scilicet Lunam & Martem ei vicinum obser-
vaturiens circumluстрaret, die Novemb. 4. S. V. mane, idque
Coburgi Saxoniæ, quod oppidum XI. Grad. Londino orienta-
lius est, sub altitudine poli 50° 20' circiter. Excitatus au-
tem, ut ait, rumoribus Cometae in Germania vidi, vultu in
Orientem verso pernoctavit, ut coelo tum forte perquam se-
reno, si quid novi orietur, situm ejus notaret. Luna vero
jam ad stellam aliquam Tychoi incognitam applicatâ, (sed
quæ in Catalogo Flamsteedii Britannico habetur, etque nume-
ro 44a Leonis) voluit diæ stellæ locum ex circumvicinis
Fixis determinare; dumque Tuum trium graduum capax
hinc inde circumrotat, incidit in Luculam quandam nebulo-
sam, speciem insolitam praee se serentem, quamque vel novum
Cometam esse, vel Stellam nebulosam ad instar ejus quæ in
Cingulo Andromede est, statim conclusit.

Primo autem Cometam vidit, Hora 4½ matutinâ. paulo
altiorem dubius stellulis Telecopicis, que in Figura I. literis
a & c signatur, cum quibus tamen Hora 6a vitus est in linea
accurate rectâ; unde conflabat eum moveri, idque motu di-
recto. Inter horas vero 5. & 6. Tubo decempedali Phen-
omenon hoc contemplatus est, vidique duas aliás stellulas
contiguas prioribus minores, literisque e & d notatas, & su-
pra hæ tertiam g. Erat autem distantia Cometae ab e paulo
minor quam ab a, major vero distantia de. 6h. 38' distantia
Cometae ab e dupla erat intervalli inter ipsas d e, ac linea
de producta reliquit Cometam infra se, sic tamen ut mar-
ginem ejus superiorem attingeret. 6h. 45'. Cometa jam sen-
sibiliter remotor erat ab e quam ab a, distabarque ab a paulo
plus quam dimidia distantia stellularum a & g.

Notandum vero est Horologium totis 14 minutis coelum
anticipasse, uti ex altitudinibus Cordis Leonis tum captis pa-
tuit.

Nobilis fane est hæc observatio, adeoque Stellumarum Co-
metae tunc adjacentium loca non una methodo perquisivimus,
exercitatisfimam manum suam & instrumenta perelegantia.

* Tubo scilicet objecta invertente.
praebente Reverendo Viro D. Jacobo Pound, R. Soc. Sodali. Unde constabar stellulas illas tum temporis intrafscriptos habisse situs, nempe

Long. Lat.
a 29° 54'. 20° 1. 29° 20 Bor.
d 29° 27'. 20 8° 10 co
e 29° 34'. 30 10° 45

Circulus autem maximus per a & c ductus, deprehensus est per Ultimam Caudam Urse maioris transfire, adeoque angulum cum circulo Longitudinis ad esse 15° 36'. Cumque distantia Cometae ab a versus c paulo major fuerat dimidio distantiae a g, (quam Tubo pedem & Micrometer deprehendimus 2 h 15 min.) ponamus eam suisse duodecim minutorum; & ex datis proveniet Cometae locus ad 29° 5 1' cum Lat. Borae 18° 17'. Hora (sicilet Horologii 6°), sed Londini 5° 2 Temp. Appar.

Deinde Novemb. 6. mane, 4° 42', Tubo bipedali deprehendi Cometam omnino in linea recta inter Martem & Stellarem N; quae quidem in Catalogo Britannico 45° Leonis est, & tunc erat in 2° 42' cum Lat. Aust. 0° 16'. Mars autem tum temporis habuit (ex collatis observationibus paulo antea & post factis) 3° 46' cum Lat. Bor. 1° 56'. Unde, ob datam ejus viam, Cometa occupavit 3° 23' cum Lat. Bor. 1° 6'. Londini Temp. App 3° 58' mane.

Novembri quoque 11° 5° 15° mane, Cometa æqualiter distabat à Stellis Leonis σ & τ Bayero, nondum vero attigit rectam eadtem jungentem, sed parum absuit ab ea. In Catalogo Britannico σ tune habuit 14° 15'. cum Lat. Bor. 1° 41', σeτ, τ vero 17° 3' cum Lat. Aust. 0° 34'. Proinde Cometae Latitudine paulo minor erat medio inter illas, nempe quam 0° 33', Bor. ac Longitudine quam 15° 39'. Sed hodie non utique fidendum, cum pendebat ab æstimata æqualitate distantiarum, quae res lubrica est. Cauda autem jam capta non nisi dimidio gradu longa Tubo decempedali visæ est. Qui plura cupit adeat Librum ipsum Germanicè editum.
II. An Account of the Book entituled Commercium Epistolicum Collinii & aliorum, De Analyti promota; published by order of the Royal-Society, in relation to the Dispute between Mr. Leibnitz and Dr. Keill, about the Right of Invention of the Method of Fluxions, by some call'd the Differential Method.

Several Accounts having been published abroad of this Commercium, all of them very imperfect: It has been thought fit to publish the Account which follows.

This Commercium is composed of several ancient Letters and Papers, put together in order of Time, and either copied or translated into Latin from such Originals as are described in the Title of every Letter and Paper; a numerous Committee of the Royal-Society being appointed to examine the Sincerity of the Originals, and compare therewith the Copies taken from them. It relates to a general Method of resolving finite Equations into infinite ones, and applying these Equations, both finite and infinite, to the Solution of Problems by the Method of Fluxions and Moments. We will first give an Account of that Part of the Method which consists in resolving finite Equations into infinite ones, and squaring curvilinear Figures thereby. By Infinite Equations are meant such as involve a Series of Terms converging or approaching the Truth nearer and nearer in infinitum, so as at length to differ from the Truth less than by any given Quantity, and if continued in infinitum, to leave no Difference.
Dr. Wallis in his Opus Arithmeticum published A.C. 1657: Cap. 33. Prop. 68. reduced the Fraction \( \frac{a}{1-r} \) by perpetual Division into the Series \( a + AR + AR^2 + AR^3 + AR^4 + \&c. \)

Viscount Brounker squared the Hyperbola by this Series
\[
\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \&c. \quad \text{that is by this,} \quad 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \&c. \quad \text{conjoyning every two Terms into one.} \quad \text{And the Quadrature was published in the Philosophical Transactions for April 1668.}

Mr. Mercator soon after published a Demonstration of this Quadrature by the Division of Dr. Wallis; and soon after that Mr. James Gregory published a Geometrical Demonstration thereof. And these Books were a few Months after sent by Mr. John Collins to Dr. Barrow at Cambridge, and by Dr. Barrow communicated to Mr. Newton (now Sir Isaac Newton) in June 1669. Whereupon Dr Barrow mutually sent to Mr. Collins a Tract of Mr. Newton’s entitled Analysis per equationes numero terminorum infinitas. And this is the first Piece published in the Commercium, and contains a general Method of doing in all Figures, what my Lord Brounker and Mr. Mercator did in the Hyperbola alone. Mr. Mercator lived above ten Years longer without proceeding further than to the single Quadrature of the Hyperbola. The Progress made by Mr. Newton shews that he wanted not Mr. Mercator’s Assistance. However, for avoiding Disputes, he supposes that my Lord Brounker invented, and Mr. Mercator demonstrated, the Series for the Hyperbola some Years before they published it; and, by consequence, before he found his general Method.

The aforesaid Treatise of Analysis Mr. Newton, in his Letter to Mr. Oldenburg, dated Oct. 24. 1676, mentions in the following Manner. *Eo ipso tempore quo Mercatoris Logarithmotechnia prodiit, communicatum est per amicum D. Barrow (tunc Matheos Professor Cantab) cum D. Collinio Compendium quoddam barum Serierum, in quo significaveram Areas & Longitudines Curvarum omnium, & Solidorum superficies & contenta ex datis*
Mr. Collins in the Years 1669, 1670, 1671 and 1672 gave notice of this Compendium to Mr. James Gregory in Scotland, Mr. Bertet and Mr. Vernon then at Paris, Mr. Alphonso Borelli in Italy, and Mr. Strode, Mr. Townsend, Mr. Oldenburgh, Mr. Dary and others in England, as appears by his Letters. And Mr. Oldenburg in a Letter, dated Sept. 14. 1669, and entered in the Letter-Book of the Royal-Society, gave notice of it to Mr. Francis Sluus at Liege, and cited several Sentences out of it. And particularly Mr. Collins in a Letter to Mr. James Gregory dated Novemb. 25. 1669. spake thus of the Method contained in it. Barrovius Provinciam suam publicè praelegendi remisit cuidam nomine Newtono Cantabrigienfi, cujus tanguam vixi acutissimo ingenio præditi in Prefatione Praælectionum Opticarum, meminit: quippe antequam ederetur Mercatoris Logarithmotechnia, eandem methodum adinvenerat, eamque ad omnès Curvas generaliter & ad Circulum diversimodo applicat. And in a Letter to Mr. David Gregory dated August 1. 1676. he mentions it in this manner. Paucos post menses quam editi sunt hi Libri [viz. Mercatoris Logarithmotechnia & Exercitationes Geometricæ Gregorii] misi sunt ad Barrovium Cantabrigiæ. Ille autem responsum dedit hanc infinitarum Serierum Doctrinam à Newtono biennium ante excogitam fuisse quam ederetur Mercatoris Logarithmotechnia & generaliter omnibus figuris applicatam, simulque transmisit D. Newtoni opus manuscriptum. The last of the said two Books came out towards the End of the Year 1668, and Dr. Barrow sent the said Compendium to Mr. Collins in July following, as appears by three of Dr. Barrow's Letters. And in a Letter to Mr. Strode, dated July 26. 1672, Mr. Collins wrote thus of it. Exemplar ejus [Logarithmotechniæ] misi Barrovio Cantabrigiam, qui quasdam Newtoni chartas extemplo remisit: E quibus & aliis qua præs ab authoræ eum Barrovio communicata fuerant, pastem illum methodum à dito Newtono aliquot annis ante ea excogitam & modo universal...
sali applicata fuisse: Ita ut ejus opè, in quavis Figura Curvilinea propofita, qua una vel pluribus proprietatibus definitur, Quadratura vel Area dictæ figūræ, accurata si possibile fit, sin minus infinitè verò propinquâ, Evolutio vel longitūdō Lineæ Curvæ, Centrum gravitatis figūræ, Solida ejus rotationale genitâ & eorum superficies; sinuella radicum extractione obtineri quæant. Postquam intellexerat D. Gregorius hanc methodum ad D. Mercatore in Logarithmotechnia usurpatur & Hyperbole quadranda adhibitam, quamque adauxerat ipse Gregorius, jam universalem redditam esse, omnibusque figuris applicatam; acritæ studio eandem acquisivit multumque in ea enoandâ desudavit. Utque D. Newtonus & Gregorius in animo hâc ab hac methodum exornare: D. Gregorius autem D. Newtonum primum ejus inventorem anticipare haud integrum ducit. And in another Letter written to Mr. Oldenburgh to be communicated to Mr. Leibnitz, and dated June 14 1676. Mr. Collins adds: Hujus autem methodi ea est praebantia ut cum tam late pateat ad nullam hæredem difficultatem. Gregorium autem aliquosque in ea suisse opinione arbitrâ, ut quicquid uspiam antea de hac re innotuit, quasi dubia diluculi lux fuit si cum meridiana claritate conferatur.

This Tract was first printed by Mr. William Jones, being found by him among the Papers and in the Hand-writing of Mr. John Collins, and collated with the Original which he afterwards borrowed of Mr. Newton. It contains the above-mention'd general Method of Analysis, teaching how to resolve finite Equations into infinite ones, and how by the method of Moments to apply Equations both finite and infinite to the Solution of all Problems. It begins where Dr. Wallis left off, and founds the method of Quadratures upon three Rules.

Dr. Wallis published his Arithmetica infinitorum in the Year 1655, and by the 59th Proposition of that Book, if the Abscissa of any curvilinear Figure be called $x$, and $m$ and $n$ be Numbers, and the Ordinates erected at right Angles be $x^m$, the Area of the Figure shall be $\frac{n}{m+n}x^{m+n}$. And this is assumed by
by Mr. Newton as the first Rule upon which he founds his Quadrature of Curves. Dr. Wallis demonstrated this Proposition by Steps in many particular Propositions, and then collected all the Propositions into One by a Table of the Cases. Mr. Newton reduced all the Cases into One, by a Dignity with an indefinite Index, and at the End of his Compendium demonstrated it at once by his method of Moments, he being the first who introduced indefinite Indices of Dignities into the Operations of Analysis.

By the 108th Proposition of the said Arithmetica Infinitorum, and by several other Propositions which follow it; if the Ordinate be composed of two or more Ordinates taken with their Signes + and —, the Area shall be compos’d of two or more Areas taken with their Signes + and — respectively. And this is assumed by Mr. Newton as the second Rule upon which he founds his Method of Quadratures.

And the third Rule is to reduce Fractions and Radicals, and the affected Roots of Equations into converging Series, when the Quadrature does not otherwise succeed; and by the first and second Rules to square the Figures, whose Ordinates are the single Terms of the Series. Mr. Newton, in his Letter to Mr. Oldenburgh dated June 13. 1676. and communicated to Mr. Leibnitz, taught how to reduce any Dignity of any Binominal into a converging Series, and how by that Series to square the Curve, whose Ordinate is that Dignity. And being desired by Mr. Leibnitz to explain the Original of this Theoreme, he replied in his Letter dated Oct. 24. 1676, that a little before the Plague (which raged in London in the Year 1665) upon reading the Arithmetica Infinitorum of Dr. Wallis, and considering how to interpole the Series \( x, x - \frac{1}{3} x^3, x - \frac{2}{5} x^5, \ldots \), he found the Area of a Circle to be \( x - \frac{\frac{1}{2} x^3}{3} - \frac{\frac{1}{8} x^5}{5} - \frac{\frac{1}{18} x^7}{7} \ldots \). And by pursuing the Method of Interpolati-
on he found the Theoreme abovemention'd, and by means of this Theoreme he found the Reduction of Fractions and Surds into converging Series, by Division and Extraction of Roots; and then proceeded to the Extraction of affected Roots. And these Reductions are his third Rule.

When Mr. Newton had in this Compendium explained these three Rules, and illustrated them with various Examples, he laid down the Idea of deducing the Area from the Ordinate, by considering the Area as a Quantity, growing or increasing by continual Flux, in proportion to the Length of the Ordinate, supposing the Abscissa to increase uniformly in proportion to Time. And from the Moments of Time he gave the Name of Moments to the momentaneous Increases, or infinitely small Parts of the Abscissa and Area, generated in Moments of Time. The Moment of a Line he called a Point, in the Sense of Cavallerius, tho' it be not a geometrical Point, but a Line infinitely short, and the Moment of an Area or Superficies he called a Line, in the Sense of Cavallerius, tho' it be not a geometrical Line, but a Superficies infinitely narrow. And when he consider'd the Ordinate as the Moment of the Area, he understood by it the Rectangles under the geometrical Ordinate and a Moment of the Abscissa, tho' that Moment be not always expressed. Sit ABD, faith he, Curva quavis, & AHKB rectangulum cujus latus AH vel KB est unitas. Et cogita rectam DBK uniformiter ab AH motam areas ABD & AK describere; & quod [recta] BK (i) sit momentum quo [area] AK (x), & [recta] BD (y) momentum quo [area curvilinea] ABD gradatim angentar; & quod ex momento BD perpetim dato possis, per precedentes [tres] Regulas, aream ABD ipsa descriptam investigare, sine cum area AK (x) momento i descripta conferre. This is his Idea of the Work in squaring of Curves, and how he applies this to other Problems, he expresses in the next Words. Jam qua ratione, faith he, superficies ABD ex momento suo perpetim
petit dato per precedentes [tres] Regulas elicietur, eadem qualibet alia quantitas ex momento suo sic dato elicietur. Exemplo res fit clarius. And after some Examples he adds his Method of Regression from the Area, Arc, or solid Content, to the Abscissa; and shews how the same Method extends to Mechanical Curve, for determining their Ordinates, Tangents, Areas, Lengths, &c. And that by assuming any Equation expressing the Relation between the Area and Abscissa of a Curve, you may find the Ordinate by this Method. And this is the Foundation of the Method of Fluxions and Moments, which Mr. Newton in his Letter dated Octob. 24, 1676 comprehended in this Sentence. Data equatione quotcunque fluentes quantitates involvente, invenire Fluxiones; & vice versa.

In this Compendium Mr. Newton represents the uniform Fluxion of Time, or of any Exponent of Time by an Unit; the Moment of Time or of its Exponent by the Letter o; the Fluxions of other Quantities by any other Symbols; the Moments of those Quantities by the Rectangles under those Symbols and the Letter o; and the Area of a Curve by the Ordinate inclosed in a Square, the Area being put for a Fluent and the Ordinate for its Fluxion. When he is demonstrating any Proposition he uses the Letter o for a finite Moment of Time, or of its Exponent, or of any Quantity flowing uniformly, and performs the whole Calculation by the Geometry of the Ancients in finite Figures or Schemes without any Approximation: and so soon as the Calculation is at an End, and the Equation is reduced, he supposes that the Moment o decreases in infinitum and vanishes. But when he is not demonstrating but only investigating a Proposition, for making Dispatch he supposes the Moment o to be infinitely little, and forbears to write it down, and uses all manner of Approximations which he conceives will produce no Error in the Conclusion. An Example of the first kind you have in the End of this Compendium, in demonstrating the first of the three Rules laid down in the Beginning of the Book.
Examples of the second kind you have in the same Compendium, in finding the Length of Curve Lines p. 15. and in finding the Ordinates, Areas and Lengths of Mechanical Curves p. 18. 19. And he tells you, that by the same Method, Tangents may be drawn to mechanical Curves p. 19. And in his Letter of Decemb. 10. 1672. he adds, that Problems about the Curvature of Curves Geometrical or Mechanical are resolv'd by the same Method. Whence its manifest, that he had then extended the Method to the second and third Moments. For when the Areas of Curves are considered as Fluents (as is usual in this Analysis) the Ordinates express the first Fluxions, the Tangents are given by the second Fluxions, and the Curvatures by the third. And even in this Analysis p. 16. where Mr. Newton faith, Momentum est superficies cum de solidis, & Linea cum de superficiebus, & Punctum cum de lineis agitur, it is all one as if he had said, that when Solids are considered as Fluents, their Moments are Superficies, and the Moments of those Moments (or second Moments) are Lines, and the Moments of those Moments (or third Moments) are Points, in the Sense of Cavallerius. And in his Principia Philosophiae, where he frequently considers Lines as Fluents described by Points, whose Velocities increase or decrease, the Velocities are the first Fluxions, and their Increase the second. And the Probleme, Data equatione fluentes quantitates involvente fluxiones invenire & vice versa, extends to all the Fluxions, as is manifest by the Examples of the Solution thereof, published by Dr. Wallis Tom. 2. p 391, 392, 396. And in Lib. II. Princip. Prop. xiv. he calls the second Difference the Difference of Moments.

Now that you may know what kind of Calculation Mr. Newton used in, or before the Year 1669. when he wrote this Compendium of his Analysis, I will here set down his Demonstration of the first Rule abovementioned. Sit
Sit Curve alique AD & Basis AB = x, perpendiculariter applicata BD = y, & area ABD = z, ut primum. Item sit \( B\beta = o, \) BK = v, \& Rectangulum \( B\beta HK \) (ov) equale spatio \( B\delta \) AD. Est ergo \( A\beta = x + o, \) \& \( A\delta = z + ov. \) His premisisse, ex relatione inter \( x \) \& \( z \) ad arbitrium assumpta, quero \( y \) ut sequitur.

Pro habitu sumatur [equation] \( \frac{1}{2} \frac{x^2}{y} = z, \) sive \( \frac{x^2}{y} = zz. \) Tum \( x + o \) (A\beta) pro x, \& \( z + ov \) (A\delta \beta) pro z substitutis, prodibit \( \frac{1}{2} \) in \( x^2 + 3 x^3 o + 3 x^2 o^2 + o^3 = \) (ex natura Curve) \( z^2 + 2 z + o + ov^2. \) Est sublatis \( \frac{1}{2} \frac{x^2}{y} \) \& \( z + 2 z + ov + o^2 v. \) Si jam supponamus \( B\beta \) in infinitum diminui \& evanescere, sive о esse nihil, erunt \( v \) \& \( y \) equales, \& termini per o multiplicati evanescent; ideoque restabit \( \frac{1}{2} x \) \( x = 2 z v, \) sive \( \frac{1}{2} x \) \( x = (z y) = \frac{1}{2} \frac{x^2}{y}, \) sive \( x z (\frac{x^2}{y}) = y. \) Quare \& contra, si \( x^2 = y, \) erit

\( \frac{1}{2} \frac{x^2}{y} = z. \)

Vel generaliter, Si \( \frac{n}{m+n} \times ax^{m+n} = z; \) sive ponendo \( \frac{na}{m+n} \)

\( = c, \) \& \( m+n = p, \) Si \( c x^p = z, \) sive \( c^n x^p = z^n. \) Tum \( x + o \)

pro x, \& \( z + ov \) sive (quod perinde est) \( z + o + y \) pro z substitutis, prodit \( c^n \times x^p + p o x^{p-1} \) \&c. \( = z^n + n o y z^{n-1} \) \&c. religuis nempte [Serierum] terminis, qui tandem evanescent, omiissis. Jam sublatis \( c^n x^p \) \& \( z^n \) equalibus, religisque per o divisitis, restat \( c^p x^{p-1} = ny z^{n-1} (\frac{nyz^n}{z} = \frac{ny c^n x^p}{c x^n}) \) sive dividiendo per \( c^n x^p, \) erit \( px^{p-1} = \frac{ny}{c x^n} \) sive \( px^{p-n} \) \( = ny; \) vel re-

stituendo \( \frac{na}{m+n} \) \( pro c \) \& \( m+n \) \( pro p, \) hoc est \( m \) \( pro p - n,\) \& \( na \)

\( 2. \)

H h pro
\[ \text{prop. c. siet } a \frac{m}{n} = y. \quad \text{Quare } e \text{ contra, } ax^{\frac{m}{m+n}} = y \text{ erit } a x^{\frac{n}{m+n}}. \]

\[ = z \quad \text{Q. E. D.} \]

By the same way of working the second Rule may be also demonstrated. And if any Equation whatever be assumed expressing the Relation between the Abscissa and Area of a Curve, the Ordinate may be found in the same manner, as is mentioned in the next Words of the Analysis. And if this Ordinate drawn into an Unit be put for the Area of a new Curve, the Ordinate of this new Curve may be found by the same Method: And so on perpetually. And these Ordinates represent the first, second, third, fourth and following Fluxions of the first Area.

This was Mr. Newton's Way of working in those Days, when he wrote this Compendium of his Analysis. And the same Way of working he used in his Book of Quadratures, and still uses to this Day.

Among the Examples with which he illustrates the Method of Series and Moments set down in this Compendium, are these. Let the Radius of a Circle be 1, and the Arc \( z \), and the Sine \( x \), the Equations for finding the Arc whose Sine is given, and the Sine whose Arc is given, will be

\[ z = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \frac{1}{9}x^9 + \&c. \]
\[ x = z - \frac{1}{3}z^3 + \frac{1}{5}z^5 - \frac{1}{7}z^7 + \frac{1}{9}z^9 - \&c. \]

Mr. Collins gave Mr. Gregory notice of this Method in Autumn 1669, and Mr. Gregory, by the Help of one of Mr. Newton's Series, after a Year's Study, found the Method in December 1670, and two Months after, in a Letter dated Feb. 15, 1671, sent several Theorems, found thereby, to Mr. Collins, with leave to communicate them freely. And Mr. Collins was very free in communicating what he had received both from Mr. Newton and from Mr. Gregory, as appears by his Letters printed in the Commercium. Amongst the Series which Mr Gregory sent in the said Letter, were these
these two. Let the Radius of a Circle be \( r \), the Arc \( \alpha \), and the Tangent \( t \), the Equations for finding the Arc whose Tangent is given, and the Tangent whose Arc is given, will be these.

\[
a = t - \frac{t^3}{3r^2} + \frac{t^5}{5r^4} - \frac{t^7}{7r^6} + \frac{t^9}{9r^8} - \&c.
\]

\[
t = a + \frac{a^3}{3r^2} + \frac{2a^5}{15r^4} + \frac{17a^7}{315r^6} + \frac{62a^9}{2835r^8} + \&c.
\]

In this Year (1671) Mr. Leibnitz published two Tracts at London, the One dedicated to the Royal-Society; the Other dedicated to the Academy of Sciences at Paris; and in the Dedication of the First he mentioned his Correspondence with Mr. Oldenburgh.

In February 1672, meeting Dr. Pell at Mr. Boyle’s, he pretended to the differential Method of Mouton. And notwithstanding that he was shewn by Dr. Pell that it was Mouton’s Method, he persisted in maintaining it to be his own Invention, by reason that he had found it himself without knowing what Mouton had done before, and had much improved it.

When one of Mr. Newton’s Series was sent to Mr. Gregory, he tried to deduce it from his own Series combined together, as he mentions in his Letter dated December 19, 1670. And by some such Method Mr. Leibnitz, before he left London, seems to have found the Sum of a Series of Fractions decreasing in Infinitum, whose Numerator is a given Number and Denominators are triangular or pyramidal or triangulo-triangular Numbers, \&c. See the Mystery! From the Series \( \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \&c. \) subduct all the Terms but the first (viz. \( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \&c. \) and there will remain \( 1 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \&c. \). And from this Series take all the Terms but the first, and there will remain \( \frac{1}{2} = \frac{2}{1 \times 2} + \frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{1}{4 \times 5} + \&c. \). And from the first Series take all the Terms but the two first, and there will remain \( \frac{3}{2} = \frac{2}{1 \times 2} + \frac{2}{2 \times 3} + \frac{2}{3 \times 4} + \frac{2}{4 \times 5} + \&c. \).
In the End of February or beginning of March 1677, Mr. Leibnitz went from London to Paris, and continuing his Correspondence with Mr. Oldenburg and Mr. Collin, wrote in July 1674, that he had a wonderful Theoreme, which gave the Area of a Circle or any Sector thereof exactly in a Series of rational Numbers; and in October following, that he had found the Circumference of a Circle in a Series of very simple Numbers, and that by the same Method (so he calls the said Theoreme) any Arc whose Sine was given might be found in a like Series, though the Proportion to the whole Circumference be not known. His Theoreme therefore was for finding any Sector or Arc whose Sine was given. If the Proportion of the Arc to the whole Circumference was not known, the Theoreme or Method gave him only the Arc; if it was known it gave him also the whole Circumference: and therefore it was the first of Mr. Newton's two Theoremes above-mention'd. But the Demonstration of this Theoreme Mr. Leibnitz wanted. For in his Letter of May 12. 1676, he desired Mr. Oldenburgh to procure the Demonstration from Mr. Collins, meaning the Method by which Mr. Newton had invented it.

In a Letter compos'd by Mr. Collins and dated April 15. 1675, Mr. Oldenburgh sent to Mr. Leibnitz Eight of Mr. Newton's and Mr. Gregory's Series, amongst which were Mr. Newton's two Series above mention'd for finding the Arc whose Sine is given, and the Sine whose Arc is given; and Mr. Gregory's two Series above mentioned for finding the Arc whose Tangent is given, and the Tangent whose Arc is given. And Mr. Leibnitz in his Answer, dated May 20. 1675, acknowledged the Receipt of this Letter in these Words. Literas suas multa fruge Algebraica referatas accipi, pro quibus tibi & doctissimo Collinio gratias ago. Cum nunc prater ordinari-as curas Mechanici is imprimit negotiis disfarbar, non potui examinare Series quas missis ac cum meis comparare. Ubi fecero perscrribam tibi sententiam meam: nam aliquot jam anni sunt quod inveni meas via quadam sic satisfinguliari.
But yet Mr. Leibnitz never took any further notice of his having received these Series, nor how his own differed from them, nor ever produced any other Series then those which he received from Mr. Oldenburgh, or numeral Series deduced from them in particular Cases. And what he did with Mr. Gregory's Series for finding the Arc whose Tangent is given, he has told us in the Acta Eruditorum mensis Aprilis 1691. pag. 178. Jam anno 1675, faith he, compositum habeam opusculum Quadrature Arithmetica ab amicis ab illo tempore lectum, &c. By a Theoreme for tranfmuting of Figures, like those of Dr. Barrow and Mr. Gregory, he had now found a Demonstration of this Series, and this was the Subject of his Opusculum. But he still wanted a Demonstration of the rest: and meeting with a Pretence to ask for what he wanted, he wrote to Mr. Oldenburg the following Letter, dated at Paris May 12. 1676.

*Cum Georgios Mohr Danus nobis attulerit communicatam sibi a Doctissimo Collinio vestro expressionem rationis inter arcum & sinum per infinitas Series sequentes; posto sinu x, arco z, radio r,*

\[
\begin{align*}
z &= x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \text{&c.} \\
x &= z - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^6}{6} + \frac{x^8}{8} - \text{&c.}
\end{align*}
\]

_Hoc, INQUAM, cum nobis attulerit ille, que mihi valde ingeniosa videntur, & posterior imprimis Series elegantiam quandam singularem habeat: ideo rem gratum mihi feceris, Vir clarissime, si demonstrationem transmiseris. Habebis vicissim mea ab his longe diversa circa hanc rem meditata, de quibus jam aliquot abhinc annis ad te perscrivisse credo, demonstratione tamen non addita, quam nunc polo. Ora ut Clarissimo Collinio multam a me salutem dicas: is facile tibi materiam suppediabit satisfaciendi desiderio meo._ Here, by the Word INQUAM, one would think that he had never seen these two Series before, and that his diversa circa hanc rem meditata were something else than one of the Series which he had received from Mr.
Mr. Oldenburg the Year before, and a Demonstration thereof which he was now polishing, to make the Present an acceptable Recompence for Mr. Newton's Method.

Upon the Receipt of this Letter Mr. Oldenburg and Mr. Collins wrote pressingly to Mr. Newton, desiring that he himself would describe his own Method, to be communicated to Mr. Leibnitz. Whereupon Mr. Newton wrote his Letter, dated June 13. 1676, describing therein the Method of Series, as he had done before in the Compendium above-mentioned; but with this Difference: Here he described at large the Reduction of the Dignity of a Binomial into a Series, and only touched upon the Reduction by Division and Extraction of affected Roots: There he described at large the Reduction of Fractions and Radicals into Series by Division and Extraction of Roots, and only set down the two first Terms of the Series into which the Dignity of a Binomial might be reduced. And among the Examples in this Letter, there were Series for finding the Number whose Logarithm is given, and for finding the Versed Sine whose Arc is given: This Letter was sent to Paris, June 26. 1676, together with a MS. drawn up by Mr. Collins, containing Extracts of Mr. James Gregory's Letters.

For Mr. Gregory died near the End of the Year 1675; and Mr. Collins, at the Request of Mr. Leibnitz and some other of the Academy of Sciences, drew up Extracts of his Letters, and the Collection is still extant in the Hand Writing of Mr. Collins with this Title; Extracts of Mr. Gregory's Letters, to be lent to Mr. Leibnitz to peruse, who is desired to return the same to you. And that they were sent is affirmed by Mr. Collins in his Letter to Mr. David Gregory the Brother of the Deceased, dated August 11. 1676, and appears further by the Answers of Mr. Leibnitz and Mr. Tschurnhause concerning them.

The Answer of Mr. Leibnitz directed to Mr. Oldenburg and dated August 27. 1676, begins thus; Litera tuae die Julii 26. data, plura ac memorabilia circa rem Analyticam continent quam multa
multa volumina spissa de his rebus edita. Quare tibi pariter ac clarissimis viris Newtono ac Collinio gratias ago; qui nos participes tot meditationum egregiarum esse voluistis. And towards the End of the Letter, after he had done with the Contents of Mr. Newton's Letter, he proceeds thus. Ad alia tuarum Literarum venio que doctissimus Collinus communicare gravatus non est. Vellem adjecisset appropiationis Gregoriana linearis demonstrationem. Fuit enim his certe studiis promovendi aptissimus. And the Answer of Mr. Tschurnhause, dated Sept. 1. 1676, after he had done with Mr. Newton's Letter about Series, concludes thus. Similia porro quae in hac re praestitit eximius ille Geometra Gregorius memoranda certe sunt. Et quidem optimè fama ipsius consulturi, qui ipsius relieta Manuscripta luci publice ut exponantur operam navabant. In the first Part of this Letter, where Mr. Tschurnhause speaks of Mr. Newton's Series, he faith that he looked over them curfiorily, to see if he could find the Series of Mr. Leibniz for squaring the Circle or Hyperbola. If he had searched for it in the Extracts of Gregory's Letters he might have found it in the Letter of Febr. 15. 1671. above-mentioned. For the MS. of those Extracts with that Letter therein is still extant in the Hand-Writing of Mr. Collins.

And tho' Mr. Leibnitz had now received this Series twice from Mr. Oldenburgh, yet in his Letter of August 27. 1676. he sent it back to him by way of Recompence for Mr. Newton's Method, pretending that he had communicated it to his Friends at Paris three Years before or above; that is, two Years before he received it in Mr. Oldenburgh's Letter of April 15. 1675; at which Time he did not know it to be his own, as appears by his Answer of May 20. 1675 above-mentioned. He might receive this Series at London, and communicate it to his Friends at Paris above three Years before he sent it back to Mr. Oldenburg: but it doth not appear that he had the Demonstration thereof so early. When he found the Demonstration, then he compos'd it in his Opusculum, and communicated that also to his Friends; and he himself has told
told us that this was in the Year 1675. However, it lies upon him to prove that he had this Series before he received it from Mr. Oldenburgh. For in his Answer to Mr. Oldenburgh he did not know any of the Series then sent him to be his own; and concealed from the Gentlemen at Paris his having received it from Mr. Oldenburgh with several other Series, and his having seen a Copy of the Letter in which Mr. Gregory had sent it to Mr. Collins in the Beginning of the Year 1671.

In the same Letter of August 27, 1676, after Mr. Leibnitz had described his Quadrature of the Circle and Equilateral Hyperbola, he added: *Vicissim ex seriebus regressum pro Hyperbola hanc inveni. Si fit numerus aliguis unitate minor \( \frac{1}{m} \), ejusque logarithmus Hyperbolicus \( I \). Erit \( m = \frac{1}{1 - \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3}} + \cdots + \frac{1}{1 \times 2 \times 3 \times 4} + \cdots \). Si numerus sit major unitate, ut \( 1 + n \), tunc pro eo inveniendo mihi etiam prodiit Regula que in Newtoni Epistolae expressa est: *seilicet erit \( n = \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \cdots + \frac{1}{1 \times 2 \times 3 \times 4 \times 5} + \cdots \). Quod regressum ex arcibus attinet, incideram ego directe in Regulam quae ex dato arcu finum complementi exhibet. Nempe sinus complementi \( = 1 - \frac{a^2}{1 \times 2} + \frac{a^3}{1 \times 2 \times 3} - \cdots \). Sed postea quoque deprehendi ex ea illam nobis communicatam pro inveniendo sinus recto, qui est \( a = \frac{a^2}{1 \times 2 \times 3} + \frac{a^3}{1 \times 2 \times 3 \times 4} + \cdots \). Sed demonstrari. Thus Mr. Leibnitz put in his Claim for the Co-invention of these four Series, tho' the Method of finding them was sent him at his own Request, and he did not yet understand it. For in this same Letter of August 27, 1676, he desired Mr. Newton to explain it further. His Words are. *Sed desperaverim ut Clarissimus Newtonus nonnulla quoque amplius explicet; ut originem Theorematis quod initio positi: Item modum quo quantitates \( p, q, r \), in suis Operationibus invenit: Ac denique quomodo in Methodo regressum se gerat, ut cum ex Logarithmo quaerit Numerum. Neque enim explicat quomodo id ex methodo sua derivetur. He pretended to have found two Series for the Number whose Logarithm was given, and yet in the same Letter declared.
fired Mr. Newton to explain to him the Method of finding those very two Series.

When Mr. Newton had received this Letter, he wrote back that all the said four Series had been communicated by him to Mr. Leibnitz; the two first being one and the same Series in which the Letter $l$ was put for the Logarithm with its Sign $+$ or $-$; and the third being the Excess of the Radius above the verified Sine, for which a Series had been sent to him. Whereupon Mr. Leibnitz desisted from his Claim. Mr. Newton also in the same Letter dated Octob. 24. 1676, further explained his Methods of Regression, as Mr. Leibnitz had desired. And Mr. Leibnitz in his Letter of June 21. 1677, desired a further Explication; but soon after, upon reading Mr. Newton's Letter a second time, wrote back July 12. 1677, that he now understood what he wanted; and found by his old Papers that he had formerly used one of Mr. Newton's Methods of Regression, but in the Example which he had then by chance made use of, there being produced nothing elegant, he had, out of his usual Impatience, neglected to use it any further. He had therefore several direct Series, and by consequence a Method of finding them, before he invented and forgot the inverse Method. And if he had searched his old Papers diligently, he might have found this Method also there; but having forgot his own Methods, he wrote for Mr. Newton's.

When Mr. Newton in his Letter dated June 13. 1676, had explained his Method of Series, he added: Ex his videre est quantum fines Analyticos per hujusmodi infinitas aequationes ampliantur: quippe que earum beneficio ad omnia pene dixerim problemata (et numeralia Diophanti & similia excipias) sese extendit. Non tamen omnia universalis evadit, nisi per ulteriores quaedam Methodos eliciendi Series infinitas. Sunt enim quaedam Problematum in quibus non licet ad Series infinitas per Divisionem vel Extraktionem radicum simplicium affectarumve pervenire. Sed quomodo in his casibus procedendum sit jam non vacat dicere; ut neque alia quaedam tradere, qua circa Reductionem infinitarum Serierum
in finitas, ubi rei natura tulerit, excogitavi. Nam parcius scribo, quod ha speculationes dim mihi fasfido esse copernit; adeo ut ab isdem jam per quinque fere annos abstinerim. To this Mr. Leibnitz in his Letter of August 27, 1676, answered:

Quod dicere videmini plerisque difficulitates (exceptis Problematisbus Diophanteis) ad series Infinitas reduci; id mihi non videtur. Sunt enim multa utque adeo mira & implexa ut neque ab equationibus pendeant neque ex Quadraturis. Qualia sunt (ex multis aliis) Problemata methodi Tangentium inverse. And Mr. Newton in his Letter of Octob. 24, 1676, replied: Ubi dixi omnia pene Problemata solubilia existere; volui de ipsis praefertim intelligi circa quos Mathematici se hactenus occuparunt; vel altem in quibus Rationcinia Mathematica locum aliquem obtinere possunt. Nam alia sane adeo perplexis conditionibus implicata excogitare liceat, ut non satis comprehenderem valcamus: & multo minus tantarum computationum onus sustiner quod ida requirerent. Attamen ne minimum dixisse videas, inversa de Tangentibus Problemata sunt in potestate, aliaque illis difficiliora. Ad quae solvenda usus sum duplici methodo, una concinnior, altera generaliori. Utamque visum est impressentia literis transpositis confignare, ne propter alios idem obtinentes, institutum in aliquibus mutare cogerer. 5 a cc d x 10 e ff h, &c. id est, Una methodus consistit in extractione fluentis quantitatis ex equatione simul involvente fluxionem ejus: altera tantum in assumptione seriei pro quantitate qualibet incognita, ex qua certa commode derivari possunt; & in collatione terminorum homologorum equationis resultantis ad errandos terminos assumpta seriei. By Mr. Newton's two Letters, its certain that he had then (or rather above five Years before) found out the Reduction of Problems to fluxional Equations and converging Series: and by the Answer of Mr. Leibnitz to the first of those Letters, its as certain that he had not then found out the Reduction of Problems either to differential Equations or to converging Series.

And the same is manifest also by what Mr. Leibnitz wrote in the Acta Eruditorum, Anno 1691, concerning this Matter.
Jan anno 1675, faith he, compositum habebam opusculum Quadrature Arithmetica ab amicis ab illo tempore lectum, sed quod, materia sub manibus crescente, limare ad Editionem non vacavit, postquam alia occupationes supervenere; præsertim cum nunc prolixius exponere vulgari morte quæ Analysis nostra paucis exhibet, non satis opera pretium videatur. This Quadrature composèd vulgari morte he began to communicate at Paris in the Year 1675. The next Year he was polishing the Demonstration thereof, to send it to Mr. Oldenburgh in Recompence for Mr. Newton's Method, as he wrote to him May 12. 1676; and accordingly in his Letter of August 27. 1676. he sent it composed and polished vulgari morte. The Winter following he returned into Germany, by England and Holland, to enter upon publick Business, and had no longer any Leisure to fit it for the Press, nor thought it afterwards worth his while to explain those Things prolixly in the vulgar manner which his new Analysis exhibited in short. He found out this new Analysis therefore after his Return into Germany, and by consequence not before the Year 1677.

The same is further manifest by the following Consideration. Dr. Barrow published his Method of Tangents in the Year 1670. Mr. Newton in his Letter dated December 10. 1672. communicated his Method of Tangents to Mr. Collins, and added: Hoc est unum particulare vel Corollarium potius Methodi generalis, quæ extendit se citra molestum illum calculum, non modo ad ducentum Tangentes ad quasvis Curvas sive Geometricas sive Mechanicas, vel quamodocunque rectas Lineas aliasque Curvas respicientes: verum etiam ad resolvendum alia abstræta Problematum genera de Curvatibus, Acre, Longitudinibus, Centris Gravitatis Curvarum, &c. Neque (quemadmodum Huddenii methodus de Maximis & Minimis) ad solas restringitur aequationes illas, quæquantitatisardis sunt immunes. Hanc methodum intertextui alteri is l quæ Equationum Exegefin instituo, reducendi eas ad series infinitas. Mr. Slusius sent his Method of Tangents to Mr. Oldenburgh Jan. 17. 1673, and the same was 2
soon after published in the Transactions. It proved to be the same with that of Mr. Newton. It was founded upon three Lemmas, the first of which was this, Differentia duarum dignitatum ejusdem gradus applicata ad differentiam laterum dat partes singulares gradus inferioris ex binomio laterum, ut \( \frac{y^3 - x^3}{y - x} = yy + yx + xx \), that is, in the Notation of Mr. Leibnitz

\[ \frac{dy^3}{dy} = 3yy. \]

A Copy of Mr. Newton's Letter of Decemb. 10, 1672 was sent to Mr. Leibnitz by Mr. Oldenburg amongst the Papers of Mr. James Gregory, at the same time with Mr. Newton's Letter of June 13, 1676. And Mr. Newton having described in these two Letters that he had a very general Analysis, consisting partly of the Method of converging Series, partly of another Method, by which he applied those Series to the Solution of almost all Problems (except perhaps some numerical ones like those of Diophantus) and found the Tangents, Areas, Lengths, solid Contents, Centers of Gravity, and Curvities of Curves, and curvilinear Figures Geometrical or Mechanical, without sticking at Surds; and that the Method of Tangents of Sluys was but a Branch or Corollary of this other Method: Mr. Leibnitz in his returning Home through Holland, was meditating upon the Improvement of the Method of Sluys. For in a Letter to Mr. Oldenburgh, dated from Amsterdam Nov. 8, 1676, he wrote thus. Methodus Tangentium a Sluysio publicata nondum rei satisfìgium tenet. Potest aliquid amplius priìtari in eo genere quod maximi fœret usus ad omnis generis Problematas: etiam ad meam (fine extractionibus) Æquationum ad series reductionem. Nimimum potest brevis quamdam calculi circa Tangentes Tabula, eousque continuanda donec progressio Tabula apparet; ut eam scilicet quisque quonque liberis fine calculo continuare posset. This was the Improvement of the Method of Sluys into a general Method, which Mr. Leibnitz was then thinking upon, and by his Words,
Potest aliquid amplius prastari in eo genere quod maximi foret usus ad omnis generis Problemata, it seems to be the only Improvement which he had then in his Mind for extending the Method to all sorts of Problems. The Improvement by the differential Calculus was not yet in his Mind, but must be referred to the next Year.

Mr. Newton in his next Letter, dated Octob. 24. 1676, mentioned the Analysis communicated by Dr. Barrow to Mr. Collins in the Year 1669, and also another Tract written in 1671: about converging Series, and about the other Method by which Tangents were drawn after the Method of Slufts, and Maxima and Minima were determined, and the Quadrature of Curves was made more easy, and this without flicking at Radicals, and by which Series were invented which brake off and gave the Quadrature of Curves in finite Equations when it might be. And the Foundation of these Operations he comprehended in this Sentence express enigmatically as above. Data equationes fluentes quocunque quantitates involvente fluxiones invenire, & vice versa. Which puts it past all Dispute that he had invented the Method of Fluxions before that time. And if other things in that Letter be considered, it will appear that he had then brought it to great Perfection, and made it exceeding general; the Propositions in his Book of Quadratures, and the Methods of converging Series and of drawing a Curve Line through any Number of given Points, being then known to him. For when the Method of Fluxions proceeds not in finite Equations, he reduces the Equations into converging Series by the binomial Theorem, and by the Extraction of Fluents out of Equations involving or not involving their Fluxions. And when finite Equations are wanting, he deduces converging Series from the Conditions of the Problem, by assuming the Terms of the Series gradually, and determining them by those Conditions. And when Fluents are to be derived from Fluxions, and the Law of the Fluxions is wanting, he finds
that Law quam proxime, by drawing a Parabolick Line through any Number of given Points. And by these Improvements Mr. Newton had in those Days made his Method of Fluxions much more universal than the Differential Method of Mr. Leibnitz is at present.

This Letter of Mr. Newton's, dated Octob. 24. 1676, came to the Hands of Mr. Leibnitz in the End of the Winter or Beginning of the Spring following; and Mr. Leibnitz soon after, viz. in a Letter dated June 21. 1677, wrote back: Clarissimi Suis methodum Tangentium nondum esse absolutum Celeberrimo Newtono assentior. Et jam à multo tempore rem Tangentium generalius tractavi, scilicet per differentias Ordinarum.

Hinc nominando, in posterum, dy differentiam duarum proximarum y &c. Here Mr. Leibnitz began first to propose his Differential Method, and there is not the least Evidence that he knew it before the Receipt of Mr. Newton's last Letter. He faith indeed, Jam à multo tempore rem Tangentium generalius tractavi, scilicet per differentias Ordinarum: and so he affirmed in other Letters, that he had invented several converging Series direct and inverse before he had the Method of inventing them; and had forgot an inverse Method of Series before he knew what use to make of it. But no Man is a Witness in his own Cause. A Judge would be very unjust, and act contrary to the Laws of all Nations, who should admit any Man to be a Witness in his own Cause. And therefore it lies upon Mr. Leibnitz to prove that he found out this Method long before the Receipt of Mr. Newton's Letters. And if he cannot prove this, the Question, Who was the first Inventor of the Method, is decided.

The Marquis's De l'Hospital (a Person of very great Candour) in the Preface to his Book De Analyse quantitatum infiniti

*parvarum*, published A. C. 1696. tells us, that a little after the Publication, of the Method of Tangents of Des Cartes, Mr. Fermat found also a Method, which Des Cartes himself at length allowed to be, for the most part, more simple than his
his own. But it was not yet so simple as Mr. Barrow afterwards made it, by considering more nearly the nature of Polygons, which offers naturally to the Mind a little Triangle, compos'd of a Particle of the Curve lying between two Ordinates infinitely near one another, and of the Difference of these two Ordinates, and of that of the two correspondent Abscissa's. And this Triangle is like that which ought to be made by the Tangent, the Ordinate, and the Sub-tangent: so that by one simple Analogy, this last Method saves all the Calculation which was requisite either in the Method of Des Cartes, or in this same Method before. Mr. Barrow stooped not here, he invented also a sort of Calculation proper for this Method. But it was necessary in this as well as in that of Des Cartes, to take away Fractions and Radicals for making it useful. Upon the Defect of this Calculus, that of the celebrated Mr. Leibnitz was introduced, and this learned Geometer began where Mr. Barrow and others left off. This his Calculus led into Regions hitherto unknown, and there made Discoveries which astonished the most able Mathematicians of Europe, &c. Thus far the Marquis. He had not seen Mr. Newton's Analysis, nor his Letters of Decem. 10. 1672. June 13. 1676, and Octob. 24. 1676: and so not knowing that Mr. Newton had done all this and signified it to Mr. Leibnitz, he reckoned that Mr. Leibnitz began where Mr. Barrow left off, and by teaching how to apply Mr. Barrow's Method without sticking at Fractions and Surds, had enlarged the Method wonderfully. And Mr. James Bernoulli, in the Acta Eruditorum of January 1691 pag. 14. writes thus: Qui calculus Barrovianum (quem in Lectionibus suis Geometricis adsumbravit Author, cujusque Specimina sunt tota illa Propositionum inibi contenatarum farrago,) intellexerit, [calculus] alterum à Domino Leibnitz inventum, ignorare vix poteris; utpote quib in priori illo fundatus est, & nihil forte in Differentialium notatione & operationis aliquo compendio ab eo non differt.

Now
Now Dr. Barrow, in his Method of Tangents, draws two
Ordinates indefinitely near to one another, and puts the Letter a for the Difference of the Ordinates, and the Letter e for the Difference of the Abscissa's, and for drawing the Tangent gives these Three Rules

1. Inter computandum, faith he, omnes abjicio terminos in quibus ipsarum a vel e potestas habeatur; vel in quibus ipse ducentur in se. Etenim isti termini nihil valebunt. 2. Post equationem constitutam omnes abjicio terminos literis constantes quantitates notas seu determinatas significantibus, aut in quibus non habentur a vel e. Etenim illi termini semper ad unam equationis partem addunti nihil adaquabunt. 3. Pro a Ordinalam, & pro e Subtangenti substituo. Hinc demum Sub-

tangentis quantitas digne-ctur. Thus far Dr. Barrow.

And Mr. Leibnitz in his Letter of June 21. 1677 above-men-
tioned, wherein he first began to propose his Differential Method, has followed this Method of Tangents exactly, excepting that he has changed the Letters a and e of Dr. Barrow into dx and dy. For in the Example which he there gives, he draws two parallel Lines and sets all the Terms below the under Line, in which dx and dy are (severally or jointly) of more than one Dimension, and all the Terms above the upper Line, in which dx and dy are wanting, and for the Reasons given by Dr. Barrow, makes all these Terms vanish. And by the Terms in which dx and dy are but of one Dimension, and which he sets between the two Lines, he determines the Proportion of the Subtangent to the Ordi-

nate. Well therefore did the Marquis de l'Hospital observe that where Dr. Barrow left off Mr. Leibnitz began: for their Methods of Tangents are exactly the same.

But Mr. Leibnitz adds this Improvement of the Method, that the Conclusion of this Calculus is coincident with the Rule of Sluys, and shews how that Rule presently occurs to any one who understands this Method. For Mr. Newton had represented in his Letters, that this Rule was a Corolla-

ry of his general Method.

And
And whereas Mr. Newton had said that his Method in drawing of Tangents, and determining Maxima and Minima, &c. proceeded without sticking at Surds: Mr. Leibnitz in the next Place, shews how this Method of Tangents may be improved so as not to stick at Surds or Fractions, and then adds: *Arbitror que celare voluit Newtonus de Tangentibus ducendis ab his non abludere.* Quod addit, ex hoc eodem fundamento Quadraturas quoque reddi faciores me in hac sententia confirmat; nimium semper figura illa sunt quadrabiles-que sunt ad equationem differentialem. By which Words, compared with the preceding Calculation, its manifest that Mr. Leibnitz at this time understood that Mr. Newton had a Method which would do all these things, and had been examining whether Dr. Barrow’s Differential Method of Tangents might not be extended to the same Performances.

In November 1684 Mr. Leibnitz published the Elements of this Differential Method in the *Atta Eruditorum*, and illustrated it with Examples of drawing Tangents and determining Maxima and Minima, and then added. *Et hac quidem initia sunt Geometriae cujusdam multo sublimioris, ad difficillima & pulcherrima quoque etiam mistica Matheos Problematum pertingentis, quae sine calculo differentiai AUT SIMILI non temere quisquam pari facilitate tractabit.* The Words AUT SIMILI plainly relate to Mr. Newton’s Method. And the whole Sentence contains nothing more than what Mr. Newton had affirmed of his general Method in his Letters of 1672 and 1676.

And in the *Atta Eruditorum* of June 1686, pag. 297: Mr. Leibnitz added: *Malo autem dx & similia adhibere quam litteras pro illis, quia istud dx est modificatio quaedam ipsius x.* &c. He knew that in this Method he might have used Letters with Dr. Barrow, but chose rather to use the new Symbols $d\ x$ and $d\ y$, though there is nothing which can be done by these Symbols, but may be done by single Letters with more brevity.
The next Year Mr. Newton's *Principia Philosophiae* came abroad, a Book full of such Problemes as Mr. Leibnitz had called *difficillima* & *pulcherrima* etiam *mi* *Ma* *th* *es* *eos* proble*ma* *ta*, qua *sine* *calc* *ulo* *differentiali* *aut* *SIMILI* *non* *temere* *qui* *quam* *pari* *facilitate* *tra* *t* *abit*. And the Marquess de L'Hospital has represented this Book *presque tout de ce calcul*; composed almost wholly of this Calculus. And Mr. Leibnitz himself in a Letter to Mr. Newton, dated from Hannover, March \(\frac{3}{2}\) 1693, and still extant in his own Hand-writing, and upon a late Occasion communicated to the Royal Society, acknowledged the same thing in these Words: *Mirifice ampliaveras Geometriam, tuis Scribus, sed edito Principiorum operè ostendisti, patere tibi etiam qua *Analyysi* receptæ non subsunt. Conatus sum ego quoque, nosis commodis adhibitis qua differentias & *summas* *exhibeant*, Geometriam illam quam *Transcendentes* appell. *Analyzi quodammodo subjecere, nec res male processit*; And again in his Answer to Mr. Fatio, printed in the *Acta Eruditorum* of May 1700, *pag. 203. lin. 21. he acknowledged the same thing. In the second Lemma of the second Book of these *Principles*, the Elements of this Calculus, are demonstrated synthetically, and at the End of the Lemma there is a Scholium in these Words. In Literis *qua mihi cum Geometra* *peritissimo* G. G. Leibnitzio annis abhinc decem intercedebant, *cum* *significarem* *me* *compotem* *esse* *methodi* *determinandi* *Maximas* & *Minimas*, *ducendi* *Tangentes* & *similia* *peragendi*, *qua* in *terminis surdis* *aque* *ac* in *rationales* *procederet*; & *literis* *transpositis* *hanc* *sententiam* *involventibus*: *[Data* *equatione* *quotcunque* *flu* *entes* *quantitates* *involvente, fluxiones* *invenire, & vice versa]* *eandem* *cclarem: re* *script* *t* *Vix* *clarissimus* *se* *quoque* *in* *ejusmodi* *methodum* *incidisse, & *methodum* *suum* *communicavit* *a* *mea* *vix* *abludentem* *praterquam* *in* *verborum* *et* *notarum* *formulis*. *Utriusque* *fundamentum* *continetur* *in hoc* *Lemmate*. In those Letters, and in another dated Decem. 10. 1672, a Copy of which, at that time, was sent to Mr. Leibnitz by Mr. Oldenburg, as is mentioned above, Mr. Newton had so far explained his Method, that it was not difficult for
Mr. Leibnitz, by the Help of Dr. Barrow’s Method of Tangents, to collect it from those Letters. And its certain, by the Arguments above-mentioned, that he did not know it before the writing of those Letters.

Dr. Wallis had received Copies of Mr. Newton’s two Letters of June 13. and Octob. 24. 1676 from Mr. Oldenburgh, and published several things out of them in his Algebra, printed in English 1683, and in Latin 1693; and soon after had Intimation from Holland to print the Letters entire, because Mr. Newton’s Notions of Fluxions passed there with Applause by the Name of the Differential Method of Mr. Leibnitz. And thereupon he took notice of this Matter in the Preface to the first Volume of his Works published A. C. 1695. And in a Letter to Mr. Leibnitz dated Decemb. 1. 1696, he gave the Account of it. Cum Prefationis (presigenda) postremum folium erat sub prelo, ejusque typos jam poluerant Typothete; me monuit amicus quidam (harum rerum gnarus) qui peregre fuerat, tum talen methodum in Belgio pradicari, tum illam cum Newtoni methodo Fluxionum quasi coincidere. Quod fecit ut (translatis typis jam positis) id monitum interfereret. And in a Letter dated April 10. 1695, and lately communicated to the Royal-Society, he wrote thus about it. I wish you would print the two large Letters of June and August [he means June and October] 1676. I had intimation from Holland, as desired there by your Friends, that somewhat of that kind were done; because your Notions (of Fluxions) pass there with great Applause by the Name of Leibnitz’s Calculus Differentialis. I had* this intimation when all but part of the Preface to this Volume was printed off; so that I could only insert (while the Press stay’d) that short Intimation thereof which you there find. You are not so kind to your Reputation (and that of the Nation) as you might be, when you let things of worth lye by you so long, till others carry away

* Extat hæc Epistola in tertio volumine operum Wallisi.
The Reputation that is due to you. I have endeavoured to do you justice in that Point, and am now sorry that I did not print those two Letters verbatim.

The short Intimation of this Matter, which Dr. Wallis inserted into the said Preface, was in these Words. In secundo Volumine (inter alia) habetur Newtoni Methodus de Fluxionibus (ut ille loquitur) consimilis naturæ cum Leibnitii (ut hic loquitur); Calculo Differentiali (quod qui utramque methodum contulerit satis animadvertat, ut ut sub locuendi formulis diversis) quam ego descripti (Algebrae cap. 91. &c. præsertim cap 95) ex binis Newtoni Literis, aut earum alteris, Junii 13. Octob. 24. 1676 ad Oldenburgum datis, cum Leibniti tunc communicandis (iisdem fere verbis, saltem leviter mutatis, quæ in illis literis habendur.) ubi METHODUM HANC LEIBNITIO EXPONIT, tum ante DECEM ANNOS medum plures [id est, anno 1666 vel 1665] ab ipso excogitatum. Quod moneo, nequis causetur de hoc Calculo Differentiali nihil a nobis dictum esse.

Hereupon the Editors of the Acta Lipsiensia, the next Year in June, in the Style of Mr. Leibnitz, in giving an Account of these two first Volumes of Dr. Wallis, took notice of this Clause of the Doctor’s Preface, and complained, not of his saying that Mr. Newton in his two Letters above-mentioned explained to Mr. Leibnitz the Method of Fluxions found by him Ten Years before or above; but that while the Doctor mentioned the Differential Calculus, and said that he did it nequis causetur de calculo differentiali nihil ab ipso dictum fuisset, he did not tell the Reader that Mr. Leibnitz had this Calculus at that time when those Letters passed between him and Mr. Newton, by means of Mr. Oldenburgh. And in several Letters which followed hereupon, between Mr. Leibnitz and Dr. Wallis, concerning this Matter, Mr. Leibnitz denied not that Mr. Newton had the Method Ten Years before the writing of those Letters, as Dr. Wallis had affirmed; pretended not that he himself had the Method so early; brought no Proof that he had it before the Year 1677;
1677; no other Proof besides the Concession of Mr. Newton that he had it so early; affirmed not that he had it earlier; commended Mr. Newton for his Candour in this Matter; allowed that the Methods agreed in the main, and said that he therefore used to call them by the common Name of his Infinitesimal Analysis; represented, that as the Methods of Vieta and Cartes were called by the common Name of Analysis Speciosa, and yet differed in some things; so perhaps the Methods of Mr. Newton and himself might differ in some things, and challenged to himself only those things wherein, as he conceived, they might differ, naming the Notation, the differential Equations and the Exponential Equations. But in his Letter of June 21, 1677 he reckon’d differential Equations common to Mr. Newton and himself.

This was the State of the Dispute between Dr. Wallis and Mr. Leibnitz at that time. And Four years after, when Mr. Fatio suggested that Mr. Leibnitz, the second Inventor of this Calculus, might borrow something from Mr. Newton, the eldest Inventor by many Years: Mr. Leibnitz in his Answer, published in the Acta Eruditorum of May 1700, allowed that Mr. Newton had found the Method apart, and did not deny that Mr. Newton was the eldest Inventor by many Years, nor asserted any thing more to himself, than that he also had found the Method apart, or without the Assistance of Mr. Newton, and pretended that when he first published it, he knew not that Mr. Newton had found any thing more of it than the Method of Tangents. And in making this Defence he added: Quam [methodum] ante Dominum Newtonum & Menullus quod sciam Geometra habuit; uti ante hunc maximi nominis Geometram N E M O specimeine publice dato se habere probavit, ante Dominos Bernoullios & Menullus communicavit. Hitherto therefore Mr. Leibnitz did not pretend to be the first Inventor. He did not begin to put in such a Claim till after the Death of Dr. Wallis, the last of the old Men who were acquainted with what had passed between the English and Mr. Leibnitz.
Forty Years ago. The Doctor died in October A. C. 1703, and Mr. Leibnitz began not to put in this new Claim before January 1705.

Mr. Newton published his Treatise of Quadratures in the Year 1704. This Treatise was written long before, many things being cited out of it in his Letters of Octob. 24. and Novemb. 8. 1676. It relates to the Method of Fluxions, and that it might not be taken for a new Piece, Mr. Newton repeated what Dr. Wallis had published Nine Years before without being then contradicted, namely, that this Method was invented by Degrees in the Years 1665 and 1666. Hereupon the Editors of the Acta Leipfienfia in January 1705, in the Style of Mr. Leibnitz, in giving an Account of this Book, represented that Mr. Leibnitz was the first Inventor of the Method, and that Mr. Newton had substituted Fluxions for Differences. And this Accusation gave a Beginning to this present Controversy.


Before Mr. Newton saw what had been published in the Acta Leipfica, he express'd himself offended at the printing of this Paragraph of Mr. Keill's Letter, lest it should create a Controversy. And Mr. Leibnitz, understanding it in a stronger Sense than Mr. Keill intended it, complain'd of it as a Calumny, in a Letter to Dr. Sloane dated March 4, 1711 N. S. and moved that the Royal-Society would cause Mr. Keill to make a publick Recantation. Mr. Keill chose rather to explain and defend what he had written; and Mr. Newton, upon being shewed the Accusation in the Acta Leipfica, gave him leave to do so. And Mr. Leibnitz in a second Letter to Dr. Sloane, dated Decem. 29. 1711, instead of making good his Accusa-
lication, as he was bound to do that it might not be deem'd a Calumny, insisted only upon his own Candour, as if it would be Injustice to question it; and refus'd to tell how he came by the Method; and said that the Acts Lipsica had given every Man his due, and that he had concealed the Invention above Nine Years, (he should have said Seven Years) that No body might pretend (he means that Mr. Newton might not pretend) to have been before him in it; and called Mr. Keill a Novice unacquainted with things past, and one that acted without Authority from Mr. Newton, and a clamorous Man who deserv'd to be silence, and desir'd that Mr. Newton himself would give his Opinion in the Matter. He knew that Mr. Keill affirmed nothing more than what Dr. Wallis had published thirteen Years before, without being then contradicted. He knew that Mr. Newton had given his Opinion in this matter in the Introduction to his Book of Quadratures, published before this Controversy began: but Dr. Wallis was dead; the Mathematicians which remained in England were Novices; Mr. Leibnitz may Question any Man's Candour without Injustice, and Mr. Newton must now retract what he had publish'd or not be quieter.

The Royal-Society therefore, having as much Authority over Mr. Leibnitz as over Mr. Keill, and being now twice pressed by Mr. Leibnitz to interpose, and seeing no reason to condemn or censure Mr. Keill without enquiring into the matter; and that neither Mr. Newton nor Mr. Leibnitz (the only Persons alive who knew and remembred any thing of what had pass'd in these matters Forty Years ago) could be Witnesses for or against Mr. Keill; appointed a numerous Committee to search old Letters and Papers, and report their Opinion upon what they found; and ordered the Letters and Papers, with the Report of their Committee to be publish'd. And by these Letters and Papers it appear'd to them, that Mr. Newton had the Method in or before the Year 1669, and it did not appear to them that Mr. Leibnitz had it before the Year 1677.
For making himself the first Inventor of the Differential Method, he has represented that Mr. Newton at first used the Letter $o$ in the vulgar manner for the given Increment of $x$, which destroys the Advantages of the Differential Method; but after the writing of his Principles, changed $o$ into $x$, substituting $x$ for $dx$. It lies upon him to prove that Mr. Newton ever changed $o$ into $x$, or used $x$ for $dx$, or left off the Use of the Letter $o$. Mr. Newton used the Letter $o$ in his Analysis written in or before the Years 1669, and in his Book of Quadratures, and in his Principia Philosophiae, and still uses it in the very same Sense as at first. In his Book of Quadratures he used it in conjunction with the Symbols, and therefore did not use that Symbol in its Room. These Symbols $o$ and $x$ are put for things of a different kind. The one is a Moment, the other a Fluxion or Velocity as has been explained above. When the Letter $x$ is put for a Quantity which flows uniformly, the Symbol $x$ is an Unit, and the Letter $o$ a Moment, and $x$ and $dx$ signify the same Moment. Printe Letters never signify Moments, unless when they are multiplied by the Moment $o$ either exprest or understood to make them infinitely little, and then the Rectangles are put for Moments.

Mr. Newton doth not place his Method in Forms of Symbols, nor confine himself to any particular Sort of Symbols for Fluents and Fluxions. Where he puts the Areas of Curves for Fluents, he frequently puts the Ordinates for Fluxions, and denotes the Fluxions by the Symbols of the Ordinates, as in his Analysis. Where he puts Lines for Fluents, he puts any Symbols for the Velocities of the Points which describe the Lines, that is, for the first Fluxions; and any other Symbols for the Increase of those Velocities, that is, for the second Fluxions, as is frequently done in his Principia Philosophiae. And where he puts the Letters $x, y, z$ for Fluents, he denotes their Fluxions, either by other Letters as $p, q, r$; or by the same Letters in other Forms as $X, Y, Z$ or $x, y, z$; or by any
any Lines as $DE$, $FG$, $HI$, considered as their Exponents. And this is evident by his Book of Quadratures, where he represents Fluxions by prick'd Letters in the first Proposition, by Ordinates of Curves in the last Proposition, and by other Symbols, in explaining the Method and illustrating it with Examples, in the Introduction. Mr. Leibnitz hath no Symbols of Fluxions in his Method, and therefore Mr. Newton's Symbols of Fluxions are the oldest in the kind. Mr. Leibnitz began to use the Symbols of Moments or Differences $dx, dy, dz$ in the Year 1677. Mr. Newton represented Moments by the Rectangles under the Fluxions and the Moment $o$, when he wrote his Analysis, which was at least Forty Six Years ago. Mr. Leibnitz has used the Symbols $sx, sy, sz$ for the Sums of Ordinates ever since the Year 1686; Mr. Newton represented the same thing in his Analysis, by inscribing the Ordinate in a Square or Rectangle. All Mr. Newton's Symbols are the oldest in their several Kinds by many Years.

And whereas it has been represented that the use of the Letter $o$ is vulgar, and destroys the Advantages of the Differential Method: on the contrary, the Method of Fluxions, as used by Mr. Newton, has all the Advantages of the Differential, and some others. It is more elegant, because in his Calculus there is but one infinitely little Quantity represented by a Symbol, the Symbol $o$. We have no Ideas of infinitely little Quantities, and therefore Mr. Newton introduced Fluxions into his Method, that it might proceed by finite Quantities as much as possible. It is more Natural and Geometrical, because founded upon the *prima quantitatum nascentium rationes*, which have a Being in Geometry, whilst *Indivisibles*, upon which the Differential Method is founded, have no Being either in Geometry or in Nature. There are rationes *prima quantitatum nascentium*, but not *quantitates prima nascentes*. Nature generates Quantities by continual Flux or Increase; and the ancient Geometers admitted such a Generation of Areas and Solids, when they drew one Line into another by local Motion.
to generate an Area, and the Area into a Line by local Motion to generate a Solid. But the summing up of Indivisibles to compose an Area or Solid was never yet admitted into Geometry. Mr. Newton's Method is also of greater Use and Certainty, being adapted either to the ready finding out of a Proposition by such Approximations as will create no Error in the Conclusion, or to the demonstrating it exactly: Mr. Leibnitz's is only for finding it out. When the Work succeeds not in finite Equations Mr. Newton has recourse to converging Series, and thereby his Method becomes incomparably more universal than that of Mr. Leibnitz, which is confin'd to finite Equations: for he has no Share in the Method of infinite Series. Some Years after the Method of Series was invented, Mr. Leibnitz invented a Proposition for transmuting curvilinear Figures into other curvilinear Figures of equal Areas, in order to square them by converging Series: but the Methods of squaring those other Figures by such Series were not his. By the help of the new Analysis Mr. Newton found out most of the Propositions in his Principia Philosophiae: but because the Ancients for making things certain admitted nothing into Geometry before it was demonstrated synthetically, he demonstrated the Propositions synthetically, that the Systeme of the Heavens might be founded upon good Geometry. And this makes it now difficult for unskilful Men to see the Analysis by which those Propositions were found out.

It has been represented that Mr. Newton, in the Scholium at the End of his Book of Quadratures, has put the third, fourth, and fifth Terms of a converging Series respectively equal to the second, third, and fourth Differences of the first Term, and therefore did not then understand the Method of second, third, and fourth Differences. But in the first Proposition of that Book he shewed how to find the first, second, third and following Fluxions in infinitum; and therefore when he wrote that Book, which was before the Year 1676, he did understand the Method of all the Fluxions, and by con-
sequence of all the Differences. And if he did not understand it when he added that Scholium to the End of the Book, which was in the Year 1704, it must have been because he had then forgot it. And so the Question is only whether he had forgot the Method of second and third Differences before the Year 1704.

In the Tenth Proposition of the second Book of his *Principia Philosophiae*, in describing some of the Uses of the Terms of a converging Series for solving of Problemes, he tells us that if the first Term of the Series represents the Ordinate $BC$ of any Curve. Line $ACG$, and $CBDI$ be a Parallelogram infinitely narrow, whose Side $DI$ cuts the Curve in $G$ and its Tangent $CF$ in $F$, the second Term of the Series will represent the Line $IF$, and the third Term the Line $FG$. Now the Line $FG$ is but half the second Difference of the Ordinate: and therefore Mr. *Newton* when he wrote his *Principia*, put the third Term of the Series equal to half of the second Difference of the first Term, and by consequence had not then forgotten the Method of second Differences.

In writing that Book, he had frequent occasion to consider the Increase or Decrease of the Velocities with which Quantities are generated, and argues right about it. That Increase or Decrease is the second Fluxion of the Quantity, and therefore he had not then forgotten the Method of second Fluxions.

In the Year 1692, Mr. *Newton*, at the Request of Dr. *Wallis*, sent to him a Copy of the first Proposition of the Book of Quadratures, with Examples thereof in first, second and third Fluxions: as you may see in the second Volume of the Doctor's Works, pag. 391, 392, 393 and 396. And therefore he had not then forgotten the Method of second Fluxions.
Nor is it likely, that in the Year 1704, when he added the aforesaid Scholium to the End of the Book of Quadratures, he had forgotten not only the first Proposition of that Book, but also the last Proposition upon which that Scholium was written. If the Word \([ut]\), which in that Scholium may have been accidentally omitted between the Words \([erit\) and \([ejus,]\) be restor'd, that Scholium will agree with the two Propositions and with the rest of his Writings, and the Objection will vanish.

Thus much concerning the Nature and History of these Methods, it will not be amiss to make some Observations thereupon.

In the Commercium Epistolicum, mention is made of three Tracts written by Mr. Leibnitz; after a Copy of Mr. Newton's Principia Philosophiae had been sent to Hannover for him, and after he had seen an Account of that Book published in the Acta Eruditorum for January and February 1689. And in those Tracts the principal Propositions of that Book are composed in a new manner, and claimed by Mr. Leibnitz as if he had found them himslef before the publishing of the said Book. But Mr. Leibnitz cannot be a Witness in his own Cause. It lies upon him either to prove that he found them before Mr. Newton, or to quit his claim.

In the last of those three Tracts, the 20th Proposition (which is the chief of Mr. Newton's Propositions) is made a Corollary of the 19th Proposition, and the 19th Proposition has an erroneous Demonstration adapted to it. It lies upon him either to satisfy the World that the Demonstration is not erroneous, or to acknowledge that he did not find that and the 20th Proposition thereby, but tried to adapt a Demonstration to Mr. Newton's Proposition to make it his own. For he represents in his 20th Proposition that he knew not how Mr. Newton came by it, and by consequence that he found it himself without the Assistance of Mr. Newton.

By the Errors in the 15th and 19th Proposition of the third Tract, Dr. Keill hath shewed that when Mr. Leibnitz wrote these
these three Tracts, he did not well understand the Ways of working in second Differences. And this is further manifest by the 10th, 11th, and 12th Propositions of this third Tract. For these he lays down as the Foundation of his infinitesimal Analysis in arguing about centrifugal Forces, and proposes the first of them with relation to the Center of Curvity of the Orb, but uses this Proposition in the two next, with Relation to the Center of Circulation. And by confounding these two Centers with one another in the fundamental Propositions upon which he grounds this Calculus, he erred in the Superstructure, and for want of Skill in second and third Differences, was not able to extricate himself from the Errors. And this is further confirmed by the sixth Article of the second Tract. For that Article is erroneous; and the Error arises from his not knowing how to argue well about second and third Differences. When therefore he wrote those Tracts he was but a Learner, and this he ought in candour to acknowledge.

It seems therefore that as he learnt the Differential Method by means of Mr. Newton's aforesaid three Letters compared with Dr. Barrow's Method of Tangents; so Ten Years after, when Mr. Newton's Principia Philosophiae came abroad, he improved his Knowledge in these Matters, by trying to extend this Method to the Principal Propositions in that Book, and by this means composed the said three Tracts. For the Propositions contained in them (Errors and Trifles excepted) are Mr. Newton's (or easy Corollaries from them) being published by him in other Forms of Words before: and yet Mr. Leibnitz published them as invented by himself long before they were published by Mr. Newton. For in the End of the first Tract, he represents that he invented them all before Mr. Newton's Principia Philosophiae came abroad, and some of them before he left Paris, that is before October 1676. And the second Tract he concludes with these Words: Multa ex his deduci possent praxi accommodata, sed no-
bis nunc fundamenta Geometrica jecisse suffecerit, in quibus maxima consistebat difficultas. Et fortasse attende consideranti vias quasdam novas satis antea impeditas aperuisset videbimur. Omnia autem respondent nostra Analyti Infinitorum, hoc est calculo Summarum & Differentiarum (cujus elementa quaedam in his Actis dedimus) communibus quoad licuit verbis hic expresso. He pretends here that the Fundamenta Geometrica in quibus maxima consistebat difficultas were first laid by himself in this very Tract, and that he himself had in this very Tract opened vias quasdam novas satis antea impeditas. And yet Mr. Newton's Principia Philosophiae came abroad almost two Years before, and gave occasion to the Writing of this Tract, and was written communibus quoad licuit verbis, and contains all these Principles and all these new Ways. And Mr. Leibnitz, when he published that Tract, knew all this, and therefore ought then to have acknowledged that Mr. Newton was the first who laid the Fundamenta Geometrica in quibus maxima consistebat Difficultas, and opened the vias novas satis antea impeditas. In his Answer to Mr. Fatio he acknowledged all this, saying Quam [methodum] ante Dominum Newtonum & me nullus quod sciam Geometra habuit; uti ante hunc maximi nominis Geometram, NEMO SPECIMINE PUBLICE DATO se habere PROBavit. And what he then acknowledged he ought in Candour and Honour to acknowledge still upon all Occasions.

Mr. Leibnitz in his Letter of May 28. 1697, wrote thus to Dr. Wallis. Methodum Fluxionum profundissimi Newtoni cognoscere methodo mea differentiali non tantum animadverti postquam opus ejus [Principiorum sicilicet] & tuum prodiit; sed etiam professus sum in Actis Eruditorum, & alias quaque monui. Id enim candor meo convenire judicavi, non minus quam ipsius merito. Itaque communi nomine designare solo Analyseos infinitesimalis; que latius quam Tetragonistica patet. Interim quem admodum & Viettea & Cartesiana methodus Analyseos speciosae nomine venit; discrimina tamen nonnulla supersunt: ita fortasse

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Newconiana & Mea differunt in nonnullis. Here also Mr. Leibnitz allows that when Mr. Newton's Principles of Philosophy came abroad, he understood thereby the Affinity that there was between the Methods, and therefore called them both by the common Name of the infinitesimal Method, and thought himself bound in candour to acknowledge this Affinity: and there is still the same Obligation upon him in point of Candour. And besides this Acknowledgment, he here gives the Preference to Mr. Newton's Method in Antiquity. For he represents that as the vulgar Analysis in Species was invented by Vieta, and augmented by Cartes, which made some Differences between their Methods: so Mr. Newton's Method and his own might differ in some things. And then he goes on to enumerate the Differences by which he had improved Mr. Newton's Method as we mentioned above. And this Subordination of his Method to Mr. Newton's, which he then acknowledged to Dr. Wallis, he ought still to acknowledge.

In enumerating the Differences or Improvements which he had added to Mr. Newton's Method; he names in the second Place Differential Equations: but the Letters which passed between them in the Year 1676, do show that Mr. Newton had such Equations at that time, and Mr. Leibnitz had them not. He names in the third Place Exponential Equations: but these Equations are owing to his Correspondence with the English. Dr. Wallis, in the Interpolation of Series, considered Fract and Negative Indices of Dignities. Mr. Newton introduced into his Analytical Computations, the Fract, Surd, Negative and Indefinitive Indices of Dignities; and in his Letter of October 24, 1676, represented to Mr. Leibnitz that his Method extended to the Resolution of affected Equations involving Dignities whose Indices were Fract or Surd. Mr. Leibnitz in his Answer dated June 21, 1677, mutually desired Mr. Newton to tell him what he thought of the Resolution of Equations involving Dig-
Dignities whose Indices were undetermined, such as were these $x^y + y^x = xy$, $x^x + y^y = x + y$. And these Equations he now calls Exponential, and represents to the World that he was the first Inventor thereof, and magnifies the Invention as a great Discovery. But he has not yet made a publick Acknowledgment of the Light which Mr. Newton gave him into it, nor produced any one Instance of the use that he has been able to make of it where the Indices of Dignities are Fluents. And since he has not yet rejected it with his usual Impatience for want of such an Instance, we have reason to expect that he will at length explain its Usefulness to the World.

Mr. Newton in his Letter of October 24, 1676 wrote that he had two Methods of resolving the Inverse Problems of Tangents, and such like difficult ones; one of which consisted in assuming a Series for any unknown Quantity from which all the rest might conveniently be deduced, and in collating the homologous Terms of the resulting Equation, for determining the Terms of the assumed Series. Mr. Leibnitz many Years after published this Method as his own, claiming to himself the first Invention thereof. It remains that he either renounce his Claim publickly, or prove that he invented it before Mr. Newton wrote his said Letter.

It lies upon him also to make a publick Acknowledgment of his Receipt of Mr. Oldenburgh's Letter of April 15, 1675, wherein several converging Series for squaring of Curves, and particularly that of Mr. James Gregory for finding the Arc by the given Tangent, and thereby squaring the Circle, were communicated to him. He acknowledged it privately in his Letter to Mr. Oldenburg dated May 20, 1675 still extant in his own Hand-writing, and by Mr. Oldenburg left entered in the Letter-Book of the Royal Society. But he has not yet acknowledged it publickly, as he ought to have done when he published that Series as his own.
It lies upon him also to make a publick Acknowledgment of his having received the Extracts of Mr. James Gregory's Letters, which, at his own Request, were sent to him at Paris in June 1676 by Mr. Oldenburgh to peruse: amongst which was Mr. James Gregory's Letter of Feb. 15. 1671, concerning that Series, and Mr. Newton's Letter of December 10. 1672 concerning the Method of Fluxions.

And whereas in his Letter of Decem. 28. 1675 he wrote to Mr. Oldenburgh, that he had communicated that Series above two Years before to his Friends at Paris, and had written to him sometimes about it; and in his Letter of May 12. 1676 said to Mr. Oldenburgh that he had written to him about that Series some Years before; and in his Letter to Mr. Oldenburgh dated Aug. 27. 1676, that he had communicated that Series to his Friends above three Years before; that is, upon his first coming from London to Paris: He is desired to tell us how it came to pass, that when he received Mr. Oldenburgh's Letter of Apr. 15. 1675 he did not know that Series to be his own.

In his Letters of July 15. and Octob. 26. 1674, he tells us of but one Series for the circumference of a Circle, and faith that the Method which gave him this Series, 'gave him also a Series for any Arc whose Sine was given, tho' the Proportion of the Arc to the whole Circumference be not known. This Method therefore, by the given Sine of 30 Degrees, gave him a Series for the whole Circumference. If he had also a Series for the whole Circumference deduced from the Tangent of 45 Degrees, he is desired to tell the World what Method he had in those Days, which could give him both those Series. For the Method by the Transmutation of Figures will not do it. He is desired also to tell us why in his said Letters he did not mention more Quadratures of the Circle than one.

And if in the Year 1674 he had the Demonstration of a Series for finding any Arc whose Sine is given, he is desired to
to tell the World what it was; and why in his Letter of May 12, 1676 he desired Mr. Oldenburgh to procure from Mr. Collins the Demonstration of Mr. Newton's Series for doing the same thing; and wherein his own Series differed from Mr. Newton's. For upon all these Considerations there is a Suspicion that Mr. Newton's Series for finding the Arc whose Sine is given, was communicated to him in England; and that in the Year 1673 he began to communicate it as his own to some of his Friends at Paris, and the next Year wrote of it as his own in his Letters to Mr. Oldenburgh, in order to get the Demonstration or Method of finding such Series. But the Year following, when Mr. Oldenburgh sent him this Series and the Series of Mr. Gregory and six other Series, he dropped his Pretence to this Series for want of a Demonstration, and took time to consider the Series sent him, and to compare them with his own, as if his Series were others different from those sent him. And when he had found a Demonstration of Gregory's Series by a Transmutation of Figures, he began to communicate it as his own to his Friends at Paris, as he represents in the Acta Eruditorum for April 1691, pag. 178, saying; Jam Anno 1675 compositum habebam opusculum Quadraturæ Arithmetica ab Amicis ab illo tempore lectum, &c. But the Letter by which he had received this Series from Mr. Oldenburgh he concealed from his Friends, and pretended to Mr. Oldenburgh that he had this Series a Year or two before the Receipt of that Letter. And the next Year, upon receiving two of Mr. Newton's Series again by one George Mohr, he wrote to Mr. Oldenburgh in such a manner as if he had never seen them before, and upon Pretence of their Novelty, desired Mr. Oldenburgh to procure from Mr. Collins Mr. Newton's Method of finding them. If Mr. Leibnitz thinks fit to obviate this Suspicion, he is in the first Place to prove that he had Mr. Gregory's Series before he received it from Mr. Oldenburgh.
It lies upon him also to tell the World what was the Method by which the several Series of Regression for the Circle and Hyperbola, sent to him by Mr. Newton June 13. 1676, and claimed as his own by his Letter of August 27. following, were found by him before he received them from Mr. Newton.

And whereas Mr. Newton sent him, at his own Request, a Method of Regression, which upon the first reading he did not know to be his own, nor understood it; but so soon as he understood it he claimed as his own, by pretending that he had found it long before, and had forgot it, as he perceived by his old Papers: it lies upon him, in point of Candor and Justice, either to prove that he was the first Inventor of this Method, or to renounce his Claim to it for preventing future Disputes.

Mr. Leibnitz in his Letter to Mr. Oldenburgh dated Feb. 3. 1673, claimed a Right to a certain Property of a Series of Numbers Natural, Triangular, Pyramidal, Triangulo-Triangular, &c. and to make it his own, represented that he wondered that Monsieur Pascal, in his Book entitled Triangulum Arithmeticum, should omit it. That Book was published in the Year 1665, and contains this Property of the Series; and Mr. Leibnitz has not yet done him the Justice to acknowledge that he did not omit it. It lies upon him therefore in Candor and Justice, to renounce his Claim to this Property, and acknowledge Mr. Pascal the first Inventor.

He is also to renounce all Right to the Differential Method of Mouton as second Inventor: for second Inventors have no Right. The sole Right is in the first Inventor until another finds out the same thing apart. In which case to take away the Right of the first Inventor, and divide it between him and that other, would be an Act of Injustice.

In his Letter to Dr. Sloane dated Decem. 29. 1711. he has told us that his Friends know how he came by the Differential Method. It lies upon him, in point of Candor, openly and plainly, and without further Hesitation, to satisfy the World how he came by it.
In the same Letter he has told us that he had this Method above Nine Years before he published it, and it follows from thence that he had it in the Year 1675 or before. And yet its certain that he had it not when he wrote his Letter to Mr. Oldenburgh, dated Aug. 27. 1676, wherein he affirmed that Problems of the Inverse Method of Tangents and many others, could not be reduced to infinite Series, nor to Equations or Quadratures. It lies upon him therefore, in point of Candor, to tell us what he means by pretending to have found the Method before he had found it.

We have shewed that Mr. Leibnitz in the End of the Year 1676, in returning home from France through England and Holland, was meditating how to improve the Method of Sufus for Tangents, and extend it to all sorts of Problems, and for this end proposed the making of a general Table of Tangents; and therefore had not yet found out the true Improvement. But about half a Year after, when he was newly fallen upon the true Improvement, he wrote back Clariss. Sufii Methodum Tangentium nondum esse absolutam Celeberrimo Newtono assentior. Et jam A MULTO TEMPORE rem Tangentium generalis tractavi, seilicet per differentias Ordinatarum. Which is as much as to say that he had this Improvement long before those Days. It lies upon him, in point of Candor, to make us understand that he pretended to this Antiquity of his Invention with some other Design than to rival and supplant Mr. Newton, and to make us believe that he had the Differential Method before Mr. Newton explained it to him by his Letters of June 13. and October 24. 1676, and before Mr. Oldenburgh sent him a Copy of Mr. Newton's Letter of Decem 10. 1672 concerning it.

The Editors of the Acta Eruditorum in June 1696, in giving an Account of the two first Volumes of the Mathematical Works of Dr. Wallis, wrote thus, in the Style of Mr. Leibnitz. Ceterum ipse Newtonus, non minus Candidior quam praecaris in rem Mathematicam meritis insignis, publice &
praeponit agnovit Leibnitàium, tum cum (intervenedente celeberrimo Viro Henrico Oldenburgò Bremeroni, Societatis Regiae Anglicaè tum Secretario) inter ipsos (ejusdem jam tum Societatis Socios) Commercium intercederet, id est jam fere ante annos viginti & amplius, Calculum summ differentiam, Seriesque infinitas, & pro iis quoque Methodos generalis habuisse; quod Wallisius in Prefatione Operum, facta inter eos communicationis mentionem faciens, præterit; quoniam de eo fortasse non satis ipsi constabat. Ceterum Differentiarum consideratio Leibnitiana, cujus mentionem facit Wallisius (ne quis seilset, ut ipse ait, causaeretur de Calculo Differentiali nihil ab ipso diemum fuisse) mediationes apernit, quæ aliunde non aqua nascebantur, &c. By the Words here cited out of the Preface to the two first Volumes of Dr. Wallis's Works, it appears that Mr. Leibnitz had seen that Part of the Preface, where Mr. Newton is said to have explained to him (in the Year 1676) the Method of Fluxions found by him Ten Years before or above. Mr. Newton never allowed that Mr. Leibnitz had the Differential Method before the Year 1677. And Mr. Leibnitz himself in the Acta Eruditorum for April 1691. pag. 178, acknowledged that he found it after he returned home from Paris to enter upon Business, that is, after the Year 1676. And as for his pretended general Method of infinite Series, it is so far from being general, that it is of little or no use. I do not know that any other Use hath been made of it, than to colour over the Pretence of Mr. Leibnitz to the Series of Mr. Gregory for squaring the Circle.

Mr. Leibnitz, in his Answer to Mr. Fatio printed in the Acta Eruditorum for the Year 1700. pag. 203. wrote thus.

Ipse [Newtonus], scit unus omnium optime, satisque indicavit publice cum sua Mathematica Naturæ Principia publicaret, Anno 1687, novâ quaedam inventâ Geometrica, quâ ipsi communia mecum fueræ, NEUTRUM LUCI AB ALTERO ACCEPTÆ, sed meditationibus quemque suis debere, & a me decennio ante [i.e. anno 1677] exposita suffisse. In the
Book of Principles here referred unto, Mr. Newton did not acknowledge that Mr. Leibnitz found this Method without receiving Light into it from Mr. Newton's Letters above-mentioned; and Dr. Wallis had lately told him the contrary without being then confuted or contradicted. And if Mr. Leibnitz had found the Method without the Assistance of Mr. Newton, yet second Inventors have no Right.

Mr. Leibnitz in his aforesaid Answer to Mr. Fatio, wrote further: Certe cum elementa Calculi mea edidi anno 1684, ne conftabas quidem mihi alium de inventis ejus [sc. Newtoni] in hoc genere, quam quod ipse olim significaverat in literis, posse fes Tangentes invenire non sublatis irrationalibus, quod Hugenius quoque se posse mihi significavit postea, et si ceterorum ejus Calculi adhuc exper. Sed majora multo consecutum Newtonum, vifo demum libro Principiorum ejus, fatis intellexi. Here he again acknowledged that the Book of Principles gave him great Light into Mr. Newton's Method: and yet he now denies that this Book contains any thing of that Method in it. Here he pretended that before that Book came abroad he knew nothing more of Mr. Newton's Inventions of this kind, than that he had a certain Method of Tangents. and that by that Book he received the first Light into Mr. Newton's Method of Fluxions: but in his Letter of June 21. 1677 he acknowledged that Mr. Newton's Method extended also to Quadratures of curvilinear Figures, and was like his own. His Words are; Arbitror que celare voluit Newtonus de Tangentibus ducendis ab his non abludere. Quod addit, ex hoc eodem fundamento Quadraturas quoque reddi faciliores me in sentimentia hac confirmat; nimiram semper figura illa sunt quadrabilia quae sunt ad equationem differentialem.

Mr. Newton had in his three Letters above-mentioned (copies of which Mr. Leibnitz had received from Mr. Oldenbergh) represented his Method to general, as by the Help of Equations, finite and infinite, to determin Maxima and Minima, Tangents, Areas, solid Contents, Centers of Gravity, Lengths
Lengths and Curvities of curve Lines and curvilinear Figures, and this without taking away Radicals, and to extend to the like Problems in Curves usually called Mechanical, and to inverse Problems of Tangents and others more difficult, and to almost all Problems, except perhaps some Numerical ones like those of Diophantus. And Mr. Leibnitz in his Letter of Aug. 27. 1676, represented that he could not believe that Mr. Newton's Method was so general. Mr. Newton in the First of his three Letters set down his Method of Tangents deduced from this general Method, and illustrated it with an Example, and said that this Method of Tangents was but a Branch or Corollary of his General Method, and that he took the Method of Tangents of Slusius to be of the same kind: and thereupon Mr. Leibnitz, in his Return from Paris through England and Holland into Germany, was considering how to improve the Method of Tangents of Slusius, and extend it to all sorts of Problems, as we shewed above out of his Letters. And in his third Letter Mr. Newton illustrated his Method with Theorems for Quadratures and Examples thereof. And when he had made so large an Explanation of his Method, that Mr. Leibnitz had got Light into it, and had in his Letter of June 21. 1677 explained how the Method which he was fallen into answered to the Description which Mr. Newton had given of his Method, in drawing of Tangents giving the Method of Slusius, proceeding without taking away Fractions and Surds, and facilitating Quadratures; for him to tell the Germans that in the Year 1684, when he first published his Differential Method, he knew nothing more of Mr. Newton's invention, than that he had a certain Method of Tangents, is very extraordinary and wants an Explanation.

At that time he explained nothing more concerning his own Method, than how to draw Tangents and determin Maxima and Minima without taking away Fractions or Surds. He certainly knew that Mr. Newton's Method would do all
this, and therefore ought in Candor to have acknowledged it. After he had thus far explained his own Method, he added that what he had there laid down were the Principles of a much sublimier Geometry, reaching to the most difficult and valuable Problems, which were scarce to be resolved without the Differential Calculus, AUTO SIMILL or another like it. What he meant by the Words AUTO SIMILL was impossible for the Germans to understand without an Interpreter. He ought to have done Mr. Newton justice in plain intelligible Language, and told the Germans whose was the Methodus SIMILIS, and of what Extent and Antiquity it was, according to the Notices he had received from England; and to have acknowledged that his own Method was not so ancient. This would have prevented Disputes, and nothing less than this could fully deserve the Name of Candor and Justice. But afterwards, in his Answer to Mr. Fatio, to tell the Germans that in the Year 1684, when he first published the Elements of his Calculus, he knew nothing of a Methodus SIMILIS, nothing of any other Method than for drawing Tangents, was very strange and wants an Explanation.

It lies upon him also to satisfy the World why, in his Answer to Dr. Wallis and Mr. Fatio, who had published that Mr. Newton was the eldest Inventor of that Method by many Years, he did not put in his Claim of being the eldest Inventor thereof, but laid till the old Mathematicians were dead, and then complained of the new Mathematicians as Novices; attacked Mr. Newton himself, and declined to contend with any Body else, notwithstanding that Mr. Newton in his Letter of Octob. 24. 1676 had told him, that for the sake of Quiet, he had Five Years before that time laid aside his Design of publishing what he had then written on this Subject, and has ever since industriously avoided all Disputes about Philosophical and Mathematical Subjects, and all Correspondence by Letters about those Matters, as tending to Disputes;
putes; and for the same Reason has forborn to complain of Mr. Leibnitz, until it was shewed him that he flood accused of Plagiary in the Acta Lipsia, and that what Mr. Keill had published was only in his Defence from the Guilt of that Crime. It has been said the Royal-Society gave judgment against Mr. Leibnitz without hearing both Parties. But this is a Mistake. They have not yet given judgment in the Matter. Mr. Leibnitz indeed desired the Royal-Society to condemn Mr. Keill without hearing both Parties; and by the same sort of Justice they might have condemned Mr. Leibnitz without hearing both Parties; for they have an equal Authority over them both. And when Mr. Leibnitz declined to make good his Charge against Mr. Keill, the Royal-Society might in justice have censured him for not making it good. But they only appointed a Committee to search out and examining such old Letters and Papers as were still extant about these Matters, and report their Opinion how the Matter stood according to those Letters and Papers. They were not appointed to examine Mr. Leibnitz or Mr. Keill, but only to report what they found in the ancient Letters and Papers: and he that compares their Report therewith will find it just. The Committee was numerous and skilful and composed of Gentlemen of several Nations, and the Society are satisfied in their Fidelity in examining the Hands and other Circumstances, and in printing what they found in the ancient Letters and Papers so examined, without adding, omitting or altering any thing in favour of either Party. And the Letters and Papers are by order of the Royal-Society preserved, that they may be consulted and compared with the Commercium Epistolicum, whenever it shall be desired by Persons of Note. And in the mean time I take the Liberty to acquaint him, that by taxing the Royal-Society with Injustice in giving Sentence against him without hearing both Parties, he has transgressed one of their Statutes which makes it Expulsion to defame them.

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The
The Philosophy which Mr. Newton in his *Principles* and *Optiques* has pursued is Experimental; and it is not the Business of Experimental Philosophy to teach the Causes of things any further than they can be proved by Experiments. We are not to fill this Philosophy with Opinions which cannot be proved by Phænomena. In this Philosophy Hypotheses have no place, unless as Conjectures or Questions proposed to be examined by Experiments. For this Reason Mr. Newton in his *Optiques* distinguished those things which were made certain by Experiments from those things which remained uncertain, and which he therefore proposed in the End of his *Optiques* in the Form of Queries. For this Reason, in the Preface to his *Principles*, when he had mention'd the Motions of the Planets, Comets, Moon and Sea as deduced in this Book from Gravity, he added: *Uti nam certa Naturæ Phænomena ex Principiis Mechanicis eodem argumentandi genere derivare liceret*. Nam multa me movent ut nonnihil suspicer ea omnia ex viribus quibusdam pendere posse, quibus corporum particula per causas nondum cognitas vel in se mutuo impelluntur & secundum figuræ regularis coherant, vel ab invicem fugantur & recedant: quibus viribus ignotis Philosophi had-termus Naturam frustra tentarunt. And in the End of this Book in the second Edition, he said that for want of a sufficient Number of Experiments, he forbore to describe the Laws of the Actions of the Spirit or Agent by which this Attraction is performed: And for the same Reason he is silent about the Cause of Gravity, there occurring no Experiments or Phænomena by which he might prove what was the Cause thereof. And this he hath abundantly declared in his *Principles*, near the Beginning thereof, in these Words; *Virium causas & sedes Physicas jam non expendo*. And a little after: *Voces Attractionis, Impulsus, vel Propensio-nis ejusdemque in centrum indifferenter & pro se mutuo promis-cue nuduro, hab Vires non Physice sed Mathematicæ tantum considerando. Unde caveat Lectore ne per hujusmodi voces cogitet me*.
Speciæm vel modum actionis, causanæve aut rationem physicam alicubi definire, vel Centris (quæ sunt punta Mathematæ) vires verè & physis tribuere, si forte aut Centra trahere aut vires Centrorum esse dixeræ. And in the End of his Opticks: Quæ causa efficiente haæ attractiones [i.e. gravitas, visque magnetica & electrica] peragantur, hic non inquiræ. Quam ego Attractionem appello, fieri sane potest ut ea efficiatur impulsu vel alio alique modo nobis ignoto. Hanc vocem Attractionis ita hic accipi velit ut in universum solummodo vim aliquam significare intelligatur quæ corpora ad se mutuo tendant, cuicunque demum causæ attribuenda sint illa vis. Nam ex Phænomenis Naturæ illud nos prius edocës oportet quænam corpora se invicem attrabant, & quænam sint leges & proprietates istius attractionis, quam in id inquirere par sit quanam efficiente causa peragatur attractione. And a little after he mentions the same Attractions as Forces which by Phænomena appear to have a Being in Nature, tho' their Causes be not yet known; and distinguishes them from occult Qualities which are supposed to flow from the speciæc Forms of things. And in the Scholium at the End of his Principles, after he had mentioned the Properties of Gravity, he added: Rationem vero barum Gravitatis proprietatem ex Phænomenis nondum posse deducere, & Hypotheses non fingo. Quicquid enim ex Phænomenis non deducitur Hypothese vocanda est; & Hypotheses seu Metaphysica seu Physica, seu Qualitatum occultarum, seu Mechanica, in Philosophia experimenti locum non habet. Satis est quod Gravitas revera excitat & agat secundum leges a nobis expositas, & ad Corporum caelestium & Maris nostris motus omnes sufficiat. And after all this, one would wonder that Mr. Newton should be reflected upon for not explaining the Causes of Gravity and other Attractions by Hypotheses; as if it were a Crime to content himself with Certainties and let Uncertainties alone. And yet the Editors of the Acta Eruditorum, (a) have told the World that Mr. Newton denies that the cause of Gravity is Mechanical, and that if the Spirit or Agent by which Electrical Attraction is performed; be not the Ethier or subtle Matter of Cartes, it is less valuable than anHypothesis, and perhaps may be the Hylarchic Principle of Dr. Henry Moor: and Mr. Leibniz (b) hath accused him of making Gravity a natural or essential Property of Bodies, and an occult Quality and Miracle. And by this sort of Railery- they are persuading the Germans that Mr. Newton wants Judgment, and was not able to invent the Infinitesimal Method.

(a) Acta 1714, mensæ Martis, p. 141, 142. (b) In trælatu de Bonitate Dei & in Epistolæ ad D. Hartocker & aliis.
It must be allowed that these two Gentlemen differ very much in Philosophy. The one proceeds upon the Evidence arising from Experiments and Phenomena, and stops where such Evidence is wanting; the other is taken up with Hypotheses, and propounds them, not to be examined by Experiments, but to be believed without Examination. The one for want of Experiments to decide the Question, doth not affirm whether the Cause of Gravity be Mechanical or not Mechanical: the other that it is a perpetual Miracle if it be not Mechanical. The one (by way of Enquiry) attributes it to the Power of the Creator that the least Particles of Matter are hard: the other attributes the Hardness of Matter to conspiring Motions, and calls it a perpetual Miracle if the Cause of this Hardness be other than Mechanical. The one doth not affirm that animal Motion in Man is purely mechanical: the other teaches that it is purely mechanical, the Soul or Mind (according to the Hypothesis of an Harmonia Praesstabilita) never acting upon the Body so as to alter or influence its Motions. The one teaches that God (the God in whom we live and move and have our Being) is Omnipresent; but not as a Soul of the World: the other that he is not the Soul of the World, but INTELLIGENTIA SUPRA MUNDANA, an Intelligence above the Bounds of the World; whence it seems to follow that he cannot do any thing within the Bounds of the World, unless by an incredible Miracle. The one teaches that Philosophers are to argue from Phenomena and Experiments to the Causes thereof, and thence to the Causes of those Causes, and so on till we come to the first Cause: the other that all the Actions of the first Cause are Miracles, and all the Laws impress on Nature by the Will of God are perpetual Miracles and occult Qualities, and therefore not to be considered in Philosophy. But must the constant and universal Laws of Nature, if derived from the Power of God or the Action of a Cause not yet known to us, be called Miracles and occult Qualities, that is to say, Wonders and Absurdities? Must all the Arguments for a God taken from the Phenomena of Nature be exploded by new hard Names? And must Experimental Philosophy be exploded as miraculous and absurd, because it afferts nothing more than can be proved by Experiments, and we cannot yet prove by Experiments that all the Phenomena in Nature can be solved by meer Mechanical Causes? Certainly these things deserve to be better considered.

ER RATA. Pag. 199. l. 37. 14. put an Afterisk (*) after the Word Letter.
PHILOSOPHICAL
TRANSACTIONS.

For the Months of March, April and May, 1715.

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II. Botanicum Hortense IV. Giving an Account of divers Rare Plants, Observed the last Summer A. D. 1714, in several Curious Gardens about London, and particularly the Society of Apothecaries Physick Garden at Chelsea. By James Petiver, F. R. S.

III. Observations of the late Total Eclipse of the Sun on the 22d of April last past, made before the Royal Society at their House in Crane-Court in Fleet-street, London. By Dr. Edmund Halley, Reg. Soc. Secr. With an Account of what has been communicated from abroad concerning the same.


Quanquam multæ varietates mutationesque inter stellas fixas, quoad apparentem earum magnitudinem in caelo contingant, nulla tamen inter omnes mutabilis apparentiæ stellas fixas mirabilior habitæ est, illâ quam Fabricius Anno 1596, in collo Ceti primus observavit. Licet enim primò pro ejusmodi nova Stella habitâ fuerit quæ nunquam ante existerit, & postquam disparuit non amplius esset redditura: nunc tamen experientia fatis comprobavit eam constantem existere, & ab òmi dubio à principio mundi in eo loco, quern nunc quoque obtinet, existitisse. Hoc solummodo in ea mirandum, quod semper quotannis in varia magnitudine spectandam exhibeat, & plerumque certis temporibus nudis oculis plane videri nequeat; qua de causa etiam a Domino Hevelio Stella Mira vocata est.

Aliam huic similem ego quoque in collo Cygni deprehendî, multo autem minorem, & quotannis breviori spatio temporis conspiciendi. Unde mirum non est eam tamdiu incognitam manuisse: Imo pro singulari felicitate reputandum, quod eo ipso tempore visibilis fuerit, ac in sua maxima magnitudine apparuerit, quo Bayerus stellas in Cygno consideravit & delineavit, ubi eam notâ x designavit, & inter constanter apparentes fixas 5* magnitudinis recensuit. Quemadmodum etiam supra memoratam in collo Ceti, cum hocce Sydus consideraret & delinearet, in quarta magnitudine deprehendit & literâ o designavit, eam-
eamque pro constanter apparente Stella fixa habuit.
Ut mutabilem apparentiam stellæ χ in collo Cygni de-
prehenderem, occasionem dedit stella capiti Cygni vicina,
quam Hevelius, Astronomus solertissimus, Anno 1670 &
7r. observavit. Cum enim spem conciperem fore ut ead-
em stella nunc iterum sæpius esset apparitura, non secus
ac stella in collo Ceti, quam Hevelio post primam dispari-
tionem mox iterum apparessisse constabat, quærebam eam
1° & 6° (11° & 16°) Julii 1686. noctibus serenis, non au-
tem reperiebam; sed potius animadvertebam stellam illam
5ta magnitudinis in collo Cygni, à Bayero græca litera χ
signatam, desiderari. Die vero 9 (19.) Octobris depre-
hendi eam nudis oculis distincte omnino. Et quia facile
adducebar ut existimarem eandem nudis nostris oculis ite-
rum disparitatem, delineavi aliquot ipsam circumstantes
stellulas, ope bipedalis magnæque capacitate Tubi, ut ex
harum cum illa comparatione magnitudinem ejus, cum de-
crecseret, expenderem, uti Fig. A. exhibetur.
Deprehensum est quoque, stellam istam paulatim decre-
visse, donec eam Tubo 8 pedum non potuerim amplius
assequi, cum tamen aliam illam in collo Ceti, quando nu-
do oculo non amplius patet, per tubum 4 pedum semper
dignoscere possim.
Ab illo tempore quærebam quidem stellam istam variis
noctibus frustra, tandem verò tamen eam reperi 6° (16°)
Aug. 1687. iterum ope octopedalis Tubi, at vero exiguum
valde. Inde de die in diem crevisse deprehendebatur; &
factum est, ut die 23 Octob. (2. Novemb.) iterum primâ
vice, nudo oculo, ut se conspiciendam præberet, valde
licet adhuc dum exigum. Die 2do (12°) Nov. optime erat
conspicua, etiam post 26um Nov. (6. Dec.) ut ut hoc ulti-
mo die jam iterum in statu decreascendi existeret. Postmo-
dum non nisi per Tubos dignoscii potuit, tandemque adeò
exigua evisit, ut iterum Tubo 8 pedum eam deprehendere
non potuerim. Atque ita animadvertsum est hac vice, ab
una disparitione usque ad alteram, annum unum, mensem unum, unamque hebdomadam circiter effluxisse. Sequentes quoque observationes docucunt hanc stellam tempus satis contans in sua apparitione servare, non ramen quavis vice ad aequalam magnitudinem pervenire. Imo aliquando accidit, ut nudo oculo planè invisibilis maneât, dum per Tubum est conspicuâ, & maximam suam magnitudinem assècuta est; prout Anno 1688 in fine, & 1689 in principio anni accidit. E contrario, sequente anno 1690 stella hâc eo melius videri poterat, & quidem notabiliter major quam sua vicina; quam Bayerus juxta collocavit extra collum Cygni, nullaque literâ notavit, quam ego solius memoriae juvandâ gratia, Hebraica literâ notavi. Et postquam hujus stellæ apparentiam & disparentiam sæpius observavi, comperì eam valde esse regularem, revolutionemque 404; dierum servare.

Cum Miscellanea Berolinensia, serius ad nos perlata sint, non ante annum ultimo elapsum hanc Stellam novam secundum D. Kirchii monitum conspeximus; idque juxta Idus Julius, st. vet cum multo clarior quam vicina 2, ac fere aequalis media in collo Cygni (Bayero) apparuit: sed post semem nudis oculis inconsipicua facta, tandem etiam Telescopio evanuit. Juxta periodum qua revolvi dicitur, mensa saltem Augusto currentis anni 1715. maximam suam claritatem adipsût debeat: unde curiosis siderum scrutaturibus de hâc sit sane notabile certa capi poterint documenta.

Ut autem facilius in Cælo inveniatur, duo Schemata adjecimus, quorum alterum exhibet Cygni collum, cum Fixis huius Novæ adjunctis, Novisque duabus aliis intra seculum in ejus vicinia emergis; quorum qua ante Pectus Cygni etiamnum conspicitur quasi quinti honoris: que vero sub Capite per biennium tantum visà haevemus latet. Altera Figura, quae est Kirchii A, Telescopicas Novæ proximas demonstrat, ut sitatur quo pracise loco primum redentis radium seduli Cælispices Tubo armâti præstolari debeant.
II. BOTANICUM Hortense IV.

Giving an Account of divers Rare Plants, Observed the last Summer A. D. 1714. in several Curious Gardens about London, and particularly the Society of Apothecaries Physick-Garden at Chelsea.

By James Petiver, F. R. S.

SECTI. EUROPEAN Plants.

   Anthyllis Valentina Clus. Hist. 186. c. 9. Fig. id. Hist. 480. 
   Fig. Park. 446. fig. 3. 
   Anthyllis maritima, Chamesyce similis CB.282. I. Phyt. 552.1. 
   L. 29. p. 374. Fig. 
   This is a low spreading Ground Plant, with roundish small Leaves, and very little four leaved blush Flowers.

Dr. John Place M. D. and publick Professor of Valentina, first observed this Plant about the Ditches of that City, and there shewed it to that accurate Botanist Carolus Clusius who has given us a very good Figure of it.

Dr. Magnol hath also found it on the Coasts of Langueock.

Monf. Riqueur Apothecary to the late Queen of Spain, sent me the Seed of this and many other curious Plants, which he collected about Madrid, several of which were the last Summer raised in our Physick Garden at Chelsea, where this Flowered.
*Pfyllium majus* erectum C B. pin. 191. 3. alterum C B phyt. 353. 2.  
*Pfyllium* sive *Pulicaris* Herba Ger. 471, fig. 1. Jonst. 587, fig. 1.  
*Pulicaris* Herba Lobel. Icon. 436, 2. id. Belg. 523, Obs. 239.  
Its Top Branches and Stalks are somewhat fat or clamy, its Leaves are like *Hyssop* and broader than the Perennial. Grows plentifully in the Fields about Montpelier.  

*Pfyllium* *Dioscoridis* sive *Indicum* foliis crenatis C. B. 199, 1. prodr. 99, 1.  
*Pfyllium* *Indicum* foliis crenatis Park. 277, 3.  
*Pfyllium* *laciniiatis* foliis Bocc. 8, Tab. 4.  
This differs from the common Annual, only in having notcht or indented Leaves.  

4. Perennial Fleawort Ray 882, 3,  
*Pfyllium* *Camer. Epit. 811, Fig. Chabr. 501, Ic. 3, I B. 3, 1. 31, p. 513, fig.  
*Pfyllium majus supinum* C. B. 191, 2. majus C. B. phyt. 353, 1.  
*Pfyllium maj. sempervirens* Park. 277, 2 & vulg. 278, second. Fig.  
*Pfyllium* *Plinianum* forte, radice perenni, supinum Lobel, Icon 437, 1; id. Belg. 523, id. Obs. 239, fig.  
Grows frequently in Italy and about Montpelier.  

/*Vertues.*/ The Seed of this Plant evacuates yellow Choler, and by its Mucilage, blunts the Acrimony of the Humors, and is therefore commended in Dysenteries and other Corrosions of the Gutts.  

Dr. *Sloane* has experienced it in Excoriations of the Uvula or Palat, and where the Tongue is parcht.  

*H. Reusnerus* says a Mucilage of its Seeds, in Rose-water with *Vinegar*, has cured great Pains in the Head, proceeding from a Hot Cause, when other Medicines have failed.  

The
The same with Camphire has been successfully applied to inflamed Eyes.


Mr. Dale was the first that observed this in England, viz. about Colchester. I find it the same with that of Triumphetti, a Specimen of it being lately sent me from Peter Antony Micheli Botanist to his Royal Highness the Duke of Florence.


Ralph first observed this Plant on the Sides of Mount Libanus, and from whom all our Figures are copied.

Dalechamp has since found it in the Valleys about Marra near Gratianople in Dauphiny.

Honorius Bellus a learned Physitian in Candy, says the Rusticks of that Island make a Tea of this Plant, which cures them of all Sorts of Fluxes.

Its called Stoibeda in most parts of Greece.


Its blew Succory-like; Flowers, with narrow dented Leaves distinguish it from all others.

Grows
Grows very common near Narbone, and in Savoy on dry stony Hills.

I gathered this elegant Plant in Flower this Summer in His Majesty’s Gardens at Hampton Court, under the care of Mr. Wise, King George’s Gardiner.

8. Yellow Cat-Succory.

Catantance Cretica fl. luteo.


Mr. Jacob Bobart Botanick Professor at Oxford, sent me the first Specimen of this, which has lately Flowred very well with us in Chelsea Garden.


Jacobea marina five Cineraria vulg. Park. 669. fig. 7.

Jacobea maritima C. B. 131. 3.

This has been long cultivated as a great Ornament in Gardens.

Vertues. Alpinus says the Ægyptians use this as a very sovereign Plant, drinking a Tea of it for the Stone, and to open Obstructions of the Bowels and Womb.

Grows on the Coasts of Tuscany.


Jacobea Sicula Chrysanthemi facie Bocc. 66. Tab. 36.

It leaves like our Corn Marygold, flowers in Chelsea Gar- den even till Christmas. Grows wild about Catania, &c.

11. Common Narrow Caffidony Ray 281. 4.

Elychryson five Stachas citrina angustifolia C. B. 264. 4. vel Gallica phyt. 513. 4.

Chrysocome vulg. 1. Clus. 326. fig.


Stachas
Stachas Citrina Dod. 268.
Stachas citrina five Amaranthus luteus Jonst. 646. Ic. 1.
Stachas citrina five Coma aurea Park. 68. fig. 7.
fig. ead. flore luteo pallecente Chabr. 369. Ic. 5.
The Leaves of this Plant are best represented in I B.
and Chabreus, being much narrower than those Figured
by Clusius.
Grows plentifully about Montpelier, where it Flowers
in April and May.
Elychrysum Creticum C B. 264. 6.
Chrysecome 5 quæ Cretica Cluf. 327.
Chrysecome five Stechas citrina Cretica Park. 69. 8.
Stachas citrina globofo & ampol flore Cretica, Barrelier pl.
987. Ic. opt. 814.
This last Author has given a very accurate Figure of
this Plant, which is so beautiful an Ornament in our
most curious Gardens.
13. Stif-rim’d Mary-gold, Ray 338 c. 4. pl. 2.
After Atticus I. Cluf. 13. Fig. 1. Massiloticus Tabern. Icon.
361. 2.
After Att. luteus vulg. Park. 128. fig. 1.
After Atticus & Inquinaria 1. Inguinalis Lob. Ic. 348. 2. Belg.
423. Obs. 188. fig.
After luteus, foliis ad florem rigidis C B. 266. I. Phyt.
518. 1.
Chrysanthemum Afteris facie, foliis ad florem rigidis H.
Leyd 144.
Its Rim of yellow Flowers is beset with stiff, long,
pointed green Leaves, by which its distinguished from
all others.
Its common in Sicily, Italy, Narbon and Spain, Flowring
in May and June.

Chameelum annuum ramosum Cotulañetide fol. amplioribus capitulis spinosis Bob. H. Ox. 3. p. 36. 12. Sert. VI. Tab. 8. fig.

We are obliged to Mr. Jacob Bobart for the first Knowledge of this Plant.


Atracytis veterum I. vera, fl. luteo Chabr. 353. Ic. 4. I B. 3. 1. 25. p. 85. fig.

Atracytis Theophrasti & Diosc. sanguineo succo Col. 19. fig. 23.

Mr. Ray has given a large Description of this Thistle p. 304. from the accurate Columna, and it is remarkable for its bloody Juice.

Its said to have the same Vertues with the Carduus Benedictus.

Grows in France, Spain and Italy as also about Geneva in Pathways and Borders of Fields.


an Chameelon niger verus Park:

This differs from the Distaff Thistle in having its upper Stalks woolly like Cobwebs. It was many Years since raised in Mr. Charles Dubois his Garden at Mitcham, from Seed I gave him brought me by Mr. Samuel Daniel, Surgeon, from the Island Coos.


Clusius first observed this about Salamanca in Spain, it hath since been found in Languedoc and other places.

18. Theophrastus his Fish Thistle Ray 315. 4. Dale Suppl. 74. 4.
Acarna Theophrast Ger. 1012. fig. 7. Jonst. 1175. f. 7.
Acarna di Theophrasto Imperati 669. fig. opt.
Acarna major caule non foliolo C. B. 273. 7. Park. 966. f. 7.

Acarna similis, Carduus polyacanthus Chabr. 356. Ic. 2.
Polyacanthus Causabona Acarna similis I. B. 3. l. 25. p. 92. fig.

Imperatus his Figure, which Barrelier has copied, very well represents this elegant Thistle. Chabr. & I B. are also better than Lobel’s, which Park. and most others have followed.

Grows on the Hills, North of Rio near the Iron Mines in the Island of Ilva.

Instit. 449.
Jacea montana incana Pini capite C. B. 272.
Jacea pumila Narbonensis Park. 471. fig. 6.
Jacea mont. capite magno Stroboli I. B. 3. l. 25. p. 30. fig.
Chabr. 343. Ic. 3.
Stoebe Pinea amplo capite Barrel. 970. Ic. opt. 138.

Some of the bottom Leaves of this are whole, which are not express in any Figure yet extant. I have received very fair Specimens of this elegant Plant from that Accurate Botanist Dr. John Salvadore at Barcelona. It Grows plentifully about Narbone and Montpelier, where it Flowers in June, as it did this Summer in Chelsea Garden.
Carduus Lustian. canescens, alato caule, capite lanuginoso
Jacea Lust. canescens alato caule, capite spinoso & lanuginoso Ray Vol. 3. p. 204. pl. 28.
Its Root-Leaves like Scabious, but on the Stalk whole and narrow, its Head woolly like a Cobweb, beset with long Thorns, in the midst of which comes a yellow Flower. Raised this Summer in Chelsea Garden.

Jacea lutea Oriental. capite spinis simplicibus armato.
The lower Leaves are lobated like the Stabe Salam. r. Clus. but on the Stalks they are plain and narrow. At the Top grow specious yellow Flowers like the Sultan, of that Colour, set in scaly Heads, each ending in a single longish Prickle.

I have as yet seen this only with Mr. Fairchild at Hoxton, raised from Seed which Dr. Sherard sent to Mr. Stonestreet.

Jacea carnea, marginibus squamarum nigris Nobis.
Jacea humilis, Hieracii folio Park. 471. 5. H. Lugd. 1193. fig.
Jacea humilis alba, Hieracii folio C. B. 271. 2. phyt. 530.

23. Austrian and Spanish Stæbe Ray. 324. 4.
Stæbe Gallica & Austriaca elatior Clus. l. 4. p. 10.
Staébe *Austriaca* elatior Park. 476.

*Staébe* major calyculis non splendentibus C. B. 273. 3.

*Staébe* Salmantica alterius, altera species *Clus.* Hisp. 362.

Centaurium majus in Muris Gesn. Hort. 252.

- *species* tenuifolia *Chabr.* 345. Ic. 6. I. B. 3. 1. 25. p. 31. fig.

*Jacea* alba Lugd. 1192. Ic. 2.


*Jacea* non spinosa, *fol. magis divisis elatior, capitulis mino-

ribus non splendentibus Bob. Oxon 140. 15.

- Its lower Leaves small and deeply cut. its *Flowers* purple

like the *Common*, with small half starr'd hairy *Scales*.

- Mons. Rigueur sent me the *Seed* of this from *Madrid*,

which Flowred in *Chelsea Garden* this *Autumn*.

24. *Pona*’s Pine-leaved *Candy Knapweed*.

*Chamaepinca* Pr. Alpin. Exot. 76. fig. ex sententia G. Sherard.

Chamæpitys Berthiolo.

Chamæpitys fruticosæ Cretica Belli.

*Jacea* fruticans *Pini* folio C. B. 271. 3. Pluk. Tab. 94. fig. 3.

*Jacea* Cretica fruticæ, *Elybrush* folio, *fl. magno purpu-

rascente T. Coral. 32.

Staébe *Rorifmarini* folio Fonst. 731. fig. 4.

Staébe capitata *Rorifmarini* folio *Pona* 329. fig. *Chabr.* 344.

Ic. 4. I. B. 3. 1. 25. p. 36. fig.

Staébe capit. overo *Chamapino* fruticæ *di Candia* *Pona* 

Ital. 75. fig.

Staébe *Cretica* fruticæ, *Picea* aut potius *Pini* angustis foliis


- This is not the *Cyanus* arboreÆcens longifolia *r. Alp*.

Exot. p. 30. as Parkinson and some others affert.

- Dr. Plukenet’s Figure (which he took from Sir George *Wheeler*’s Specimen) very well agrees with the Pattern which Dr. Sherard sent me from Smyrna A. D. 1705. Prosper *Alpinus*’s also is well cut. *Pona*’s amongst thole of

Mount.
Mount Baldus was taken from a Garden Plant, but that in the Italian Edition from a Native.

This elegant Plant I have only seen with Mr. Fairchild at Hoxton, raised from the Seed which Dr. Sherard sent to the Reverend Mr. Stone-street.

Scabiosa Fraxinella folio Institut. Rei Herbar. 666.

This is a specious Plant and grows in Chelsea Garden near two Foot high, its lower Leaves are much deeper dened than the Fraxinella, and more resembles our Manna Ash. It Flowers in July and August.

Umbelliferous Plants, &c.

27. p. 143. fig.
Angelica Casalp. 307. c. 48.
Angelica sylv. montana C. B. 156. 5. phys. 273. 4. Moriff. Umb. 9. pl. 3.
Grows on the Alps and other Mountains.

Daucus 3. Diosc. 2. Plinii Col. 109, fig.
Selinum montanum Offic. Dale Suppl. 103. 2.
Selinum five Apium peregrinum i. Clus. Park. 929. fig.
Saxifraga 3 Casalp. 315.

Visnaga minor quorundam, Selinum peregrinum Clus. semine hirsuto I. B. 3. 1. 27. p 94. fig. Chabr. 396. Ic. 2.
Clusus observed this about Salamanca in Spain, Columna in Italy, and Mr. Ray in the Hedges about Messina in Sicily.

Laserpitium fol. latoribus lobatis Moris. Umb. 29.
- - majus Almagest. Botan. 207.
Libanotis latifolia altera C. B. Phyt. 277. 3.
- - five vulgaris C. B. pin. 157. 2.
Libanotis Theophr. major Jonst. 1010. Ic. 1.
Seseli Æthiopicum Herba Dod. 313. Fig.

This Grows plentifully on the Hills about Geneva.

Astrantia Clus. 194. fig. major Moris. Umbell. 7. & 10.
- maj. coronâ florâ purpurascente Inst. Rei Herbar. 314.
Belg. 829. Obs. 388.
Astrantia nigra major Bob. H. Oxon. 279. 1.
Helleborus niger Sanicula folio major C. B. 186. 5. phyt.

340. 4.
Imperatoria nigra Tab. Hist. 300. fig. 1. Sanicula fem. Ic. 831.
- Ranunculoides Sanicula foliâ major Alm. Botan. 198.
Sanicula femina Fuchsi 670. fig.
- quibusdam aliis Elleborus niger I. B 3. 1. 34. p. 638. fig.
Veratrum nigrum Dioec. Dod. 38. fig.

I have seen the Tops of this mixt with some vulnerary Herbs from Germany.

It Grows on the Alps and the Hills about Geneva.

30. Shrub Hartwort. Ray 476. c. 5
Ad. 284. Belg. 771. fig.


Sefeli *Æthiopicum Salicis* folio C. B. 161. 7.

--- Herbariorum C. B. phyt. 286. fig.


This Grows on the Sea-Coast at *Marseilles* and about *Montpelier*.


*Heliotropium* Dod 70. fig.

*Heliotropium majus* Diosc. C. B. 253. r. phyt. 487. r.


*Heliotropium majus* fl. albo I. B. 3. l. 33. p. 60. fig.

*Heliotropium* Herba Cancri Chabr. 521. Ic. 1.


Grows wild in many Places of *France*, *Italy*, *Germany*, &c.


*Alysum Galeni* Clus. 35. fig. Hisp. 387. Dod. 88. Park. 590. f. 4.


*Alysum verticillatum*, foliis profundē incisis C. B. 232.


Dr. Salvadore hath sent me this from *Barcelona*: It grows also about *Madrid* and other parts of *Spain*.

33. Galen’s *Horehound* with more deep cut Leaves.

*Alysum Galeni* foliis altius incisis Nobis.

Like the Common, but the Leaves much deeper cut, and stand
stand on longer footstalks. Both these I have observed in Chelsea Garden.

34. Spanish Silver Horehound.
*Inst.* 192.
*Marrubium album* *Hispan. majus* *Barrel*. 263. *l. 886.
This was raised in Chelsea Garden from Seed which Monsieur Ricqueur sent me from Madrid, and the Plant is very expressive in Barrelier's Icons.

*Galeopsis Anguillara* 278. five *Pseudo-Dittamnus nigrum* *Siculum* *Boc. Mus.* 151. *Tab.* 114.

Dr. Laurence Heister Professor of Anatomy at Altorf sent me formerly a Specimen of this, amongst divers curious Plants he had gathered in the Physick-Gardens at Amsterdam and Leyden.

Dr. Herman's Figure very accurately agrees with this Plant.


*Inst.* 188.

*Pseudo-Dittamnus nigro rotundo crispis folio* *Bocc. Mus.* 152. *Tab.* 1.
This chiefly differs from the Common in having thinner and larger Flower-cups; its Leaves more pointed and somewhat denting.

37. Common Cup Horehound Ray 557. xi.
*Pseudo-Dittamnus verticillatus inodorus* C. B. 222. 2. *phyt.*
424 2.
Pseudodictamnus fol. non crenatis, verticillatus inodor
Bob. H. Ox. 379. 1.
Pseudodictamnum Cam. Epit. 474 fig. opt Dod. 281. fig.
Ger. 651. f. 2. Fonst. 795. f. 1.
Pseudodictamnum floribus verticillatis Lob. 502. Ic. 2. Belg.
592. Obs. 267. fig.
This is known from the last, in having smaller Cups, plain and rounder Leaves on very woolly Stalks.

38. Broad Phlome, Yellow or French Sage, Ray 511. 13.
Phlomis fruticosa, Salvia folio latiore & rotundiore

177.
Salvia frut. lutea, latifolia, sive Verbasceum sylv. &c. Park.
52. fig xi.
Verbasceum latis Salviae foliis C. B. 240. 1. phyt.
455. 1.
Verbasceum sylv. Matth. Clus. 28. fig. 1.
The French call this Plant, Sauge Sauvage or Wild Sage.
It Grows plentifully on Sierra morena or the black mountain supposed the Mons Marianus of the Antients, situate between Portugal and Andalusia, where the Natives call this Plant Matulera. My worthy Friend Mr. Charles du Bois tells me the Country People about Mitcham use this as a certain Remedy in the Quinsey.

Phlomis fruticosa, Salvia folio longiore & angustiore

177.
The Leaves of this are very like Common Sage, but paler above and whiter underneath, and much lesser than the last and narrower. That accurate Botanist Dr. Salvadore hath sent me this from Barcelona.

40. Aleppo Phlome.
Pseudo-Salvia Chalepensis ampliore folio cordiformi
Bobart. H. Ox. 397. 2. Sect. xi. Tab. 16. fig.

These
These Leaves differ from the Broad Phlome in being thicker, more rugged and cordated at the Footstalk: I am obliged to Mr. Jacob Bobart for the first Knowledge of this Plant, which I have since observed with Mr. Thomas Fairchild at Hoxton.

41. Samos Phlome.

Phlomis Samia Herbacea, folio Lunaria T. Coral. 10.

The Flowers pale, buff or whitish, the inside or lower Lip point or shaded with brown, the Bottoms of each Calyx are guarded with two or three long slender Thorns: its Root or lower Leaves, in Shape, resemble Garden Hospital, but are stiffer, and underneath soft and whitish.

I have as yet observed this Plant only in Chelsea Garden where it Flour'd in July.

42. True Old Time Ray 519. 3. c. 7. Lecaan 43. p. 80.

Thymus Capitatus qui Dioscoridis C. B. 219. 3. phyt. 414. 3.
Thymus legitimate Clus. 357. fig. opt.
Thymus legitimate capitatum Park. 7. fig. 1.
Thymus Creticum Jonst. 574. fig. 3. opt.

This fragrant Time of the Antients I first received from Coos; it grows also about Sevill and Cales.

43. Broad Candy Savory, Ray 519. 4.

Satureia Cretica C. B. 218. 4. phyt. 413. 4. Jonst. 576. f. 4.
Satureia Cretica Jonst. 576. f. 4. latiore folio Bob. H. Ox.

412. 6.

Thymbra legitimate Clus. 358. fig. r. opt.
Thymbra legitimate Dioscoridis Ponæ 104.
Thymbra f. Satureia Cretica legitimate Park. 5. fig. 4.
Thymum Creticum Pone verticillatum Barrel. 278. l. c. 898.
Tragoriganum Clusi Ger. 543. fig.

It's distinguish'd by its broad Time leaves and close Whorles.
Tragoriganum P. Alp. 78. fig. c. 36. Dod.
Tragoriganum Creticum C. B. 223. 4. Park. 17. fig. 1.
Tragoriganum Cretense Jonst. 668. Ic. 3.
Tragoriganum altera species Clus. 355. fig. 3.
Alpinus and Clusius his Figures, which are both Originals, very well agree with this Plant, which Jacob Bobart not long since sent me a Sample of, and has much narrower and smaller Leaves than the broad Candy Savory.

Tragoriganum Ger. 543. fig. 1.
Tragoriganum Clus. Jonst. 668. fig. 2.
Tragoriganum alterum Clus. 355. fig. 2. Hisp. 240. fig. Dod. 286.
Tragoriganum angustifolium C. B. 223. 3.
Tragoriganum Hispanicum Park. 17. f. 3.
Tragoriganum tenuioribus foliis fl. candido Chabr. 421.
Ic. 4. I. B. 3. l. 28. p. 261. fig.

Mon. Ricqueur sent me the Seed of this elegant Plant from Madrid, which Flowred with us in Chelsea Garden.

Sideritis marina Salvifolia nostra Donati 84.
Sideritis Heraclea Dioscoridis, five marina Salvifolia nostra Donato Park. 1681. fig. 16.
Betonica maritima, flore ex luteo pallecente Instit. 203.

Dr. Magnol found this on the Stoney Sea Shores in Languedoc: and Dr. Salvador hath sent it me from Barcelona: It much resembles the Sideritis glabra arvensis Chabr. 473.
Ic. 1. but has yellowish Flowers and softer Leaves. It flowers with us in June and July.

III.

N. B. The Rest will be incerted in the next Transactions.
III. Observations of the late Total Eclipse of the Sun on the 22d of April last past, made before the Royal Society at their House in Crane-Court in Fleet-street, London. By Dr. Edmund Halley, Reg. Soc. Secr. With an Account of what has been communicated from abroad concerning the same.

Though it be certain from the Principles of Astronomy, that there happens necessarily a Central Eclipse of the Sun in some part or other of the Terraqueous Globe, about Twenty Eight times in each Period of Eighteen Years; and that of these no less than Eight do pass over the Parallel of London, Three of which Eight are Total with continuance: yet, from the great Variety of the Elements whereof the Calculus of Eclipses consists, it has so happened that since the 20th of March, Anno Christi 1140, I cannot find that there has been such a thing as a Total Eclipse of the Sun seen at London, though in the mean time the Shade of the Moon has often past over other Parts of Great Britain.

The Novelty of the thing being likely to excite a general Curiosity, and having found, by comparing what had been formerly observed of Solar Eclipses, that the whole Shadow would fall upon England, I thought it a very proper Opportunity to get the Dimensions of the Shade ascertained by Observation; and accordingly I caused a small Map of England, describing the Track and Bounds thereof, to be dispersed all over the Kingdom, with a Request to the Curious to observe what they could.
could about it, but more especially to note the Time of Continuance of total Darkness, as requiring no other Instrument than a Pendulum Clock with which most Persons are furnish'd, and as being determinable with the utmost Exactness, by reason of the momentaneous Occultation and Emerision of the luminous Edge of the Sun, whose least part makes Day. Nor has this Advertisement failed of the desired Effect, for the Heavens having proved generally favourable, we have received from so many Places so good Accounts, that they fully answer all our Expectations, and are sufficient to establish several of the Elements of the Calculus of Eclipses, so as for the future we may more securely rely on our Predictions; though it must be granted, that in this our Astronomy has lost no Credit.

The Day of the Eclipse approaching, I received the Orders of the Society to provide for the Observation to be made at their House in Crane-Court, and accordingly I procured a Quadrant of near 30 Inches Radius, exceedingly well fixt with Telescope Sights, and moved with Screws so as to follow the Sun with great Nicety; as also a very good Pendulum Clock well adjusted to the mean Time, and several Telescopes to accommodate the more Observers.

In order to examine both Clock and Quadrant, I, on the 20th of April, observed the Distance of the upper Limb of the Sun from the Zenith 36°. 16', and the next Day 35°. 58'; by which it appeared that the Distances from the Zenith taken by this Quadrant ought to be increased by about one Minute: and that Allowance being made, by several Observations taken before and after Noon on the said 21st Day, the Clock was found to answer the apparent Time or Hour of the Sun with sufficient Exactness, as not going above 10" too fast. The next Day April 22°, just before the Eclipse began, we took three Distances of the Sun from the Zenith, viz. at 7h. 42'. 52". A. M. the correct
correct Distance of the Sun's Center a vertex was 62°. 1'. 40". At 7h. 45'. 48". it was 61°. 34'. 40". And again at 7h. 48'. 55". it was 61°. 6'. 40": which with the given Declination of the Sun and Latitude of the Place shew the true Times respectively to have been 7h. 42'. 38", 7h. 45'. 35". and 7h. 48'. 39": all concurring that the Clock was only 14 Seconds too fast, and had gained scarce any thing sensible in a Day's time: so that it might be entirely depended upon during the Continuance of the Eclipse.

Having computed that the Eclipse would begin at 8h. 7'. I attended soon after Eight with a very good Telescope of about Six Foot, without stirring my Eye from that part of the Sun whereat the Eclipse was to begin: and at 8h. 6'. 20". by the Clock, I began to perceive a small Depression made in the Sun's Western Limb, which immediately became more conspicuous; so that I concluded the just Beginning not to have been above five Seconds sooner; that is, exactly at 8h. 6' 00" correct Time.

From this time the Eclipse advanced, and by Nine of the Clock was about Ten Digits, when the Face and Colour of the Sky began to change from perfect serene azure blew, to a more dusky livid Colour having an eye of Purple intermixt, and grew darker and darker till the total Immersion of the Sun, which hapened at 9h. 9'. 17". by the Clock, or 9h. 9'. 3". true time. This Moment was determinable with great nicety, the Sun's light being extinguish'd at once; and yet more so was that of the Emerision, for the Sun came out in an Instant with so much Lustre that it surpriz'd the Beholders, and in a Moment restored the Day, viz. at 9h. 12'. 26". true time, after he had been totally obscured for 3', 23" of Time. And as near as I could estimate the Points on the Moon's Limb; where the last Particle of the Sun vanished was about the middle of the South East Quadrant of her Limb, or about 45 Degrees from her Nadir to the Left-Hand: And the first Emersi-
Emersion was about Ten Degrees below the Horizontal Line through the Moon’s Center on the West side; and at 14 Minutes past Nine, correct Time, I judged the Horns of the Eclipse to have been exactly perpendicular, and by consequence, the Centers of the Sun and Moon to be in equal Altitude.

It was universally remarked, that when the last part of the Sun remained on his East side, it grew very faint, and was easily supportable to the naked Eye, even through the Telescope, for above a Minute of Time before the total Darkness; whereas on the contrary, my Eye could not endure the Splendour of the emerging Beams in the Telescope from the first Moment. To this perhaps two Causes concurred; the one, that the Pupil of the Eye did necessarily dilate itself during the Darkness, which before had been much contracted by looking on the Sun. The other, that the Eastern parts of the Moon, having been heated with a Day near as long as Thirty of ours, could not fail of having that part of its Atmosphere replete with Vapours, raised by the so long continued action of the Sun; and by consequence it was more dense near the Moon’s Surface, and more capable of obstructing the Lustre of the Sun’s Beams. Whereas at the same time the Western Edge of the Moon had suffered as long a Night, during which there might fall in Dews all the Vapours that were raised in the preceding long Day; and for that reason, that part of its Atmosphere might be seen much more pure and transparent. But from whatever cause it proceeded, the thing itself was very manifest and noted by every one.

About two Minutes before the Total Immersion, the remaining part of the Sun was reduced to a very fine Horn, whose Extremities seemed to lose their Acuteness, and to become round like Stars. And for the Space of about a Quarter of a Minute, a small Piece of the Southern Horn of
of the Eclipse seemed to be cut off from the rest by a good interval, and appeared like an oblong Star rounded at both Ends, in this Form : which Appearance could proceed from no other Cause but the Inequalities of the Moon's Surface, there being some elevated parts thereof near the Moon's Southern Pole, by whose Interposition part of that exceedingly fine Filament of Light was intercepted.

A few Seconds before the Sun was all hid, there discovered itself round the Moon a luminous Ring, about a Digit or perhaps a tenth Part of the Moons Diameter in Breadth. It was of a pale whiteness or rather Pearl colour, seeming to me a little tinged with the Colours of the Iris, and to be concentrick with the Moon, whence I concluded it the Moon's Atmosphere. But the great height thereof far exceeding that of our Earth's Atmosphere; and the Observations of some, who found the Breadth of the Ring to encrease on the West Side of the Moon, as the Emission approached; together with the contrary Sentiments of those whose Judgment I shall always revere, makes me less confident, especially in a Matter whereunto, I must confess, I gave not all the Attention requisite.

Whatever it was, this Ring appeared much brighter and whiter near the Body of the Moon than at a Distance from it; and its outward Circumference, which was ill defined, seemed terminated only by the extream Rarity of the Matter it was composed of; and in all Respect it resembled the Appearance of an enlightned Atmosphere viewed from far: but whether it belonged to the Sun or Moon I shall not at present undertake to decide.

During the whole time of the Total Eclipse I kept my Telescope constantly fixt on the Moon, in order to observe what might occur in this uncommon Appearance: and I found that there were perpetual Flashes or Coruscations of Light, which seemed for a Moment to dart out
from behind the Moon, now here, now there, on all Sides; but more especially on the Western Side a little before the Emersion: And about two or three Seconds before it; on the same Western Side where the Sun was just coming out, a long and very narrow Streak of a dusky but strong Red Light seemed to colour the dark Edge of the Moon; tho' nothing like it had been seen immediately after the Immersion. But this instantly vanished upon the first Appearance of the Sun, as did also the aforesaid luminous Ring.

As to the Degree of Darkness, it was such that one might have expected to have seen many more Stars than I find were seen at London: The three Planets, Jupiter, Mercury and Venus were all that were seen by the Gentlemen of the Society from the Top of their House, where they had a free Horizon; and I do not hear that any one in Town saw more than Capella and Aldebaran of the Fixed Stars. Nor was the Light of the Ring round the Moon capable of effacing the Luster of the Stars, for it was vastly inferior to that of the full Moon, and so weak that I did not observe that it cast a Shade. But the under Parts of the Hemisphere, especially in the South East, under the Sun, had a crepuscular brightness: and all round us, so much of the Segment of our Atmosphere as was above the Horizon and was without the Cone of the Moon's Shadow, was more or less enlightened by the Sun's Beams: and its Reflection gave a diffused Light which made the Air seem hazey, and hindered the Appearance of the Stars. And that this was the real Cause thereof, appears by the Darkness being more perfect in those Places near which the Center of the Shade past, where many more Stars were seen, and in some not less than Twenty; though the Light of the Ring was to all alike.

During the Time whilst the Sun recovered his Light, several Altitudes were taken to examine the Regularity of the
the Clock's Motion; and though the Sun now rose much
flower than at the beginning, yet they all conspired within a
very few Seconds that the Clock went still one Quarter of
a Minute too fast. And the End of the Eclipse approaching, I attended the Moment thereof with all the Care I
could, and concluded the compleat Separation of the
Sun and Moon at 10h. 20'. 15". by the Clock, or exactly
10h. 20'. correct time.

Hitherto I exhibit only what myself saw, but there were
with us a great many of the Members of the Society; and
the Right Honourable the Earl of Abingdon and the Lord
Chief Justice Parker were of the Number: the latter of
which shewed an uncommon Curiosity and Desire of Ex-
actness, his Lordship doing us the Honour to assist at most
of the Observations made for determining the Error of
the Clock; and did himself, at the Moment of the Emer-
sion from total Darkness, observe the Distance of the Pla-
net Jupiter from the Zenith 48°. 29'. by which the Time
thereof is verified.

There were also present several foreign Gentlemen, and
among them Monsieur le Chevalier de Louville and
Mr. Monmort, both of them Members of the Royal Acade-
my of Sciences at Paris: the first whereof came purposely
to observe this Eclipse with us, and having seen the Be-

inning applied himself to take Digits with his Microme-
ter, and to observe the Occultations of three Spots at that
time seen in the Sun; and he was pleased to communicate
the following Notes, viz.

At 8 28 20 Four Digits were Eclipsed.
8 32 57 The First and bigger Spot touched the
Moon.
8 33 18 The same was wholly hid.
8 34 08 The first of the two lesser Spots was hid.
8 34 58 The Second of them was hid.

R r 2

At
At 9. 36. 01 Emersion of the greater Spot.
9. 38. 26 Emersion of the first lesser Spot.
9. 40. 25 Emersion of the second lesser Spot.
10. 20. 04 The End of the Eclipse.

And he determined the time of the total Darkness 3′.22″, or one Second less than by my Account.

The Heavens were all the while very propitious to us, and there was very little or no Wind, and not so much as one Cloud interrupted our View from the Beginning to the End; but no sooner was the Eclipse over, but a great Body of Clouds hid the Sun for many Hours after.

These Observations having been made with all the Care we could, are not, ’tis hoped, far from the Truth.

What we have received from other Places is as follows,

The Reverend Mr. James Pound Rector of Wansted in Essex and R. S. S. gives the following Account of the principal Phenomena observed there; he being furnish’d with very curious Instruments, and well skill’d in the Matter of Observation, and having rectified his Clock by several Altitudes of the Sun taken both before and after, viz.

At 8. 6. 37 The Eclipse first perceived.
9. 9. 28 The Total Immersion.
9. 12. 48 The Emersion.
10. 20. 32 The just End of the Eclipse.
6. 3. 20 The Continuance of total Darkness.

The near Agreement of this Observation with our own (the Difference being only what is due to the Difference of our Meridians) makes us the less solicitous for what was noted at the Royal Observatory at Greenwich, from whence we can only learn that the Duration of Total Darkness was 3′. 11″.

The
The Reverend Mr. William Derham Rector of Upminster in Essex and Reg. Soc. Sed. assisted by Samuel Molyneux Esq. Secretary to his Royal Highness the Prince, and other Persons of Quality, made the following Observations there, which he has lately communicated, viz.

At 8. 7. 41 The Eclipse began.
8. 33. 46 The Moon touched the greater Spot.
8. 34. 36 She touched the middle Spot.
8. 35. 41 She touched the third Spot.
9. 10. 58 The total Darkness began on a sudden, and Aldebaran appeared.
9. 14. 6 The Emerson or End of total Darkness.
9. 42. 41 The third and last Spot discovered.
10. 21. 45 The End of the Eclipse, by a 13 1/2 Foot Glass.

And a little before the Beginning of the Eclipse, he found the greater and preceding Spot to be more Northerly than the Sun's Center 373; such Parts as the Sun's Diameter was 1647, and that it followed his Western Limb 0. 43" of Time: by which data the Situation of that Spot is well determined.

Our Professors of Astronomy in both Universities were not so fortunate. My worthy Colleague Dr. John Keill by reason of Clouds saw nothing distinctly at Oxford but the End, which he observed at 10h. 15'. 10". As to the total Darkness, he could only estimate it by the sudden Change of the Light of the Sky; and reckoned its Continuance but 3'. 30"; which was certainly too little, the Center of the Shadow having without doubt past very near Oxford. And the Reverend Mr. Roger Cotes at Cambridge had the misfortune to be oppressed by too much Company, so that, though the Heavens were very favourable, yet he missed both the time of the Beginning of the Eclipse and that of total.
total Darkness. But he observed the Occultations of the
three Spots, viz, of the first and greatest at 8h. 34'. 11". of
the second at 8h. 35'. 15", and of the last at 8h. 36'. 55".
He noted also the End of total Darkness at 9h. 14'. 37"
and the exact End of the Eclipse at 10h. 21'. 57".

We have received several Accounts from some Places
which lay near the Track of the Center of the Shade,
and which might have been very proper to determine the
greatest Continuance of the Darkness; as from Plymouth,
Exeter, Weymouth, Daventry, Northampton and Lynn regis,
all agreeing that the whole Sun was obscured at those Pla-
ces full four Minutes, and at some of them rather more.
But these Observers give us no Account how they measured
this Time, and therefore it may well be supposed they
took it in a round Number, and perhaps from pocket Mi-
minute-Watches. What I think may best be relied on for this
Purpose, are two corresponding Observations made, the
one at Barton near Kettering in Northamptonshire, where
by the Observation of John Bridges Esq; Treasurer of his
Majesty's Revenue of Excise, and R. S. S. with a good
Pendulum-Clock and all due Care, the whole Sun was hid
no more than 3'. 53". The other was by Mr. John Whit-
side, A. M. Keeper of the Ashmolean Museum at Oxford, and
a skilful Mathematician, who observed after the same man-
ner, at King's-Walden in Hertfordshire near Hitchin, that
the total Eclipse continued but 3'. 52". Hence it follows
that the Center of the Shade past near the middle between
these two Places, which are but 30 Geographical Miles
afunder, and situate near at right Angles to the Way of
the Shade, and therefore that the total Obscurity, where
longest, could last but about 3'. 57", or perhaps a second or
two more at Lynn and less at Plymouth: the Velocity of the
Progress of the Shade gradually decreasing, and its Dia-
meter encreasing as it past on to the Eastwards. And this
Situation of the middle Line is confirmed by an Observa-
tion made at the Seat of the Right Honourable the Lord
Foley at Witley eight Miles beyond Worcester, by his Order,
and communicated by his Lordship to the Royal Society;
whereby it appears that the total Darkness lasted there
3. 15". Hence it follows that Witley was about three or
4 Miles farther from the Center of the Shade on the North-
side than London on the South; and Witley being by Ogilby's
Menfurations, 118 measured Miles from London, it is plain
that the Center past over Ipsip, which is, by the same Ad-
measurement, 57 such Miles on that Road, and about five
Miles almost due North from Oxford; so that the Center
of the Shade left Oxford but very little upon the right Hand.
This Situation agrees perfectly well with the former be-
tween Barton and King's Walden, and as far as the Geogra-
phy of our Country may be relied on, I conclude the
Center to have entred upon England about Plymouth, and
to have past over Exeter, the Devizes, Ipsip, Buckingham
and Huntingdon, leaving Oxford and Bedford on the Right,
and Lynn on the Left, and to have quitted the Coast of
Norfolk about Wells and Blakeney.

As for the Limits of the Shade, both on the North and
South side, we have by Enquiry gotten them with all the
Exactness the thing is capable of, and we should have been
glad the French Astronomers had done the like for the To-
tal Eclipse that past over Languedoc, Provence and Dauph-i-
ny on the First of May 1706. But as this is the first Eclipse
of this kind that has been observed with the Attention
the Dignity of the Phenomenon requires, we hope those
which may happen for the future to traverse Europe, may
not pass by so little regarded as hitherto.

As to the Southern Limit or Term where the Eclipse
ceased to be Total on the South side of the Sun, we have
received an Account of an Observation made at Norton-
court about Ten Miles on this side Canterbury, by the Revere-
dend Dr. John Harris, S. T. P. Prebendary of Rochester
and
and R. S. S. assisted by that accurate Observer Mr. Stephen Gray; by which we learn that the Eclipse began there at 8h. 8'. 55". and ended at 10h. 24'. 47"; and that the Total Darkness continued but about one Minute or rather less, the middle thereof being at 9h. 13'. 52". From this Duration it will follow that Norton-court was but about 3 or 4 Miles within the Shade. And that it was really so is confirmed by the Relation of the Inhabitants of Boston, about Midway between Norton-court and Canterbury, who assured Mr. Gray, as he was returning home that same Day, that the Eclipse was not Total there, but, as one of them expressed it, before the Sun had quite lost his Light on the East-side he recovered it on the West: and that there was a small Light left on the lower part of the Sun that appeared like a Star. And from Cranbrook in Kent, we are informed, by the Relation of the curious William Tempest Esq; R. S. S. that he observed there the Sun to be extinguished but for a Moment, and instantly to emerge again: So that the Limit past exactly over this Town, which is about 38 Geographical Miles from London, and very near the right Angle where the Perpendicular from London falls on the Line of the Limit, being 3'. 00 of Time to the Eastwards of London in the Latitude of 51°. 6', as near as I can gather.

How it past over Sussex we have not so authentick Relations, but have learnt that it was Total at Wadhurst beyond Tunbridge-wells, as also for some short time at Lewis; but that it was not so at Brighton, which Place being situated on an Eminence that has a commanding Prospect, all the Country to the Northward was seen in Darkness, whilst they there had some Benefit of a small Remainder of the Sun.

From these Observations we may conclude that this Limit came upon the Coast of England, about the middle between Newhaven and Brighton in Sussex, and passing by
by Cranbrook and Boston, left Canterbury about 4 Miles on
the Right hand, and quitted the Coast of Kent, not far
from Hern toward the antient Regulbium, now called Reculver. So that it seems scarce one third part of Kent, and
not so much of Sussex, out of all the South Coast of Great-
Britain, escaped being involved in this Darkness.

The Northern Limit, having past over a much greater
Space, has had more Observers, and is not less curiously
determined than the other. We find by the Account given
by the Reverend Mr. Roger Proster, Rector of Haverford-
Weft, that the Eclipse was total there a Minute and half,
whence it follows that Haverford was but about 6 Miles
within the Limit; and therefore that it entred on Pembrok-
shire about the middle of St. Brides Bay, leaving St. David’s
and Cardigan on the left Hand: and having traversed those
two Counties and Montgomery-shire, it entred on Shropshire,
leaving the Town of Shrewsbury 1° 40′ in the Shadow, as
was observed there by Dr. Hollings: whereby it appears
that Shrewsbury was about 8 Miles within the Limit. Thence
it proceeded by the East-side of Cheshire, leaving Whitchurch
and Nantwich a very little without; and passing by Congleton
went over the Peak of Darbyshire into Yorkshire, and crost
the great Northern Road between Pontefract and Doncaster,
somewhat nearer the former than the latter. For by the
Observations of that curious Gentleman Theophilus Shelton Esq; at Darrington about two Miles on this side
Pontefract, (in Lat. 53° 40′ and Long. West from London
4° 40′ of time, as may be concluded from Norwood’s Measure of a Degree) the Sun at 9 h. 11′ was reduced al-
most to a Point, which both in Colour and Size resembled
the Planet Mars; but whilst he watched for the Total
Eclipse, that Point grew bigger and the Darkness dimi-
nished; whence he argued the Limit to have been very lit-
tle more Southerly. And since he has been informed that it
was just Total in Barnsdale, three Miles South from thence.

And
And that it was so at Badsworth about the same Distance from Arrington, we are told by a Letter of the Reverend and Learned Mr. Daubuz, that he has a certain Account from that Place, that the luminous Ring round the Moon was seen there, which was no where visible but while the Eclipse was Total. From these Data we may securely determine the Remainder of this Track, and that the Edge of the Shadow having past over the rest of Yorkshire, went off to Sea about Flamborough head.

So that of the forty Counties into which England is subdivided, only the five most Northerly have not had the Sun wholly hid from them; and six others have escaped but in part, viz. Shropshire, Chester and Yorkshire, and the extream part of Derbyshire on the North, and Kent and Sussex on the South; all the rest of the Kingdom having more or less suffered an Interval of Total Darkness.

I shall not at present consider this Eclipse as universal, but only as it related to England; and it shall suffice to say, that the Shadow came out of the Atlantic Ocean, having past over the Islands Azores; and that the Southern Limit thereof reach’d the Isle of Ulhants, and the Northwest Coasts of Brittany between Brest and Morlaix; and dividing our Islands of Guernsey and Jersey, just touched upon the Promontory of Normandy called Cape de Hague. And that after it had quitted England and traversed the German Ocean, it fell on Jutland on the Southside, and Norway on the North; and thence proceeded to the Eastwards over Sweden, Finland, &c.

It remains now to consider the Figure, Position, Direction, Velocity and Magnitude of the Shadow at is past over us. And first as to the Figure, ’tis obvious that the Shadow of the Moon being a Cone and the Earth’s Surface sufficiently Spherical, the apparent Shadow on the Earth will be the common Interseccion of a Cone and Sphere, which is a Figure hitherto little considered by Geometers; and
and not being in Plano is not to be exactly described but in the Spherical or Conical Surface. How to find the Points of this Curve in all Cases is taught by P. Courfier, in a very scarce Latin Book printed at Dijon in Burgundy, and published at Paris in the Year 1663: nor do I hear of any other Author that has handled the same Subject since, though capable and worthy of further Improvement. By what he there delivers, Prop. II. 12. Lib. I. it will be easily understood, that the Convexity of so small a part of the Earth's Surface as the Shadow commonly occupies, can produce but an inconsiderable Effect; so that without sensible Error we may take it for a Plain, and the Section for a true Apollonian Ellipsis, whose transverse Axis, by reason of the smallness of the Angle of the Cone, will be to its Conjugate nearly as Radius to the Sine of the Sun's Altitude at its Center, especially if he be considerably elevated. But when he is near the Horizon, it will be necessary to have regard to the true Figure, by reason of the great Length to which the Transverse Axe is extended, and particularly when the Shade is entering upon or leaving the Earth's Disk: Of these perhaps a fuller Account may be given upon a further occasion.

As to the Position of the Axis of the Shadow, it is manifest that it must always lie in the Plane of a great Circle of the Earth passing through the Axis of the Cone of the Shade; and therefore it will be only requisite to obtain the Azimuth and Altitude of the Sun at the Place where the Center of the Shade at any time is found, to determine the Situation of the Axe and Species of the Ellipse required. Thus the middle of the Eclipse at London having been observed at 9h. 10' 45", by the given Latitude and Declination we find his Azimuth about 59° 00' and Altitude 40° 46'; that is just 40 Degrees high at the Center of the Shadow. Wherefore the Transverse Axe of the Ellipse was to its Conjugate very near as Rad. to the Sine of 40°,
or as 1000 to 643 proxime; and did make an Angle of 59°, or very little more, with the Meridian passing at that time through the Center of the Shade.

Next the Direction and the Velocity of the Motion, wherewith the Center of the Shade past over England comes to be considered, wherein the Reader is to be told that the Shadow passes in a very compound Curve, which, as the former is not in plano, and only describable on the Surface of the Sphere: nor is its Motion equable, but compounded of very many Elements producing a great Variety. By what Method its Points, and its Tangents in those Points, are to be obtained, I refer to the next Opportunity; this Account being designed for the Curious in general: only I must acquaint them, that for so small a part of the Curve as went over England, it may be esteemed a right Line, with more Exactness than we usually find in most of our Geographical Charts. And the like may be said for the Velocity, which, though in our present Instance it was continually decreasing, may, for so short a time, be supposed to have been the same, without sensible Error.

By a careful Calculation I have determined the Velocity of the Motion, at the Time of the Middle of the Eclipse at London, to have been 29 Geographical Miles in a Minute of Time quam proxime: and that its Way made an Angle of 50°. 45’ with the Meridian towards the Eastwards of the North; wherefore the said Way made an Angle with the Axis of the Ellipsis of 68°. 15’. And the greatest Duration of Total Darkness having been 3°. 57”, (as was before shewn) it will follow, that that Diameter of the Elliptick Figure according to which the Shade past, was no less than 114 1/4 Geogr. Miles: And from the Elements of the Conicks ’tis easy to be proved, that supposing the Figure of the Shade a true Ellipse, whose Axes are as Radius to the Sine of 40 Degrees, the greater Axis would be 171 Geographical Miles, and the lesser 110; and the nearest distance between the Limits supposed Parallel 164 such Miles.
And this Length of the Axis of the Shade, derived purely from the Continuance of Total Darkness, is fully confirmed by the observed Distance of the Parallel Limits; the one passing by Badsworth in Yorkshire, the other by Cranbrook in Kent. For by the two Latitudes 53° 37' and 51° 6', with the Difference of Longitude 7' and 40'' of Time, or 1° 55', the Distance of these two Places is given 166 1/2 Geogr. Miles; with the mean Angle of Position 25 Degrees from the North Westwards; wherefore this Arch makes an Angle with the Track of the Shade of 77° 1/2: and hence the nearest Distance of the Parallels becomes 163 such Miles, which by the other Way was found 164.

If therefore we conclude the Axis of the Shadow, when the Sun was just 40 Degrees high, to have extended over 2° 50' of a great Circle, we may securely determine the Difference of the Sun and Moon's Diameters at this time. For the Difference of the Horizontal Parallaxes of the Sun and Moon being found to be 60° 38'' (as shall be hereafter shewn, but is not required with extream exactness for this Purpose) the Difference of the Parallaxes in Altitude at both Ends of the Axis, will be found to be 1° 56''; and by so much did the Diameter of the Moon when forty Degrees high exceed that of the Sun: Hence the Horizontal Diameter of the Moon in this Anomaly is found 33'-27'', which may serve for a Rule in all other Cases.

I forbear to mention the Chill and Damp which attended the Darkness of this Eclipse; of which most Spectators were sensible, and equally Judges. Nor shall I trouble you with the Concern that appear'd in all Sorts of Animals, Birds, Beasts and Fishes upon the Extinction of the Sun, since ourselves could not behold it without some sense of Horror.

Lastly, I have added the following Synopsis of such Observations as have hitherto come to my Hands: acknowledging the Favour of all those, who have been willing to promote our Endeavours to perfect the Doctrine of Eclipses.
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IV.

The Author of this Treatise, whose admirable Skill in the Practice of Dissections, as well as in the Theory of the Structure of the Parts, leaves him not many Equals, in order to discover what Improvements and Progress Anatomy has met with, and with what Industry the Study of it has been cultivated, has with much Application perused a very great Number of Authors who have advanced the Science; observing therein who have the Honour of being the first Discoverers, and who have unjustly arrogated to themselves that Title, that each may receive a due Proportion of Praise according to his Merit. And in this Decision, he has impartially weighed their Deserts, the better to lay before the Reader the Increase of these Studies, and to determine more exactly the Differences that have arisen about who are first Inventors; which the Book, Chapter and Page where they are treated of will easily manifest.

The History, Lives and Elogies ascrib'd to Anatomists, which he has inserted either from their own Writings, or their Editors, or Commentators, will afford a great Variety of Pleasure, in which he has been particularly care-

Tt2
ful to set down the Names, Sur-Names, Country, Time of their Birth, what Year they died in, under what Masters educated, where they flourish'd, and in what part of Anatomy they excell'd.

Nor has he been less diligent in the Account he has given of the Books of Anatomy, which his Friends supply'd him with in great Number. The Reader will see here laid before him, all the several Editions, in what Language, what Volume they were printed in, with the Place and date of the Year they were published at; and which are the first Impressions, and which copied from them. Nor has he judged it improper to give some Account of the Figures dispers'd up and down in Anatomical Books; as whether they were Originals or Copies, cut in Wood or Copper, &c. To these he has added three Indexes, whose Use will be seen by the Titles. As for the difference of Style remarkable in this Treatise, it is chiefly owing to the Variety of Authors made use of, he thinking himself not at Liberty to vary the Expression of them whose Authority he quotes.

He says he would willingly have recounted the great Advantage Anatomy has received from the English Nation: but out of just Regard to their Merits, he has resign'd this Province to his Friend Mr. William Becket; whose Industry in collecting their Writings will not in his Opinion exceed his Talent and Abilities to recommend them to the World.

He hopes the Reader will pardon him in this, that as several Books and Editions came late to his Hands, he was forc'd to add the omissa separately: which being in greater Number than at first expected, the Author earnestly desires the Favour of those who have in their Collections any thing of this kind here omitted, that they would please to communicate the same, in order to render this first Specimen still more complete.

FINIS.
PHILOSOPHICAL
TRANSACTIONS.

For the Months of June, July and August, 1715.

The CONTENTS.


II. Botanicum Hortense IV. Giving an Account of divers Rare Plants, Observed the last Summer A. D. 1714, in several Curious Gardens about London, &c. By James Petiver, F. R. S.

III. OBSERVATIONES COELESTES BRITANNICÆ, Grenovici in Observatorio Regio habita, anno MDCCXIII.

IV. An Account of an Experiment made by Dr. Brook Taylor assisted by Mr. Hawkesbee, in order to discover the Law of the Magnetic Attrition.

V. A short Account of the Cause of the Saltness of the the Ocean, and of the several Lakes that emit no Rivers; with a Proposal, by help thereof, to discover the Age of the World: Produced before the Royal Society by Edmund Halley, R. S. Secr.

Accounts of BOOKS. I. Linear Perspective, or a New Method of representing justly all manner of Objects, &c. By Brook Taylor, L. L. D. and R. S. Secr.

II. DUCATUS LEODIENSIS, Or, The Topography of the ancient Town and Parish of LEEDS and Parts adjacent, in the County of YORK, &c. By Ralph Thoresby, Esq; Fellow of the Royal Society, London.

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D. 14. Sept. h. 12. intra ipsum Venam Chalybis Sarunetanam, 300. incircum passus ab ostio, coelo foris sereno. Barometri totius altitude 24. 4. & 24. 3. 8. 3. 18. 6. 7. 25. 3. 27. 3. 29. 0. 30. 5. 31. 6. 21. 3.

Extra hanc Venam metallicam sub dico eandem altitudinem observavi Mercurii in Barometro integro, item in 3 & 9 digg. Aereis in tubo reliqui. Sed notandum est aerem in intimis fodicibus partibus, ubi experimenta feci, fuiffe ob ignem præterito die accendum (quo venam durissimam coquent foles) raresfactum, & locum hypocauti inflar moderatè calefactum.

N. B. Multis experimentis coram R. Societate factis, comper-tum est, Aereis compressis Elasticae esse ut pondera compro-mentia dierett. His C. Scheuchzeri observatis patet eandem in Aere rarifacto obtenere regulam quam proxime; Nam licet differentia aliqua reperiri, tanta non est, ut ab inequalitate diametri Tubi non facile oriatur. Ut autem experimenta hact-rite sint, operetib Tubi capacitatem, immisso uncitam Mer-curio, in æquales partes dividi, loco partium longitudine æqualium.
Botanicum Hortense IV.

Giving an Account of divers Rare Plants, Observed the last Summer A. D. 1714. in several Curious Gardens about London and particularly the Society of Apothecaries Physick-Garden at Chelsea.

By James Petiver, F. R. S.

Continued from the last Transactions.

47. Planta Nasiiflorae. Snout-Flowers.
   Cymbalaria Hedera terrestris folio, flore maximo.
   Batav.
   Asarina Lob. Lugd. 915. fig. 2.
   Asarina Lob. flore Hederae terrestris i. B. 3. App. 840. fig.
   Asarina sterilis, Sevena Narbonensis agri Lob. Obs. 329. lc.
   Asarina flava saxatilis Hederula Lob. Icon. 601.
   Heder saxatilis Jonst. 856. fig. 2.
   Heder terrestris, magno flore C. B. 306. Phyt. 609. 3.

   Its Flowers, Seeds and Capsules plainly shew it to be a kind of Snapdragon. Lobel says this grows wild in Narbone and Provence (from whence I should be glad to see it,) but his Figure does not express the Hoariness with which our Leaves are endowed, nor are its Flowers so large or exact. I first saw this elegant Plant in the

X x

Amster
Amsterdam Physick-Garden, flowering in July, and since at Chelsea.

48. English Yellow Pansey. Ray's British Herbal Tab. 37. fig. 10.

Viola montana lutea grandiflora nostra Ray 1051. 5. Syn. ed. 2. 15. 5. 8.


Mr. Ray observed this in divers boggy mountainous Pastures in Derby-shire, Yorkshire, and Wales. It flowers very well in our Gardens, and makes a beautiful edging.

49. Bog Violet. Rays British Herball Tab. 37. Fig. 5.

Viola palustris rotundifolia Plot, Hist. Oxon. 144. 4. Tab. 9. fig. 2. Ray 1050. 4. Synopf. 152. 4. edit. 2. p. 214. 4.

Viola palustris rotundifolia glabra Moris. Oxon. 475. 5.

Dr. Plot first discovered this in the Bogs about Stowe-wood in Oxfordshire, and Chiswell in Berkshire, since which it hath been found in the like Places between Wickham and Croydon in Surry plentifully, from whence it was transplanted into Chelsea Garden. It flowers in April and May.

50. Hairy Wood Violet. Rays British Herball Tab. 37. fig. 7.

Viola Trachelii folio Morret. pin. 125.


Viola Martia major, hirsuta inodora Moris. Oxon. 475. 4. S. Tab. 35. fig.

Grows plentifully in Charlton and other Woods in Kent, &c. It flowers in March and April.

51. Upright Tree-Violet. Ray 1052. 4.

Viola arboreescens vel potius erecta Camer. Epit. 511. fig. opt.

Viola aequalis tricolor Ger. 703. Ic. 2. Jonst. 854. fig.

Viola elatior Clus. 309.

Viola Martia arboreescens purpurea C. B. 199. 2.

Viola Martia surrectis caulisculis L. Ic. 610. 2.
Jacea tricolor, fursectis caulibus, quibusdam arborea dicit
Chabr. 510. Ic. 3. I. B. 3. l. 32. p. 547. fig.

Clusius observed this to grow common in the Woods of
Austria and Styria. Dr. Morison in his Hist. Oxon. p.
476. says this is found in England on the Tops of Moun-
tains and in Woods, which has not as yet been observed,
either by the inquisitive Mr. Ray or any other.

52. Long Horse-tongue Ray 663. r.
Bonifacia five Bilingua Chabr. 45. Ic. 5. & 6. I. B. 1. l. 5.
p. 575. fig.
Hippoglossum Camer. Epit. 919. fig.
Hippoglossum Dioscoridis & Laurotaxa Plinii Col. 165.
fig. opt.
Hippoglossum five Bilingua Barrel. pl. 5. Ic. 249. Park.
702. Ic.
lc. 1. & 2.
Radix Idea, Hippoglossum Col. phyt. 64. fig.
Ruscus angustifolius, fructu folio innscente El. Bot. 70.
Inst. 79.

This and the next are accurately Figured by Columna
& Barrellier. The peculiarity of this Plant is to have a
small tongue like the Key or Fruit of the Ash-tree, growing
from the middle Rib on the under side of each Leaf.

Grows on shady Mountains in Italy and Hungary.
53. Round Horse-tongue Ray 663. 2.
B. r. l. 5. p. 574. fig.
Laurus Alexandrina & Chamadaphne Col. 165. fig. opt.
Laurus Alexandrina genuina Park. 700. fig. 1.
250.
Laurus Alex. fructu folio innsidente C B. 305.

Ruscus
Ranunculus montis subhirsutus, Gerani folio C.B. 182. 13.
Ranunculus montanus subhirsutus, Gerani folio C.B. pr. 96.
6. fig.
Ranunculus mont latifol. hirsutus alter C.B. phyt. 323. 15.
C.B. in his Phytinax lays he first found this on Mount Mutet near Basel, and afterwards on Rosberg near Tubing in Wirtenbergh. I received the first Specimen of this from Mr. Jacob Bobart at Oxford.

Herbæ Tetrapetale. Quaterpetals.
60. Babylonion Cress, Ray 821. 1.
Draba prima umbellata C.B. phyt. 174. 1.
Draba vulgaris Park. 849. fig. 1.
Draba Dioscoridis Jonst. 274. fig. 1.
Draba five Arabis Chabr. 295. le. 6.
Draba multis fl. albo. 1 B. 2. 1. 22. p. 939. fig.
Draba umbellata vel Draba maj. capitulis donata C.B. 109. 1.
Arabis five Draba & Nafturtium Babylonicum Lob. le. 224.
1 Belg. 267: fig Obs. 111.
Growing in the Borders of Fields about Vienna plentifully, as also in the like Places in Italy and France. Flowing in May and June.

Eruca Monspeliaca filiqua quadrangulâ. Park. 82. 1. fig.
149. 15. prodr. 41. 7. fig. opt.
Raphanistrum disaperon Monspel. fil. quadrangulâ echinatâ H. Leyd. 529.
Sinapi echinatum Lugd. 647. fig. Chabr. 275. le. 3. I. B. 2.
1. 21. p. 858. 8. fig.
Growing about Montpelier in Corn. It Flowers and Seeds in June, &c.

Thlaspi fruticofum Lencovii folio latifol. C.B. 108. 1. prodr.
49. 9. Park.
Thlaspi frut. umbellatum Persicum, fol. Leucotii in far sem-
pervirentibus H. Oxon 296. 23.
Thlaspi latifol. platycarpon, Leucotii folii Bock. 55, fig.
optr.
Thlaspi sempervirens & florens Dodart. 115, fig.
Thlaspidium frutic. Leucotii folio sempervirens E. B. 1
Inst. 214.
Thlaspio fruticoso di Persia, con foglia di Keiri, di fiore odo-
rato Zanon 106. Tab. 74. fig.
This is always green, Flowers long and especially to-
wards the Winter, its said to come from Persia, Boccone
found it wild about Palermo in Sicily.
Scorodo-thlaspi Ulissis aldrovandi Chabr. 294. t. I. B. 2. 1.
22. p. 932. fig.
Thlaspi Allium redolens H. Oxon. 297. 28. Tab. 18. fig.
That very great and laborious Naturalift Ulisses Aldro-
bandus first discovered this Plant growing about Bononia.
It Flowers and Seeds in Chelsea Garden early in the Spring:

Plantae A. pentapetalato. Cinquepetals.
64. Dwarf Gentian with a large Flower Ray 7 18. 2.
Gentiana Apina magno flore I B. 3. l. 31. p. 523. fig.
96.
Gentiana Alp. magno fl. caeruleo violaceo Mariana Chabr.
503. Ic. 6.
Gentiana 5. Gentianella major verna Clus. 314. fig.
Gentianella Alpina verna major Park. 402. fig. 1.
Gentianella Alp. latifol. magno flore C B. 187. 1. phyt 347.
16 prod. 97. c. x. 1.
Gentianella Helvetica Lob. Icon. 310. 1. Belg. 381.
Gentianella Campanula flore purp. Helvetica Adv. 130. fig.
I B. makes this and the Angustifolia to be the same
Plant.

Grows
Grows on most Mountains in Austria, Styria and Switzerland, where it Flowers in their Spring, viz. in June, or as soon as the Snow there melts.

This elegant Plant is pretty common in most of our Country Gardens.

Cervicaria Valerianoides caerulea C B 95. 20.
Rapunculus Valerianoides caeruleus umbellatus Boer. 104. 3.
Rapuntium umbellatum Col. phyt. 2. p. 22.
Trachelio azurum umbiliferum Pone Ital. 44. fig.
Trachelium umbiliferum caeruleum Park. 645. 8.

This is very different from the Telephium fl. purpureus
Lobel. Ic. 389. 2. as Columna and C B. from him take to be the same.

Pona; in his Italian Edition of the Plants on Mount Baldus, gives the first Figure and Description of this elegant Plant, which he received from Signior Contareni at Venice, and supposes it to come from Candy; but Pere Barrelier says it grows in moist shady Places not only about Rome but Valantia in Spain, where its also sometimes found with a white Flower. Its blew umbelliferous Tufts make a beautiful Shew in our Physick Garden where it Flowers from Midsummer till the end of September.

Caryophyllus sylv. prolifer C B. 209. 6. phyt. 393. 18.
Caryophylus prolifer Park. 1338. fig. 1.
L. 29. p. 335. fig.
fig. 5.

Grows in barren Pastures in many Parts of France, Italy, Germany, &c.
67. Gray Stitchwort Ray 1020. 7. H. Ox. 546. 44.
Holosteurum Hort. incanum vulgar Nobis.
Caryophyllus Holosteurum Ger. 277. xi. Jonst. 595. fig. 15.
Caryophyllus Holosteurum tomentosus I B. 3. 1. 29. p. 360. fig.
Park. 1339. fig. 7.
Caryophyllus Holost. tomentosus latifolius CB. 210. 2.
pr. 104. 5.
Lychnis maritima incana & tomentosa H. Oxon. 546. 44.
Myosotis tomentosa Linaria folio amplo. E B. 211. Inst. 244.

The Description of I B. and others much better agree
with this Plant than that of C B. prodr. p. 104. 5.

There is no good Figure of this Herb altho a Common
Edging in every Country Garden.

68. Penny's Myrtle Cistus Ray 1011. 13.
Alycyrum Balearicum frutescens, magno flore luteo, fol.
minoribus lucidis subitus verrucosis J. Salvadore.
Myrto-Cistus Pennei Clus. 68. fig. Pan. 67. fig. Park. 665. fig.
Myrto-Cistus, Pennei, fl. luteis oblongis Chabr. 103. Ic. 2.
I B. 2. 1. 13. p. 21. fig.

Dr. Penny a famous Physitian of London and a very curious
Naturalist of that time, first communicated this
Plant A. D. 1580. which he had gathered on the Island
Majorca to Carolus Clusius, who gave a Figure of it, and
from whence all other Authors have since copied it. We
are lately obliged to Dr. John Salvadore at Barcelona for
the more perfect Knowledge of it, who amongst many
other very curious and Rare Plants gathered in that Island
and Minorca, sent me this under the Name as above, of
which it is a true Species.

69. Matthiolus his Coris or St. John's-wort Ray 1018. 4.
Coris C B in Matth. 669. fig opt. Cam. 678. fig. Park. 570.
220. fig.

Coris lutea C B. 280. 1. lutea major phytr. 548. 1.

Y y Hype-
Grows on most Mountains in Austria, Styria and Switzerland, where it Flowers in their Spring, viz. in June, or as soon as the Snow there melts.

This elegant Plant is pretty common in most of our Country Gardens.

Cervicaria Valerianoides caerulea C B. 95. 20.
Rapunculus Valerianoides caeruleus umbellatus Boer. 104. 3.
Rapuntium umbellatum Col. phyt. 2. p. 22.
Trachelio azuro umbelliferus Pona Ital. 44. fig.
Trachelium umbelliferum caeruleum Park. 645. 8.

This is very different from the Telephium fl. purpureis Lobel. ic. 389. 2. as Columna and C B. from him take to be the same.

Pona, in his Italian Edition of the Plants on Mount Baldus, gives the first Figure and Description of this elegant Plant, which he received from Signor Contareni at Venice, and supposes it to come from Candy; but Pere Barrelier says it grows in moist shady Places not only about Rome but Valen
tia in Spain, where its also sometimes found with a white Flower. Its blue umbelliferous Tufts make a beautiful Shew in our Physick Garden where it Flowers from Midsummer till the end of September.

Caryophyllus sylv. prolifer C B. 209. 6. phyt. 393. 18.
Caryophyllus prolifer Park. 1338. fig. 1.
Betonica Coronaria squamosa sylv. Chabr. 446. ic. 6. IB. 3.
L. 29. p. 335. fig.

Grows in barren Pastures in many Parts of France, Italy, Germany, &c.

67. Gray
67. Gray Stitchwort Ray 1020. 7. H. Ox. 546. 44.
Holosteum Horst. incanum vulgare Nobis.
Caryophyllus Holosteus Ger. 277. xi. Jonst. 595. fig. 15.
Caryophyllus Holosteus tomentosus I B. 3 1. 29. p. 360. fig.
Park. 1339. fig. 7.
Caryophyllus Holost. tomentosus latifolius C B. 210. 2.
pr. 104. 5.
Lychnis maritima incana & tomentosa H. Ovon. 546. 44.
Myosotis tomentosa Linaria folio aplo. E B. 211. Inst. 244.

The Description of I B. and others much better agree
with this Plant than that of C B. prodr. p. 104. 5.

There is no good Figure of this Herb altho a Common
Edging in every Country Garden.

68. Penny's Myrtle Ciftus Ray 1011. 13.
Acyrum Balearicum frutescens, magno flore luteo, fol.
minoribus lucidis subus verrucosis F. Salvadore.
Myrto-Ciftus Pennei Clus. 68. fig. Pan. 67. fig. Park. 665. fig.
Myrto-Ciftus Pennei, fl. luteis oblongis Chabr. I03. Ic. 2.

Dr. Penny a famous Phisitian of London and a very curi-
ous Naturalist of that time, first communicated this
Plant A. D. 1580. which he had gathered on the Island
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the more perfect Knowledge of it, who amongst many
other very curious and Rare Plants gathered in that Island
and Minorca, sent me this under the Name as above, of
which it is a true Species.

69. Matthiolus his Coris or St. John's-wort Ray 1018. 4.
Coris C Bin Mat. 667. fig opt. Cam. 678. fig. Park. 570.
220. fig.

Coris lutea C B. 280. 1. lutea major phyt. 548. 1.
Hypericoides Coris quorundam & Coris legitima Cretica Chabr. 456. Lc. 5. 1. B. 3. 1. 29. p. 384. fig.

Hypericum feu Coris legitima. Eriæ fimilis H. Oxon 469. 4.


Matthiolus says this grows in dry Grounds and Hills in many Places of Italy, and by Carara a Village of Liguria, as Camerarius asserts.

I take this by its Figure to be different from Bellus and Pona their Coris legitima Cretica.

70. Succory Mullein.

Blattaria Cichorei folio villoso Nobis.

an Blattaria Orient, Agrimonia folio. T. Coral.

Its Leaves spread on the Ground, are deeply cut long and hoary, from the midst of these rise slender Stalks, much branched and full of small yellow Flowers which soon fall off.

I have as yet seen this in Chelsea Garden only.


Lotus Hæmorrhoidalis five Trifolium Hæmorroid. majus Park. 1102. fig. 8.

Lotus pentaphyllos siliquolus villosus C.B. 332. 9.

Lotus polyceirates frutescens incana alba, siliquis curtis eremoribus & brevioribus eretis H. Ox. 177. 14.

Oxytrophium alterum Scribonii Herbariorum Lob. Adv. 381. fig.

Trifolium album, rectum hirsutum. valde Chabr. 150. Lc. 1. B. 2. 1. 17. p. 360. fig.

Trifolium Hæmorroidale Lob. Illustr. 151.

Grows plentifully. about Montpelier, Lobel says, its also found about Lyons and Turin.

The Leaves or rather the Seed given in Powder from 30 Grains to 60 in Red Wine, is said to be very effectual in cutting the Piles.

Lotus Hamorrhoidalis alter minus five Lotus Lybica Dal.
Park. 110. f. 9.
Lotus siliquosus glaber, flore rotundo C B. 332. 10.
Lotus Lybica Dalechampii Lugd. 509. fig.
Trifolium rectum Montpellieranum i B. 2. l. 17. p. 359. fine fig.
Trifolium rectum Montsp. fl. rotundo C B phy. 666. 46.

Grows by River sides and moist Places about Messina, Calabria and Montpelier.

73. Camerarius his Birds Foot Trefoil. Ray 970. 20.
Lotus siliquis Ornithopodii C B. 332. 2. phy. 667. 36. i B.
2. l. 17. p. 359. fig.
Lotus peculiariis, siliquis Ornithopodii Camer. Hort. 91. Ic.
25. opt.
Lotus Ornithopodii siliquis, f. luteo, nigris lincis striato
Chabr. 158. Ic. i.
Lotus polyceratos annuus procumbens latifolius, siliquis
Ornithopodii H. Ox. 176. 8.
Lotus sylvestris Creticus Park. 1101. fig. 4. ex Pena Ital. 153.
That Reverend Divine and Botanist Sir George Wheeler,
observed this in the Island Corfu.

Fumaria tenuifolia Ger. 928. fig. 3. Jons. 1088. fig. 3.
Fumaria minor five tenuifolia Park. 287. fig. 2.
Fumaria minor tenuifolia C B. 143. 3. phy. 245. 2.
Fumaria minor five tenuifolia surrecta Chabr. 277. Ic. 6. i B.
3. l. 26 p. 203. fig.
Capnos tenuifolia Clus. 208. fig. 2. Hisp. 375. fig. Lab. Ic.
757. 2. Belg. 914 Obs. 438.
Grows amongst Corn and by Pathways about Montpelier,
and in divers parts of Spain where its called Palomilla, and
flowers in April and May.

75. Barcelona Hart Fumitory.
Fumaria henneaphyllos Hispanica faxatilis Bocc. Mus. 83.
Tab. 73.

2 Y y 2 Fumaria
Fumaria heneaphyllos Hisp. faxatilis, f. vario Barret. 866. Ic. 42.
Dr. Salvador observed this in the Fissures of Mount Serrat, and in the Hermits Walls about Barcelona where it Flowers in May, June, and July.
The curious Dutchess Dowager of Beaufort shewed me very fair Specimens of this Plant, which her Grace raised from Seed I had given her.

Plantæ fructi tricocco. Plants with Treble Husks and Seed.

76. Garden Spurge Ray 866. 18.
Lathyris C B. in Matth. 868. fig. 1. Cam. 968. fig. opt.
Lathyris major C B. 293. 1. phyt. 577. 1.
Lathyris major Hortensis Park. 191. fig. 1.
Cataputia minor five Lathyris angustifolia. Tab. 587. Ic 2.
Tithymalus latifolius Cataputia dictus H. Leyd. Grows spontaneously in many Gardens, where it Flowers and Seeds in July and August.

77. Red Flowred Spurge Ray 864. 2.
Tithymalus Amygdaloides five Characias Chabr. 534. Ic. 2. I B. 3. l. 34. p 672. fig.
Tithymalus Characias 1. Clus. 188. fig. 1. Hisp. 436. fig.
Tithymalus Characias Monspelienfium Park. 186. 2. Jonst. 499. fig. 8.
Tithymalus Characias rubens peregrinus C B. 290. 2. phyt. 572. 1.
Clusius observed this on the rough Mountains of Spain, its also found in Italy and about Montpelier in Stony Places. Its dark red flowers, distinguishing it from others like it.


An Tithymalus Tingitanus Linariae folio, lunato flore H. Leyd.
An Tithymalus Linariae folio, lunato flore H. Blek. 313.

Dr. Nissole that curious Botanist sent me this plant among many others gathered about Montpelier.


Its stalks brown and hoary as are its leaves which grow alternately and not thick set. Its flowers and seeds in July and August.

Monte Vaillant sent me a Specimen and Seed of this from Paris.


It grows in the Corn beyond Kingston-wood in the Road to Gamlingay in Cambridge-shire, where I found it in company with Mr. James Sherard, an accurate Botanist and well versed in the Knowledge of our English Plants.

Tithymalus Pinea Ger. 402. fig. 6. opt.
Tithymalus Cyparissias minor C. B. phyt. 575. 15.
Tithymalus Cyparissias vulg. Park. 193. descr. 3. but its Figure is of the Pytula or Eftula major.

Vertues. Dr. Tournefort says the Root of this Spurge is a proper Cathartick in Hydroptic and Cachetic Bodies, as also for intermittent Feavers given from 3s to 3d with 20 Grains
Grains of *Mercurius dulcis* and 30 of *Cream of Tartar*, 10.
15 or 20 of its *Seed* is a strong *Purge*, and the *Boils of Fernelius* his *Pil. ex Esula*, its *Dose Eij*. The *Roots* of this
Plant are also used in the *Benedicta Laxativa*. *Hydrgo-
sum eximium Renodei*. *Extrae. eximium & Chologogum*
Rolfinchii.

It grows pretty common in many Gardens.

82. *Common Sea Spurge* *Ray Brit. Herbal. Tab. 53. fig. 8.*
*Tithymalus maritimus* *C B 291. l. phyt. 574. xi.*
*Tithymalus Paralius* *Chabr. 534. l. 6. l. B. 3. l. 34. p. 674.*
fig. opt.
*Tithymalus paralius seu maritimus* *Park. 184. fig. Ger. 401.*
fig. 1. *Font. 498. fig. 1. opt. Matth. Lugd. 1643. fig. 2.*
& 47. fig. *1. Tab. 593. lc 1. Hist. 2. p. 292. f. 3. C B in*
Matth. 864. fig. 3.

This elegant *Spurge* is found on most of the *Sea Coasts*
of Europe.

83. *Small Portland Spurge*.
*Tithymalus maritimus minor* *Portlandicus*.

From a small downright single *Root*, rise *slender Stalks*
with *longish oval Leaves* not thick *set* (as in the *common*
*Sea-Spurge*) at the Top come *small Umbels of Flowers* and
*Seeds* like other *Spurges*. It grows not a *Span high* even in
our Gardens where it is very luxuriant.

The Reverend Mr. *William Stonestreet*, that accurate
*Botanist*, first discovered this about a Year since on a nar-
row Neck of Land covered with *Peebles* which joins
*Portland* with the *Coast of Dorset-shire*.

*Arbores Pisiolorae*. *Trees with Pea-blooms*.

*Anagyris* *Cam. 671. fig. Ger. 1239. fig. 1. Font. 1427. fig. 1.*
*Anagyris* *Tabern. 1089. lc. 2. Hist. Vol. 2. p. 794. f. 1.*
*Anagyris non fadica major vel Alpina* *C B. 391. 3.*
*Anagyris live Laburnum majus* *Park. 245 fig. 2. opt.*

-Anagy-
(283)

Anagyris minus fastens vel Laburnum Lob. Ic. 2 p. 49. 2

Belg. 2 p. 56. fig:

Laburnum Chabr. 78. Ic. 1.

Laburnum arbor trifolia Anagyris similis I B r. I. xi. p. 36 e. f.

It Flowers in May, June, &c. On Saleve & Jura two Mountains near Geneva where Mr. Ray observed it, as also on the Alps of Savoy.


Colutea Scorpioides Lob. Ic. 86. 2. pr. 2. Belg. 100. T. 2.


Ger. 1116. fig. 2. Jonst. 1299. fig. 2.


Colutea Scorpioides elatior Clus. 97. fig.

Colutea Scorpioides major Park. 227. fig. 2.

Colutea Scorpioides elatior & major frutescens H. Ox. 122. 1.

Colutea filiquosa five Scorpioides major C B 397. 2.

Emerus vulgaris Casalp. 117.

Mr. Ray has observed this about Mompellier, Geneva, &c.


Colutea Ger. 1116. fig. 1. Jonst. 1299. fig. 1. Tab. 1090. Ic. 2.


Emeri alterum genus Casalp. 117.

Grows about from plentifully, as also on Mount Vesuvius, &c.

Arbores Bacciferae. Berry trees.


Chamelea Dod. 633. fig. Cam. 573. fig. opt.


fig Chabr. 46 Ic. 3. 1 B r. 1. 15. p. 58. 4 fig. Park. 262. fig. 5.

Chamelea Arabum tricoecos Ger. 1215. fig. Jonst. 1402. fig.

I. Tab. 1075. Ic. 2.

Chamelea vera alis Tricoecos, Mezereon Cam. Hort. 39.
fig. Belg. 448. fig.
Grows plentifully in Olive Grounds and on the Rocky
Hills about Montpelier.

88. Plain Oval Mock-Privet.

Phillyrea folio subrotundovix ferrato.
These Leaves much resemble the Figure in Camerarius his
Epitome p. 95, marked thus * but its notches less visible.

89. True Mock-Privet.

Phillyrea vera, folio acuto, basi lato.
This also is very lightly notcht.

310. fig. 5.

These Leaves are oblong, pointed and wholly plain.

91. Fine dented. small leaved Mock-Privet.

Phillyrea folio minori argute ferrato.
The only Tree of this I have as yet seen is in the
Wilderness of the Bishop of London's Garden at Fulham.


Phillyrea fol. Ligustri C. B. 476. 4.
Phillyrea latiore folio Ger. 1209. fig. Jonst. 1395. fig. 2.
Phillyrea latiusculo fol. Chabr. 42. I. B. 1. 15. p. 539. fig.
Phillyrea 3 Cluf. 52. fig. 2. Hisp. 68. fig.

Phillyrea angustifolia 1 Park 1443 fig. 4. secundum Iconem.
Grows wild in Spain and about Montpelier.


Phillyrea angustifolia Ger. 1209. fig. 1. Jonst. 1595. fig. 1.
Chabr. 42. I. B. 1. 15. p. 538. fig. Lob. 1c. 2. p. 132.
Belg. T. 2. p. 154. fig. Obs. 565. fig.

Phillyrea angustifolia C. B. 476. 5.
Phillyrea 4 Cluf. 52. fig. 3. Hisp. 64. fig.

Frequent about Montpelier.

We shall Conclude with the Indian Herbs and Trees in the
next Transactions.
III. Observationes Coelestes Britannicae, Grenovici in Observatorio Regio habita, anno MDCCXIII.

Observationes SATURNI.

<table>
<thead>
<tr>
<th>Temp. per Horolog.</th>
<th>Tempora correcta</th>
<th>Die Solis, Januarii 25.</th>
<th>Distantiae a Vertice</th>
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<td>Geminorum π τετεντανον transit</td>
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<td>Pes Castoris μ transit</td>
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<td>Leonis &amp; Bayero transit</td>
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<td>Leonis ν transit</td>
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<td>Saturni centrum transit</td>
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<td>Dist. a Polo Bor.</td>
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<td>Longitudo β</td>
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<td>Latitudo Bor.</td>
<td>1°31'27&quot;</td>
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<td>Temp. per Horolog.</td>
<td>Tempora correcta</td>
<td>Die Jovis Febr. 5.</td>
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Die Veneris Febr. 6.

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Die Mercurii Febr. 18.

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<td>Temporae per Tempora Horolog. correta</td>
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<td>Latitudo Bor.</td>
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Die Martis Aprilis 7.

| 7 43 1 | 7 41 1 5 | Leonis \( \uparrow \) transit | 36 9 25 |
| 50 46 7 49 00 | Saturni centrum transit | 35 31 10 |
| 57 36 55 50 | Leonis \( \text{ v } \) Bayero transit | 37 40 20 |
| 8 6 39 8 4 53 | Leonis \( \text{ v } \) in collo \( \text{ v } \) transit | 33 19 25 |
| 16 7 14 2 | Ejusdem \( \text{ IIIa } \) Cat. Br. transit | 36 19 30 |
| 8 21 17 | 8 19 3 | Ejusdem \( \text{ IIIa } \) Cat. Br. transit | 35 35 30 |
| | Ascensio recta \( h \) 143 57 45 | 35 35 30 |
| | Distantia a Polo | 74 3 15 |
| | Longitudo \( \alpha \) 21 3 32 |
| | Latitudo | 1 31 20 |

Die Mercurii Aprilis 8.

| 7 47 43 | 7 46 00  | Saturni centrum transit | 35 31 15 |
| 54 34 52 51 | Leonis \( \text{ v } \) transit | 37 40 20 |
| 8 3 3 7 | 8 1 54 | Leonis \( \text{ v } \) transit | 33 19 25 |
| 13 4 | 11 2 | Leonis \( \text{ IIIa } \) transit | 36 19 35 |
| 8 18 14 | 8 16 3 | Leonis \( \text{ IIIa } \) transit | 35 3 45 |
| | Ascensio recta \( h \) 143 57 30 | 35 3 45 |
| | Distantia a Polo | 74 3 20 |
| | Longitudo \( \alpha \) 21 3 20 |
| | Latitudo | 1 31 10 |
| | Saturno pene stationario | Die |
### Observationes Jovis

**Anno MDCCXIII.**

<table>
<thead>
<tr>
<th>Temp. per Horolog.</th>
<th>Tempora correcta.</th>
<th>Die Jovis Novemb. 5.</th>
<th>Distantiae a Vertice</th>
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### Observationes Solis

**Anno MDCCXIII.**

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<td>12 40 4</td>
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<td>Jovis centrum transitit</td>
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<td>12 52 36</td>
<td>12 49 59</td>
<td>Aquariri 73ta Cat. Brit. prima ad h transitit</td>
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<td>Aquarii in aqua λ transitit</td>
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Die Lunæ Octobris 26.

| 7 29 16 | 7 28 42 | Aquarii in Clune σ transit | 63 34 40 |
| 7 36 34 | 7 36 | Jovis centrum transit | 63 00 5 |
| 8 14 34 | 8 14 | Aquarii 80ma prima ad † transit | 62 5 20 |
| 8 17 45 | 8 17 11 | Aquarii 84ta sec. ad † transit | 62 37 5 |
| Ascensio recta x 335 41 30 | Distantia a Polo 101 33 20 |
| Longitudo Jov. x 3 16 00 | Latitudo Aust. 1 19 8 |

Die Martis Octob. 27.

| 7 25 40 | 7 23 34 | Aquarii Clunis σ transit | 63 34 35 |
| 7 33 6 | 7 31 00 | Jovis centrum transit | 62 59 15 |
| 8 11 00 | 8 8 54 | Aquarii prima ad † transit | 62 5 15 |
| 8 14 10 | 8 12 4 | Sequens ad † transit | 62 37 10 |
| Ascensio recta x 335 43 20 | Distantia a Polo 101 32 30 |
| Longitudo Jov. x 3 17 58 | Latitudo Aust. 1 19 00 |

Die Jovis Octob. 29.

| 7 18 29 | 7 15 19 | Aquarii σ transit | 63 34 40 |
| 26 10 | 7 23 00 | Jovis centrum transit | 62 57 20 |
| 8 3 47 | 8 0 37 | Prima ad † transit | 62 5 20 |
| 6 55 | 8 3 45 | Sequens ad † transit | 62 37 10 |

Ascen-
### Observationes Martis

**Anno MDCCXIII.**

<table>
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<td>In ancone Alae (\gamma) transit</td>
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<td>13 3 3</td>
<td>Virginis (\sigma m a) Cat. Brit. (\tau) transit</td>
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<td>13 13 18</td>
<td>13 6 0</td>
<td>Martis centrum transit</td>
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<td>13 21 8</td>
<td>13 13 50</td>
<td>In cervice Virginis (\epsilon) transit</td>
<td>46 33 30</td>
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Die Martis Martii 3.

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<th>Longitudo</th>
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<tr>
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<td>11 53</td>
<td>43 20 00</td>
<td>83 46 45</td>
<td>3 43 37</td>
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<td>12 4 50</td>
<td>12 2 0</td>
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<td>45 14 20</td>
<td>27 46 00</td>
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<td>10 54</td>
<td>8 14</td>
<td>In vultu Virginis (\tau) transit</td>
<td>43 15 25</td>
<td>3 30</td>
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<td>20 8</td>
<td>17 18</td>
<td>Undecima Virginis transit</td>
<td>44 3 30</td>
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<td>Virginis 16. in Cervice (\epsilon) transit</td>
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<td>Virginis 17ma Cat Br. transit</td>
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Die
### Temp. per Horolog. | Tempora correcta | Die Martis Aprilis 7. | Distantiae a Vertice
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<td>9 16</td>
<td>Prima Virg. Cat. Br. w transit</td>
<td>41 44 50</td>
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<td>9 38</td>
<td>9 36</td>
<td>Borea in Vertice w transit</td>
<td>41 37 15</td>
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<td>Distantia a Polo 81 15 10</td>
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<td>Longit. Martis w 13 30 40</td>
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<td>Virginis g transit</td>
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<td>In Vultu Virginis p transit</td>
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<td>8 28</td>
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<td>Longitudo w 15 15 00</td>
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**Observationes L. U. NÆ.**

*Anno MDCCXIII.*

| 8 9 33 | 8 7 23 | Telecopica a transit. | 28 26 20 |
| 8 15 5 | 8 12 55 | Tauri 12 34 a Cat. Brit. transit. | 27 2 30 |
| 8 20 20 | 8 18 10 | Lunea limbus præced. transit, centro a Vertice. | 27 32 40 |
| 8 21 23 | 8 19 13 | Lunea centrum transit, limbo remoto a Vertice. | 27 47 40 |
| 8 22 52 | 8 20 42 | Lunea cupis Bor. a Vertice | 27 17 40 |
| 8 30 15 | 8 28 5 | Geminorum transit. | 28 13 20 |
| 8 41 7 | 8 38 57 | Pes Castoris transit. | 28 54 50 |
| 8 49 8 | 8 46 58 | Calx ejusdem transit. | 28 50 40 |
| Asc. rect. cent. d | 84 26 55 |
| Dist. a Polo viva | 66 4 40 |
| Sed adhibit. Paral. 65 39 50 |
| Longit. Lunea | 24 56 30 |
| Latitudo Bor. | 0 57 00 |
### Observationes SATELLITUM IOVIS.

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<td>Adhibita Parallaxi 66 51 15</td>
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**Die Veneris Octob. 30.**

6 56 30 6 52 35 Quartus Satelles visus est emergens ab umbra, diametro Jovis distans a tertio et proximo ad dextram, Tubo feil. octo pedum.

7 4 00 7 00 00 Clare explenduit, & linea duxta a proximo illo per centrum Jovis emergentem reliquit ad Austro, fita sectiles in verso.

7 36 31 7 32 30 Pegasi \(\mu\) transit per planum Arcus meridionalis.

**Die: Saturni Novemb. 7.**

7 13 2 7 5 00 Secundus Satelles emergebat, vel potius emergere incipiabat. Tubo octo pedum

2 5 11 8 57 00 Piscium \(\delta\) in Lino australi transit. N.B.
N.B. Stella illa Telecopica a quae die Januarii 25° Lunam precedit, Ascensionem rectam tunc habuit 81° 28½, & distabat a Polo 66° 58' 20", unde sit Longitudo ejus 22° 9½ cum Latitudine Australi 0° 13½. Hac autem est ea ipsa stella ad quam applicabatur Jupiter in Statione secunda, anno 1634 Febratii 6, eamque non nisi tribus suis corporis diametris ad Austros reliquit, observante Gassendor. ut habetur inter Observationa ejus pag. 174. Et ad eandem Mars observatus est Septembris 6to anno 1644 mane, ut videre est in Prolegomenis Selenographiae Heveianae pag. 65 & Fig. 1. Veram multum usui erit, ad accuratam Nodi Jovis determinationem, ejusque motus, si modo inter stellas fixas planum orbita Jovialis non hæreat immobile. Etenim post decursum 83 annorum, quibus Jupiter satis accurate septem absoluit periodos, anno fæil. 1717. Januarii 10. mane. Planeta stellam illam corporaliter tegit vel saltem stringet, spectaculo quidem raro neque hæritis quod sciam Astronomis in Jove concessò.

Stella autem ipsa, etiam Telecopica vocetur, sude caelo & absente Lune inermis occul aciem non fugit; comitemque habet sequentem ad Austros, & semidiametro Solis etroter distantem, apud quam conspicetur Jupiter arcissime conjunctus. Die vicecesimo Julii anni proximi 1716 mane.

IV. An Account of an Experiment made by Dr. Brook Taylor assisted by Mr. Hawkesbee, in order to discover the Law of the Magnetical Attraction.

By Order of the Royal Society Mr. Hawkesbee and my self made an Experiment with the great Loadstone belonging to the Royal Society, in order to discover the Law of the Magnetical Attraction; and not long after
after I gave an account of it to the Society in a Letter to Dr. Sloane, (who was then Secretary) dated June 25. 1712. Since that, Mr. Hawkesbee made another Experiment of the same nature with a smaller Loadstone; which he has given an account of in the Philosophical Transactions No. 335. But upon comparing the Numbers of that Experiment with those of the other, I find the Numbers of the first Experiment to be very much more regular. Wherefore I conclude that to be the best Experiment, and since no notice has been taken of the Account I gave of it, and I have reason to believe Mr. Hawkesbee lost the Table I left with him for the Society, of the Numbers relating to it, I take this occasion to present the Society with the following Account of it.

We placed the great Loadstone belonging to the Royal Society so, that its two Poles lay in the Plane of the Horizon, and were in a Line exactly at right Angles with the natural Direction of the Needle we made use of, (which was that Dr. Halley had made to observe the Variations with). And by means of a Carriage contrived for that purpose, the Stone was easily moved to and fro, the Poles continuing always in the same Line. The Needle was so placed, that the Center it play’d upon was in the same Line with the Poles of the Stone; the North Pole being towards the Needle. We measured the Distances from the Center of the Needle to the Extremity of the Stone; and we found the Variations of the Needle from its natural Position to be as in the following Table.

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<td>3000</td>
<td>6</td>
<td>535</td>
<td>9</td>
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</table>
V. A Short Account of the Cause of the Saltiness of the Ocean, and of the several Lakes that emit no Rivers; with a Proposal, by help thereof, to discover the Age of the World. Produced before the Royal-Society by Edmund Halley, R. S. Secr.

There have been many Attempts made and Proposals offered, to ascertain from the Appearances of Nature, what may have been the Antiquity of this Globe of Earth; on which, by the Evidence of Sacred Writ, Mankind has dwelt about 6000 Years; or according to the Septuagint above 7000. But whereas we are told that the Formation of Man was the last Act of the Creator, 'tis nowhere revealed in Scripture how long the Earth had existed before this last Creation, nor how long those five Days that preceded it may be to be accounted; since we are elsewhere told, that in respect of the Almighty a thousand Years is as one Day, being equally no part of Eternity; Nor can it well be conceived how those Days should be to be understood of natural Days, since they are mentioned as Measures of Time before the Creation of the Sun, which was not till the Fourth Day. And 'tis certain Adam found the Earth, at his first Production, fully replenished with all sorts of other Animals. This Enquiry seeming to me well to deserve Consideration, and worthy the Thoughts of the Royal Society, I shall take leave to propose an Expedient for determining the Age of the World by a Medium, as I take it, wholly new, and which in my Opinion seems to promise Success, though the Event cannot be judged of till after a long Period of Time; submitting the same to their better Judgment.
What suggested this notion was an observation I had made, that all the Lakes in the World, properly so called, are found to be Salt; some more some less than the Ocean Sea, which in the present case may also be esteemed a Lake; since by that term I mean such standing Waters as perpetually receive Rivers running into them, and have no Exite or Evacuation.

The number of these Lakes, in the known parts of the World is exceeding small, and indeed upon Enquiry I cannot be certain there are in all any more than four or five, viz. first, The Caspian Sea; secondly, The Mare Mortuum or Lacus Asphaltites; thirdly, The Lake on which stands the City of Mexico, and fourthly, The Lake of Titicaca in Peru, which by a Channel of about fifty Leagues communicates with a fifth and smaller, call'd the Lake of Paria, neither of which have any other Exite. Of these the Caspian, which is by much the greatest, is reported to be somewhat less Salt than the Ocean. The Lacus Asphaltites is so exceedingly Salt, that its Waters seem fully lated, or scarce capable to dissolve any more; whence in Summer-time its Banks are incrusted with great Quantities of dry Salt, of somewhat a more pungent nature than the Marine, as having a Relish of Sal Armoniac; as I was informed by a curious Gentleman that was upon the place.

The Lake of Mexico properly speaking is two Lakes, divided by the Causways that lead to the City, which is built in Islands in the midst of the Lake, undoubtedly for its Security; after the Idea, tis probable, its first Founders borrowed from their Beavers, who build their Houses on Damms they make in the Rivers after that manner. Now that part of the Lake which is to the Northwards of the Town and Causways, receives a River of a considerable magnitude, which being somewhat higher than the other, does with a small Fall exonerate itself in the Southern part.
part, which is lower. Of these the lower is found to be
take; but to what degree I cannot yet learn; though the
upper be almost fresh.

And the Lake of Titicaca, being near eighty Leagues
in circumference, and receiving several considerable fresh
Rivers, has its Waters, by the Testimony of Herrera and
Acofa, so brackish as not to be potable, though not ful-
ly so salt as that of the Ocean; and the like they affirm
of that of Paria, into which the Lake of Titicaca does in
part exonerate itself, and which I doubt not will be found
much saltier than it, if it were enquired into.

Now I conceive that as all these Lakes do receive Riv-
ers and have no Exits or Discharge, so it will be neces-
sary that their Waters rise and cover the Land, until such
time as their Surfaces are sufficiently extended, so as to ex-
hale in Vapour that Water that is poured in by the Rivers;
and consequently that Lakes must be bigger or lesser ac-
cording to the Quantity of the fresh they receive. But
the Vapours thus exhaled are perfectly fresh, so that the
saline Particles that are brought in by the Rivers remain
behind, whilst the fresh evaporates; and hence it is evident
that the Salt in the Lakes will be continually augmented,
and the Water grow saltier and saltier. But in Lakes that
have an Exit, as the Lake of Genesaret, otherwise called
that of Tiberias, and the upper Lake of Mexico, and in-
deed in most others, the Water being continually running
off, is supply'd by new fresh river Water, in which the
saline Particles are so few as by no means to be perceived.

Now if this be the true Reason of the Salines of these
Lakes, 'tis not improbable but that the Ocean it self is
become salt from the same Cause, and we are thereby fur-
nished with an Argument for estimating the Duration of
all Things, from an Observation of the Increment of Sal-
ines in their Waters. For if it be observed what Quanti-
ty of Salt is at present contained in a certain Weight of the
Water of the Caspian Sea, for example, taken at a certain Place, in the driest Weather; and after some Centuries of Years the same Weight of Water, taken in the same place and under the same Circumstances, be found to contain a sensibly greater Quantity of Salt than at the time of the first Experiment; we may by the Rule of Proportion take an estimate of the whole time wherein the Water would acquire the Degree of Saltness we at present find in it.

And this Argument would be the more conclusive, if by a like Experiment a similar Encrease in the Saltness of the Ocean should be observed: for that, after the same manner as aforesaid, receives innumerable Rivers, all which deposit their saline Particles therein; and are again supplied, as I have elsewhere shewn, by the Vapours of the Ocean, which rise therefrom in Atoms of pure Water, without the least admixture of Salt. But the Rivers in their long Passage over the Earth do imbibe some of the saline Particles thereof, though in so small a Quantity as not to be perceived, unless in these their Depositories after a long Tract of time. And if upon repeating the Experiment, after another equal Number of Ages, it shall be found that the Saltness is further encreased with the same Increment as before, then what is now proposed as Hypothetical would appear little less than Demonstrative. But since this Argument can be of no use to Ourselves; it requiring very great Intervals of time to come to our Conclusion, it were to be wished that the ancient Greek and Latin Authors had delivered down to us the degree of the Saltness of the Sea, as it was about 2000 Years ago; for then it cannot be doubted but that the Difference between what is now found and what then was, would become very sensible. I recommend it therefore to the Society, as opportunity shall offer, to procure the Experiments to be made of the present degree of Saltness of the Ocean, and of as many of these Lakes as can be
come at, that they may stand upon Record for the benefit of future Ages.

If it be objected that the Water of the Ocean, and perhaps of some of these Lakes, might at the first Beginnings of Things, in some measure contain Salt, so as to disturb the Proportionality of the Encrease of Saltness in them, I will not dispute it: But shall observe that such a Supposition would by so much contract the Age of the World, within the Date to be derived from the foregoing Argument, which is chiefly intended to refute the ancient Notion, some have of late entertained, of the Eternity of all Things; though perhaps by it the World may be found much older than many have hitherto imagined.

Accounts of Books

I. Linear Perspective, or a New Method of representing justly all manner of Objects, &c. By Brook Taylor, L. L. D. and R. S. Secr. 8vo. London, 1715.

The Author of this Book, finding the Art of Perspective to be very imperfect in the Books that have hitherto been publish'd on that Subject, thought it worth his while to consider the whole matter anew, and from a careful Examination of the Principles this Art is founded upon, he has endeavoured to establish some Theorems, by means of which the Practice of it might be rendered more general and easy than has yet been done. In order to this, at first sight he found it necessary to make use of new Terms of Art; the old ones seeming not to be
be expressive enough of what is meant by them, and being adapted to too confined an Idea of the Principles of this Art. In the old Perspective the chiefest regard is had to the Ground Plane, that is, the Plane of the Horizon; from whence is derived the Horizontal Line, and by means of that Line the Representations of some Figures are found by good simple Constructions. But then the Figures in all other Planes are drawn by reducing them to the Horizontal Plane by means of Perpendiculars; which is an inartificial round-about way, makes a great Confusion of Lines, and is not capable of so much Exactness. This confined way of treating this Subject, proceeds from the strong Possession the Mind is bred up in of the Notions of Upwards and Downwards, which makes one apt to refer all other irregular Positions to those principal ones. But the Minds of all Artists should be drawn as much as can be from such confined Ways of thinking, and they should be taught to accustom themselves, as much as may be, to consider Nature in its general View, without minding those particular Relations which things have with respect to themselves. For this reason our Author has rejected the Term of Horizontal Line, because it confines the Mind too much to the particular consideration of the Horizontal Plane; but he considers all Planes alike, and all Figures as they are in themselves, without considering their Relation to us; leaving the Artist to do that, when he comes to apply the general Rules of practice to any particular Design.

This Treatise is very short, because the Author has confined himself only to give the general Rules of practice, leaving the Reader to himself or to a Master to find our particular Examples to exercise himself in. Yet he hopes he has omitted nothing that is material to the understanding of this Art in the full extent of it. The whole Book consists of five Sections.
The first Section contains an Explanation of the fundamental Principle of this Art, with the Definitions of other Terms, and four Theorems. The fundamental Principle of this Art, is, that the Representation of any Point is a Point on the Picture where it is cut by a Line drawn from the original Point really placed where it ought not seem to be, to the place of the Spectator's Eye: and consequently, the Representation of any Line is the Intersection of the Picture with a Surface made by drawing Lines from the place of the Spectator's Eye, to the several Points of the original Line to be represented, really placed where it ought to seem to be. For these Lines which come from the several Points of the original Objected be placed in its proper Situation, to the Spectator's Eye, and as to many visual Rays which make the Object sensible.

When a Right Line is continued in infinitum, the Visual Ray becomes at last parallel to it, and an Object of any given Bigness, if it goes still further and further off on that Line, will at last seem to vanish; and at that time the place of its Representation on the Picture is the Point where the Ray parallel to the original Line cuts the Picture. For this reason our Author has thought it proper to call that Point the Vanishing Point of such an original Line (and consequently of all others parallel to it) (Def. 5.) And for the same Reason he calls that Line on the Picture a Vanishing Line (Def. 6.) which is produced by the Intersection of the Picture with a plane passing through the Spectator's Eye parallel to an original Plane.

There are ten Definitions in all, but these are the principal. And in our Author's Method these Vanishing Points and Vanishing Lines are of great use for the Representation of any Line passing through its vanishing Point. (Prop. 11.) Having found the Representation of one Point in any Line, by any Method whatsoever, he finds the Representation of the whole Line by its vanishing Point, which he shows an
ealy
easy Way to find in Propp 6, 8, 12. which are in the second Section. And by this means he solves several problems in Perspective, which it is not possible to do by the common Way, at least without a great deal of Difficulty, and a great Confusion of Lines. And by this Method he shews how the compleat Representations of any proposed Figures may be found, having given the Representation only of some principal parts of them. The second Section contains several Propositions to that purpose, shewing how to find the vanishing Points and Lines of proposed Lines and Planes, according to the several Circumstances proposed; and by the means of them, how to find the Representation of any given Figure. In the End of this Section there are some Examples, in the Description of the regular solids and some other Figures.

The third Section shews how to find the Representation of the Shadows of all Objects.

The fourth Section shews how to find the Representations of the Reflexions of Figures made by polish'd Planes.

The fifth Section contains a few Propositions relating to the inverse Method of Perspective; or the manner of examining a Picture already drawn; so as to find out what Point the Picture is to be seen from, or having that given, to find what the Figures are which are described on the Picture.

Our Author has observed that there may be a very good Expedient made use of in painting of large Rooms and Churches, which is drawn from the Nature of those Rays which produce the Vanishing Points. This not being mention'd in the Book it self, he thinks it not improper to take notice of it here: The Expedient is this, Having some way or other found the Representation of one Point of a Line that is wanted in the Picture, to find the whole Line, pass a Thread stretch'd through the place of the Spectator's Eye, in a Direction parallel to the Direction the original
original Line ought to be in; and the Shadow of that Thread cast by a Candle, so as to pass through the given Point on the Picture will be the Representation sought. The reason of this Construction is, because the Rays of Light that pass from the Candle to the Thread stretch'd, make the Plane which generates the Representation sought, (see Prop. 1.) And there may be other Expedients of the like nature gather'd from the same Principle.

II. DUCATUS LEODIENSIS,

Or,


THO the diligent and curious Author of this Work do not professedly treat of any Place but the ancient Town and Parish of Leeds, and the Regio Leodis, or adjoining Territory called Elmet; yet not only the Preface is more general, relating to the County, but there are many Passages in the Book itself, wherein he takes occasion to insert the Pedigrees of such of the Nobility and Gentry, as have had any Estates within the prescribed Limits, tho' the chief Seat of the Family be distant; as esteeming all Provinceales, who have but Domicilium in Provincia: to some of these he hath premised several Descents from ancient Deeds yet remaining in the respective Families; and to most of those that are inserted in the Visitations in the College at Arms, London, he hath added the Dates from Original Deeds, Registers, &c. and continued them to the present time, which hath rendered
it so acceptable to the learned Gentlemen of that Faculty, that Four Kings at Arms, and some eminent Heralds, have not only subscribed, but since their Perusal thereof, bought others for their absent Friends, expressing great satisfaction in that part of the Performance: as many learned Antiquaries have done in the other Parts relating to the Topography and Etymology of the Names of Places, &c, which he hath been very particular in, as finding the Name to be frequently a brief Description of the Place; and hath been thereby enabled to discover the Vestigia of some considerable Antiquities, in the actual Survey that he made of those Places to render the Work more compleat: He hath, by the ancient Names and the Situation of the places, been enabled to describe, in a very particular manner, the Transactions between the Pagans and Primitive Christian Saxons, relating to that noted Battle upon Win-moor, An. Dom 655. There are also many very considerable Benefactions, and lately Edifices erected of later times, particularly a magnificent Church built and endowed by Mr. Harrison; whose Nephew the Reverend Mr. Robinson hath most generously promised to endow another Church, which, it is hoped, will be shortly erected in that populous Town of Leeds, to the building of which several of the Magistrates, particularly Mr. Milner (who hath adorned the Market-place with a most noble Marble Statue of Her late Majesty placed in the Front of the Guild-hall) and other Inhabitants have subscribed very liberally. Here is also a Charity-School for an Hundred poor Children, who are cloathed and taught here, &c.

But what relates more immediately to these Philosophical Transactions, is the annexed Catalogue of the Authors Museum, justly celebrated for Antiquities and for natural and artificial Curiosities. The Catalogue of the Coins and Medals
Medals is surprizingly copious and valuable. To the ancient Greek and Confular, or Family-Monies of the Romans, he hath added above a thousand Imperial, several of which are noted by the learned Baron Spanhemius as very rare; and so likewise are those justly esteemed that relate more immediately to Britain, whether minted by the Romans or Britains. That of Thor with Runic Letters is inestimable, being the only known Piece in the World with those ancient Characters upon it. This was first deciphered by the Right Reverend Dr. Nicholston Lord Bishop of Carlisle, and after by Dr. Hicks, the two great Revivers of that sort of Literature. Upon which single Medal a learned Foreigner hath printed a distinct Treatise. *And the ingenious Sir Andrew Fountain in his Differ-tatio Episcopalis to the Right Honourable Thomas Earl of Pembroke, faith expressly: * Numismatum omnium quod aut "Anglo-Saxonibus, aut Anglo-Danis in usu suisse videntur, "nullum notari dignius est, quam id literis Racnicis inscrip-" tum, quod possidet vir genere & ingenio clarus Radulphus "Thoresbeius, Leodiensis." Those of the Saxon Kings begin with a very choice one of Edwin the ancientest Coin of the English Nation, and of the first Christian King of Northumberland; and are succeeded by those of the Danish and Norman Lines, and continued to the present Age, in a great Variety of current Monies and Medals in Gold, Silver and Copper. Those of Ireland and the English Plantations in America, are interspers'd, in the several Reigns: but those of Scotland, from the first of the Alexanders, are so numerous and valuable as to merit a particu.

* De Argento insignito Runic seu literis Gothicis &c. Sententia Nicolaev Kederi, Regii Antiquitatum Collegii, quod Helmiæ est, Assessoris, 4to 1703. Lipsiae.
lar Description. All along are very instructive Directions how to distinguish the Kings of the same Name from one another, before the Numbers were added upon their Monies. The Roman Emperours and Saxon Kings being well engraved before, the chief Difficulty is in those from William I. to Henry VII. which are therefore delineated here from the Originals. To these are prefixed the most ancient Consular Monies, which many Ages preceded the Incarnation of our Blessed Saviour, because never yet extant in any English Author. The other Medals and Monies of Popes, Emperours, Kings and Republicks, must be omitted for brevity’s sake, tho’ some of them (particularly that of the Siege of Leyden in pastboard) be very rare.

The Natural Curiosities are ranked in the following Method, 1. Human Rarities, 2. Quadrupeds, Viviparous (multifidous and bisidous) and Oviparous, with an Account of certain Balls and Stones found in the Stomachs of several Animals. 3. Serpents. 4. Birds, Land and Water-Fowls with their Eggs. 5. Fishes, viviparous and oviparous, scaled and exanguious. 6. Shells, whirled and single, double and multiple. 7. Insects, with naked and with sheathed Wings, and creeping Insects. 8. Plants, which begin with Dr. Nicolson’s Collection of above 800 dry’d Plants; the rest are reduced to the accurate Method of Dr. Sloane, in his Cat. Plant. in Insula Jamaica, proceeding from the Corals and other Submarines to the Fruits and Parts of Trees. 9. Formed Stones, which are ranged according to Mr. Lloyd’s curious Tract, Lithophylac. Britan. only to the Crystals and Diamonds are premised the Margarite Cumbrenses, some of which have as good a Water as the Oriental. After the fossil Shells and Stones of the turbinated Kind, the Bivalves and Shells amassed together into great Stones by a petrifyed Cement.
follow the Marbles and other Stones irregular. 10. The Metals Ores, Salts and Ambers, of which one with a Fly, another with a Spider enclosed.

The Artificial Curiosities relate to War, as Indian and Persian Bows, Arrows, Darts, Armour, Shields, Targets, Tomahaws, poisoned Daggers: to the Mathematics, to Household-stuff, Habits, &c. from the remotest Parts of the habitable World; not neglecting those that are obsolete of our own Nation. Then follow Statues, Bass-Relieves, Seals, Impressions, Copper-plates, Heathen Deities, Amulets, Charms and Matters relating to Romish Superstitions.

Of enamel'd Curiosities, that of General Fairfax and the fatal Battle at Naseby is perform'd with so exquisite Art, that it infinitely transcends the Metal, tho' Gold. And for Paintings, the Misery of War is admirably express'd, as to the various Passions, upon a Copper-plate about two Foot broad. To these may be added the Collection of printed Heads, and the Effigies of illustrious and learned Persons, beginning with the Royal-Family: then the Nobility, Warrior, Gentry, &c. in a Chronological Series. In the Ecclesiastical State, the Archbishops and Bishops are introduced by the Martyrs and Confessors of their venerable Order, and succeeded by other learned Dignitaries and pious Divines of both Denominations. The Judges are attended by the Literati in all Faculties, Physicians, Philosophers, Historians, Poets, Painters and other Artists. Some learned and pious Ladies are interspers'd. There are Volumes of the Saints, Popes, Emperors, and other Foreigners, amounting to the Number of 15 or 1600, many of which are done by the most celebrated Hands. Original Designs drawn by the Pen of noted Virtuoso's. Writings and Drawings by the Blind or Lame, as born without Hands. Some by other persons so
to admirably small yet legible, that in one there are 21, in another 28 Lines in the compass of an Inch. Papers of different Materials, Colour, Fineness, &c. ancient and modern: one Sheet of transparent Indian Paper a Yard in length. Inkherns from Muscovia, and Turkey, with Reed-pens painted and gilt. A Turkish Commission and Seal, a Mancks Warrant, the former impress'd with Ink not Wax, the latter upon blew Slate not Paper. Books printed in seven several Languages that are spoken in the English Dominions, not including what may now be added by the Accession of His present Majesty. A Catalogue of the various Editions of the Bible in this Museum; of the Concordances also, and Common-Prayer Books in different Languages; of the Manuscripts also, it being considerably increased since that inserted in the Oxford Catalogue anno 1697. To these are added a List of Books published in the Infancy of the Art of Printing, and others that later Controversies have rendered remarkable. And also a large Catalogue of Autographs begun of late Years by the Author, yet by his general Correspondence furnished with the Signs Manual of many of the Kings of England before the Reformation; and the proper Hand-Writing of every one since: with those of a vast Number of the Lords Spiritual and Temporal in several Reigns, and of the learned Authors, &c. The like also of foreign Potentates, Warriors, Literati &c. of these some are very remarkable, being subscribed by the Lords of the Privy Council at Whitehall, by the Lord President and Council at Tork, and the Lord Deputy and Council at Dublin, from Queen Elizabeth's Reign to the last Day of King James II. when the Warrant could not be executed. Oliver Cromwell's Instructions to the Lord Faulconberg when sent Ambassadour to the French King. The Warrants of the several Governments that so hastily supplanted one another in that Year of Confusion 1659, (which occasioned
ned the Restoration) all under their proper Hands and Seals. To these he hath since added Richard Cromwell's original Letters Patents to dissolve the Parliament; and another rare Album with many learned Hands, to those before mentioned. Then followeth a Catalogue of several Manuscript Rolls, Letters Patents, Diploma's, Charters and ancient Deeds of Gift to Religious Houses, which would be of use towards another Volume of the Monasticon Anglicanum. Bede-Rolls, Dispensations, &c. Lastly, a Description of other Antiquities here deposited, as Roman Deities, Altars, Sepulchral Monuments, Urns of different Forms and Colours, Cornelian Signets, a Roman Triumph in Basrelief, and the Story of Adonis slain by a Boar. Besides these there are Clay Coining-Moulds for counterfeiting the Roman Coyns when current, Fibula Vestiaris, Rings or Bracelets of Jet, tessellated Pavements, Lamps, Bricks with Inscriptions, of which one very instructive is mentioned in the Oxford Edition of Livy. To which are added Brass-Swords found in England, Ireland and the Isle of Man; British Arrowheads of Flint; a Danish Sacrificing Mallet of Marble, Antique Spurs, Shields, &c. of later Ages, tho' now antiquated. The Figures of many of these are very well engraven, as also the Churches and Prospects in the Book.

By the Appendix it appears what considerable Additions the indefatigable Author is continually making to this Museum. A Medal of Jo Kendall is especially remarkable, because retrieving the Memory of that noted Warrior, representing his Head in a noble Relieve, who was Turcopellerius or Colonel of the Cavalry (which Office belonged to the English Nation) at the memorable Siege of Rhodes, when Mahomet the Great was worsted. To the Autographs is added one impressed with a Stile upon a Palmeto Leaf, and folded up as a missive Letter in the East Indies.
by one Timothy a converted Malabarian. Through the whole Work he is particularly grateful, in writing the Names of his Benefactors that have sent him any Curiosities. And concludes with an account of unusual Accidents that have attended some Persons in their Births, Lives, and Deaths, of which many are very very remarkable, but I fear to be too tedious.

FINIS.

LONDON: Printed for W. INNYS at the Prince's Arms in St. Paul's Church-yard. MDCCXV.
ADVERTISEMENT.

Just Publish'd,

Methodus Incrementorum Directa, & Inversa, Autore Brook Taylor, L. L. D. & R. S. Secr. In two Parts. In the first Part are explain'd the Principles of the New Incremental Method, and by the means of that the Method of Fluxions is more fully explain'd than has yet been done; it being shew'd how this Method is deduced from the former, by taking the first and last Ratio's of the nascent and evanescent Increments. In the second Part the Usefulness of these two Methods is set forth by several Examples, viz. 1. In the summing up of Arithmetical Series. 2. In finding all the figurate Numbers. 3. In the finding of Tangents, Rays of Concavity, and the Quadrature of all sorts of Curves. 4. The Catenaria. 5. The Velaria. 6. The Fornix. 7. The Vibration of a Musical String. 8. The Centers of Oscillation and Percussion. 9. The Density of the Atmosphere. 10. The Refraction of Light passing thro' the Atmosphere. 4to. Printed for W. Innys at the Prince's Arms in St. Paul's Church Yard.

An Account of this Book will be given in a following Transaction.
PHILOSOPHICAL TRANSACTIONS.

For the Months of September and October, 1715.

The CONTENTS.

I. Some Accounts of the late great Solar Eclipse on April 22, 1715. Communicated to the Royal-Society from abroad.

II. An Account of a Journey from the Port of Oratava in the Island of Teneriff to the Top of the Pike in that Island, in August last; with Observations thereon by Mr. J. Edens.

III. Vetriculus cordis sinister stupendæ magnitudinis, lately communicated to the Royal-Society by James Douglass, M. D. and R. S. S.

IV. A ready Description and Quadrature of a Curve of the Third Order, resembling that commonly call’d the Foliate. Communicated by Mr. Abr. de Moivre, F. R. S.

V. An easy Mechanical Way to divide the Nautical Meridian Line in Mercator’s Projection; with an Account of the Relation of the same Meridian Line to the Curva Catenaria. By J. Perks, M. A.


II. Ludovici Ferdinandi Marsili, Dissertatio de Generatio- ne Fungorum. Rom. 1714. 4to.
I. Some Accounts of the late great Solar Eclipse on April 22. 1715. Communicated to the Royal-Society from abroad.

Since the Publication of the large Account we gave in Phil. Trans. No 343. of what was observed in England, and particularly at London, of this Eclipse, we have received from foreign Parts the following Observations; which seem not unworthy the Acceptance of the Curious. And first Mr. John Edens, who has obliged us with the following most particular Relation of the Pike of Teneriff and of the Ascent thereto, being on his Voyage to that Island, observed the Eclipse at Sea, in Latitude, by Observation 34°. 20', and Longitude 0h. 54'. West from London, as he concluded by their Distance and Position from the Island Forte ventura, which they soon after fell with. He writes that it began at vi h. 49', and ended at vii h. 47'. this latter very exactly, tho' not quite so nice as to the Beginning.

Had this Observer signified what Difference of Meridians there was found between the Place of Observation and the West End of Forte ventura, we might, without sensible Error, have concluded the true Longitude, not only of that Island, but also of the Pike of Teneriff, where Our Geographers and the Dutch have fixed their first Meridian. But this Gentleman being both able and desirous to render the Publick this sort of Service, we hope from him such further Observations as may put the matter past dispute. He adds that the greatest Darkness was about ⅓ of the Sun's Diameter, or nine Digits on the North side.

From
From Germany we have received the following Accounts.

At Nuremberg.

The Beginning and greatest Obscurity could not be seen for Clouds, but the End happen’d at xi h. 10'.

At Hamburg.

The Beginning was observed at viii h. 57'. The greatest Obscurity at x h. 5'. 30'', when xi ½ digg. were darkned. The End could not be seen for Clouds.

At Kiel in Holstein.

The Beginning ix h. 4'. The greatest Obscurity x h. 19'. 20'', and the Quantity then eclipsed xi. digg. 20'. The End was at x h. 29'.

At Berlin.

The Beginning could not be seen for Clouds, but the greatest Obscurity was at 22 min. past Ten, when xi digg. were eclipsed. The just End was at x h. 34'.

At Frankfort on the Meine.

The Eclipse began at vii h. 50'. The greatest Darkness at x h. 11', but perhaps should be x h. 01 min. the Digits being x. and 34 min. The End was observed at 10 min. past Eleven.

By whom these Observations were made, and with what Instruments, we are not as yet informed, but hope they may be exact enough to confirm the Longitudes of those several places, which are at present reasonably well known.

Since these there is lately come to Hand a Dutch Print entitled Nouvelles Litteraires, publish’d at the Hague, wherein, pag. 404. 405, there is an Account of the Observation of this Eclipse at Upsal in Sweden, made by M. Jo. Waller, Professor of Mathematicks in that University, who was very careful to observe it exactly; the Times being verified by three Clocks perfectly agreeing with one another and with the Sun: but more especially by
by a Quadrant of 5 Foot Radius for taking the Sun's Altitude. By this Instrument he has determined the Height of the Pole at Upfall 59°. 51' 54''. And by the same, a little before the Beginning of the Eclipse he found the Height of Sun 39°. 36'. 42''. His Clocks then shewing the Hour 1x h. 47'. 50'', which proves that they were very near the true Time. At x h. 58'. 15'', the Altitude of the Sun being 44°. 17'. 29'', was the Beginning of the total Darkness, and at xi h. 2'. 24''. was the End thereof, also sole 44°. 29'. 13''. So that here the Duration of the total Eclipse was 4'. 9'', and the Middle thereof but one third of a Minute after Eleven. And lastly the End is said to have happen'd about 4 Minutes before Noon, the Sun being 45°. 42'. 6''. But in this is a manifest Mistake, for it makes the Time of Emerfion, or from the Middle to the End, but 55'. 20''; whereas being so near the Meridian, 'tis certain that this Emerfion was the greater part of the Duration of the whole Eclipse, and consequently more than an Hour. Perhaps the Times might be deduced from the Altitudes only, and then the Mistake might be in supposing the End so much before Noon as it was really after it. However, to prevent all Doubts, we have compared this Observation with what we observed of this Eclipse at London, and find that in the Latitude of 59°. 50', the Place where the Middle of total Darkness was at xi h. 0'. 20'', was near 19 Degrees more Eaftely than London (that is exactly in the Meridian of Dantzick) and that the Eclipse began there at 1x h. 52'. 1/2, and ended at xi h. 10''. Wherefore the Duration could not be 2h. 7'. 50'', as the Editor of the said Nouvelles has publish'd; not considering that the Beginning could not be seen for Clouds, as in the very next Words he assures us.

As to the Darkness, it was such that they could scarce distinguish one another: and besides Jupiter, Mercury and Venus; of the Fixt Stars Cassiopea, Capella, Oculus Tauri and Orion, (Sirius not being yet risen) were visible.
II. An Account of a Journey from the Port of Oratava in the Island of Tenerife to the Top of the Pike in that Island, in August last; with Observations thereon by Mr. J. Edens.

On Tuesday August the 13th N. S. at Half an Hour past Ten in the Evening, I, in company of four more English and one Dutch-Man, with Horses and Servants to carry our Provision, together with a Guide (which is the same that has conducted all those that have been this Journey for many Years) set forward from the Port of Oratava. The Night being somewhat cloudy, and the Moon in the full at 12 the Night following.

At half an Hour past Eleven we came to the Town of Oratava, which is about two Miles from the Port, where we stoppt for about half an Hour, to get walking Staves to assist us in our ascending the Steep of the Pike.

At One a-Clock on Wednesday Morning we came to the Foot of a very steep Rising, about a Mile and half above the Town of Oratava, where it began to clear up; and we saw the Pike with a white Cloud covering the Top of it like a Cap.

At Two a-Clock we came to a plain place in the Road which the Spaniards call Dormajito en el Monte verde (the little Trough in the green Mountain) so call'd I suppose because a little below this Plain, on the Right hand as we went, there is a deep Hollow; at the upper End of which Hollow, there is a Spout of Wood placed in a Rock, through which there runs very clear and cool Water, which comes from the Mountains; and at a Descent a little lower than the Spout there is a Trough into which the Water comes. E e e

At
At Three, after travelling a Road, which was sometimes pretty smooth and at other times very rough, we came to a little wooden Cross, by the Road side on the Left-hand, which the Spaniards call la Cruz de la Solera (The Cross of the Solera) A Solera is a long Pole with a Hole at each end, which the Spaniards use to draw Wood with, by fastening one end to the Wood and the other to the Oxen. This Cross was made with a Piece of a Solera, and for that reason is so call’d, but why it was set up in this place I can’t tell, unless it was because somebody was kill’d thereabouts. At this place we also saw the Pike before us; and although we had come up hill quite from the Port, yet to our thoughts it seem’d almost as high here as when we were there, the white Cloud still hiding the greatest part of the Sugar-loaf.

After riding about half a Mile further, we came to the side of a Hill which was very rough and steep, (the place call’d Caravala ; where are a great many Pine Trees that grow on both sides the Road for a great way, both on the Right-hand and the Left, one of which that was close to the Road, on the Right-Hand as we went, our Guide desired us to observe; it having a great Branch growing out, which with the Boughs that were upon it look’d like the Forepart of a Ship. And from the likeness this Tree has to a Ship I suppose the Place took its Name, for Caravela signifies an old-fashioned Vessel formerly much used in Spain, sharp before, ill shap’d every Way, and all the Masts slooping forwards ; their Sails are all Mizen Sails, that is, Triangular; they will lye nearer the Wind than other Sails, but are not so commodious to handle. Amongst these Trees, not a great Height in the Air, we saw the Sulphur discharge itself like a Squib or Serpent made of Gun-powder, the Fire running downwards in a Stream, and the Smoak ascending upwards from the place where it first took Fire; and like this we saw another,
ther, whilst we lay under the Rocks the next Night at la Stancha, part of the way up the Pike; But I could not ob-
serve whether either of them gave any Report as they discharg'd.

At three Quarters after Four we came to the Top of
this high rough and steep Mountain, where grows a Tree
which the Spaniards call el Pino de la Merenda, The Pine-
Tree of the Afternoon's Meal. This is a large Tree, and
is burnt at the Bottom, as having had Fires made against
it; and in the burnt place there issues out Turpentine, a
little of which I brought with me. At a few Yards di-
stance from this Tree we had a Fire made, where we
stay'd and baited our Horses, and breakfasted our selves.
These Hills are very sandy, and there are a great many
Rabbets which breed there; There is also much Sand
found a great way up the Pike itself, and not a great way
below the Foot of the Sugar-loaf, some of which I brought
down with me.

At Three Quarters after Five we set forwards again,
and at Half an Hour past Six came to the Portillo, which
in Spanish signifies a Breach or Gap. We saw the Pike
about two Leagues and a half before us, cover'd still
with a Cloud at Top; and the Spaniards told us we were
come about two Leagues and a half from the Port.

At half an Hour past Seven we came to las Faldas, that is
the Skirts of the Pike; from whence all the way to la
Stancha, which is about a Quarter of a Mile up from the
Foot of the Pike, we rode upon little light Stones, for
the most part not much bigger than ones Fift; and a great
many not much broader than a Shilling: and if we kept
the beaten Track which was used before, it was not so
deep, but if we turn'd out of it the Horses went almost
over their Feet. I lighted and made a Hole there, think.
ing to find how deep these little Stones lie, but could

E e e 2
not find the Bottom; which makes me conclude they may cover the Ground for a great thickness.

There are a great many vast Rocks, some of them two Mile or thereabouts from the Foot of the Pike, which the Pike-Man told us was cast out from the Top of the Pike at the time it was a Vulcano; many of them lye in Heaps of above threescore Yards long, and I observ’d that the further these Rocks lye from the Foot of the Pike, the more like they are to the Stone of other common Rocks: But the nearer we went to the Pike we found them more black and solid; and some of them, tho’ not many, were glossy like Flint, and all extream heavy. Thofe that fhone so, I suppose, retain’d their natural Colour, but there are some that look like Dross that comes out of a Smith’s Forge, which without doubt was occasion’d by the extream Heat of the place they came from.

Some of these great Rocks were thrown out of the Cal-dra or Kettle in the Top of the Pike; and others from a Cave or Ciftern which is a pretty way up the side of the Pike, and has by fome been thought to have no Bottom, more of which I shall lay anon.

At Nine on Wednesday Morning we arrived at la Stanche, about a Quarter of a Mile above the Foot of the Pike on the East-side, where are three or four large hard and solid black Rocks lodg’d: under some of these we put our Horses, and under others we lay down ourfelves to fleep, after having refresh’d ourfelves with a little Wine: and we had a fire made in order to get our Dinner ready, where a Cook we took along with us both roasted and boil’d our Meat and Fowls very well. We flept here for about two Hours, then rose again, and at about Two in the Afternoon went to dinner.

There are feveral Mountains that lye Eastward from the Pike at four or five Miles Distance, call’d the Malpefes, and one more lying a little more to the Southward call’d
la montaña de rejada; all which were formerly Vulcanos, tho' not so great as that of the Pike, as appears by the Rocks and small burnt Stones that lye near them, just in the same manner as about the Pike.

Still being at la Stancha, after we had dined we lay down again to take a Nap, under the Rocks as before Dinner, but not sleeping very well we all got up again, the rest of them spending the Afternoon at Cards, &c. But I made it my business to admire the strangeness and vastness of that great Body, which indeed is very wonderful, insomuch that its impossible to express to one that has never seen it, in what a manner that confused Heap of Rubish lyes; for it may very well be stiled one of the greatest Wonders in the World. About Six at Night we saw Grand Canaria from la Stancha bearing from us E by N.

At Nine at Night, after having had our Suppers, we retired to our former Lodgings, where laying Stones for our Pillows and our Cloaks for Bed-cloaths, we endeavored to get to sleep, but all in vain for a great while. Some lying pretty nigh a Fire complain'd of being burnt on the one side and froze on the other (for the Air was very cutting and sharp) Others happening to lye in a place where there was a great many Fleas; though it be something strange that Fleas should be found there, the place being so cold in the Night; perhaps the Goats sometimes get under these Rocks and so leave them; and I am inclin'd to believe it, because the Guide and I found a dead Goat in a Cave at the very top of the Pike. I suppose this Goat straggling up here by chance was benighted, and so finding the Cold got into this place for Heat, where meeting with too much of it and a very strong Sulphurous Vapour it overcame him; for he was almost dried to Powder. But to proceed, betwixt Eleven and Twelve we got to sleep, and slept till One, when waking, our Guide told us.
us, 'twas time to prepare for our Journey. We immediately rose, and by half an Hour past One we were all upon the march; and leaving our Horses and some of our Men behind, we went away fasting, excepting about two Mouthfulls of Wine apiece, which we took at our uprising. Betwixt la Stanchez and the Top of the Pike there are two very high Mountains and the Sugar-Loaf, each of which Mountains is almost half a Mile's walking; on the first of them the Rubbish is more small, and we were apt to slip back as we went upwards. But the uppermost is all composed of hard loose rocky great Stones, cast one among another in a very confused Order. After resting several times, we came to the Top of the first Mountain, where we drank every one of us a little more Wine, and eat each of us a Bit of Ginger-Bread we had amongst us. Then being pretty well refresh'd, we set forwards again to ascend the second Mountain, which is higher than the first, but is better to walk on, because of the firmness of the Rocks. After we had travel'd for about half an Hour up the second Mountain, we came within sight of the Sugar-Loaf, which before we could not see by reason of the Interposition of these great Hills. After we were arriv'd to the Top of this second Mountain we came to a way that was almost level, but bearing some small matter up-hill; and about a Furlong farther is the Foot of the Sugar-Loaf, which we soon after came to. Then looking upon our Watches found it to be just three a-Clock. The Night was clear where we were, and the Moon shone very bright, but below over the Sea we could see the Clouds, which look'd like a Valley at a prodigious Depth below us. We had a brisk Air, the Wind being S. E. by S. as it was for the most part whilst we were upon our Journey.

Whilst we sat at the Foot of the Sugar-Loaf, resting and refreshing our selves as before in other places, we saw
the Smoak break out in several places, which at first look'd like little Clouds, but they soon vanish'd, others not long after coming in their room from the same or other places.

We set forwards to ascend the last and steepest part of our Journey, viz. the Sugar-Loaf, exactly at half an Hour past Three, and after we had rested twice or thrice, I left the Guide and the rest of my Company, and ran forwards; and when I was got very nigh the Top (which was at three quarters after Three) two more of the Company deserted, and came up about Five Minutes after me; the rest of the Company and the Guide coming up to the Top just at Four.

The Shape of the Top of the Pike is partly oval, the longest Diameter lying N. N. W. and S. S. E, and is as nigh as I could guess, about 140 Yards long; the Breadth the other way being about 110. Within the Top of the Pike is a very deep Hole call'd the Caldera (or Kettle) the deepest part of which lyes at the South End: It is I believe 40 Yards deep, reckoning from the highest side of the Pike; but it is abundance shallower reckoning from the side opposite to Garachica. The sides of this Kettle are very steep, in some places as steep as the Descent on the outside of the Sugar-Loaf. At the Bottom of this Kettle we all were, where lye a great many very large Stones, some of them higher than our Heads. The Earth that is within side the Kettle, being roll'd up long and put to a Candle, will burn like Brimstone. Several places within side the Top of the Pike are burning, as on the Outside; and in some places if you turn up the Stones you'll find very fine Brimstone or Sulphur sticking to them. At these Holes where the Smoak comes out there also comes forth a great Heat, so hot that one cannot endure one's Hand there long. At the N. by E. side within the Top is the Cave where we found the dead Goat; in which Cave
Sometimes the true Spirit of Sulphur distills, as they say, but it did not drop whilst I was there.

The Report is false about the Difficulty of breathing upon the Top of this place; for we breath'd as well as if we had been below; we eat our Breakfast there, and I was up in all for about two Hours and a quarter.

Without doubt the Quicksilver would have fell very much upon this high place, if I had had but the good fortune to have got a couple of Barometers to try. But there is no such thing in this Island, and I was fearful of not getting Company in the mind to go up with me another Year (for to go up by ones self is very chargeable) else I would have sent to England to have been supply'd, tho' the Expence had come all out of my own Pocket.

Before the Sun rose I think the Air was as cold as I have known it in England, in the sharpest Frost I was ever in; I could scarcely endure my Gloves off. There was a great Dew all the while we were there till Sun rising, which we could find by the Wetness of our Cloaths; but the Sky look'd thereabouts as clear as possible.

A little after Sun rising we saw the Shadow of the Pike upon the Sea, reaching over the Island of Gomera; and the Shadow of the upper part, viz. of the Sugar-loaf, we saw imprinted like another Pike in the Sky itself, which look'd very surprizing: but the Air being cloudy below us, we saw none of the other Islands but Grand Canaria and Gomera.

At six on Thursday Morning we came down from the Top of the Sugar-Loaf; at seven we came to the Cistern of Water which is reported to be without Bottom: this the Guide saith is false, for about seven or eight Years ago, when there was a great Vulcano in this Country, the Cave was dry and he walk'd all about it, and said that the deepest part of Water, when we were there, was not above two Fathoms.
The Dimensions of this Cave I guess to be as follows.

Length about 35 Yards
Breadth —— 12
Ordinary Depth 14 from Top to Bottom.

Upon the furthest side grows white stuff, which the Pike-man told us was Salt-Peter. There was both Ice and Snow in it when we were there: and the Ice was of a great Thickness covered with Water about Knee deep. We let down a Bottle at the End of a String for some of the Water, in which we put some Sugar and drank it, but it was the coldest I ever drank in my Life. The Ice was broken just under the Mouth of it, where we could see the Stones lye at the Bottom, for it was very clear. A little to the Right-hand within this Cave the Ice was risen up in a high heap, in form of a Spire Steeple or like a Sugar-Loaf; and in this place I believe the Water comes in. I should have been glad to have come at it, to let down a Line to try whether there may not be some Hole that the Guide knows not of, that may be of a great Depth.

In our way home, we came by a Cave three or four Miles from the Pike, where are a great many Skeletons and Bones of Men; and some say there are the Bones of Giants in this Cave, but we know not how many Bodies are laid here, nor how far the Cave may go. I intend (God willing) to go again before I leave the Island, and then I'll take a Light with me and see what Discoveries I can make.

We came home to the Port at about six a-Clock this Evening, being Thursday August 15, 1715. N S.

Whoever reads this, I hope, will pardon the Faults my Pen may have committed, for I was forc'd to write all Night; the Ship being to fail the next Morning, and I have not time to examine it.
III. Ventriculus cordis sinister stupendae magnitudinis, lately communicated to the Royal-Society by James Douglas, M. D. and R. S. S.

I lately opened a young man in St. Bartholomew's Hospital, that died of the Palpitation of the Heart, whose violent beating and prodigious subsultory Motion, for some months before his Death, was not only easily felt by laying the Hand on the Region of the Heart; but seen to rise and fall by raising the Bedcloaths that covered it. And, which is almost incredible, at sometimes the trembling and throbbing made such a Noise in his Breast, as plainly could be heard at some Distance from his Bed-side. This was accompanied with frequent Deliquiums; sometimes slow, sometimes swift, and often intermitting.

Johannes Fernelius in his Pathologia lib. 5. cap. 12: gives us an Observation of a very uncommon and surprising Case of this kind; where he says the frequent Concussion of the Heart was so violent and powerfull, as not only to displace or luxate, but even to break some of the adjoining Ribs.

Franciscus de la Boe Sylvius, another Writer of unquestionable Integrity, has a parallel Observation in his Account of this Disease.

Theodorus Kerkringius relates the History of a Woman he opened, whose Heart was of a prodigious Bigness; in his Spicilegium Anatomicum, Obs. 16.

And to mention no more, Monsieur Dionis, at the End of his Anatomy, gives a large Description of a very uncommon Case, in which the right Auricle of the Heart was prodigiously dilated to the Bigness of the Head of a new born child.
In the Dissection of this morbid Heart I observed the following remarkable Particulars.

1. That the Pericardium or Capsula Cordis was very thick, and firmly adhered or grew by a fibrous Connexion to all the outer Surface of the Heart.

2. Instead of the Water called Liquor Pericardii, there was only in some places about the Basis of the Heart a mucilaginous clear Substance like a Gelly.

3. In the right Auricle lay'd open there was nothing preternatural. The ascending and descending Cava opened into the same as usual. The Vestigium or Mark of the Foramen ovale with its semicircular limbus was very plain. And the Orificium of the Vena Cordis Coronaria was extramly large, yet its Valve was less than usual.

4. In the right Ventricle layed open, the Valvulae called tricuspides were configure after the usual manner. The sides of this Cavity were thin and full of small fleshy Columnæ as they commonly are, with great variety of Furrows and little holes. The three sigmoide or semilunar Valves in the Mouth of the arteria pulmonalis, were as they always are in a natural State.

5. The left Auricle was not much bigger than ordinary: but its muscular Appendage, called the Bulb of the Pulmonary Vein by the late Mr. Cooper, was extraordinariy dilated and enlarged beyond any thing that I ever saw.

6. The left Ventricle, whose Capacity in a natural State is always less than the right, was here considerably larger. And if the Experiment had been made, before Dissection, of filling both with any Liquor, this had certainly contained three times more than the other.

7. The Valvulae called Mitræ, placed at the Orifice of this Ventricle, are much thicker in Substance than ordinary; and the two fleshy Columns, called by Nicolaius Massa, almost 200 Years ago, duo parvi musculi, which send out

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abundance of small Tendons to be inserted into these Valves, were proportionally augmented in Bigness.

8: The semilunar Valves in the Mouth of the Aorta, or of that great Vena pulsatilis, that dispenses the Blood to all the several parts of the human Body, were very much preternaturally affected; as would easily appear upon comparing them with those in the Orifice of the pulmonary Artery, in which they are thin and very broad, so as to be able to shut the Cavity of that Vessel, and hinder the Blood from returning back into the Ventricle, and likewise transparent: but in this they are very thick, contracted as it were, and furled together, and of a whitish Colour; and in all appearance, if the Person had lived longer, they had turned boney, or undergone a Petrification.

This uncommon Structure of the Heart being thus demonstrated, let us endeavour to account for the following Phænomena. The first is the Palpitation of the Heart, which was the chief Symptom and Complaint of the sick Person. The second is the preternatural Dilatation and Enlargement of the left Ventricle. It is not improbable but the firm adhesion of the Capsula Cordis membranosæ to the substance of the Heart, occasioned that uncommon trembling and throbbing thereof: its free and easy Motion being hindered by that thick involucrum which surrounded it so close on each side. The learned Dr. Lower, in his elaborate Treatise de Corde humano gives us such an instance; and explains the Palpitation after this manner:

As for the second, viz. the Dilatation of the left Ventricle and muscular Bag of the Pulmonary Vein; that is altogether owing to the ill configuration of the Valves we have now described: for as the great Artery or Aorta arises out of this Ventricle, it has three Valves which separating give passage to the Blood from the Ventricle into the
the Vessel; and in a natural State they shut that Passage, and so prevent the Blood from recoiling into the same, if it should endeavour to return. But in this case, by reason of its contracted Narrowness and Thickeness, not being able to close or shut the Passage, the Blood flowed back again into the Cavity, which it had gradually enlarged, and dilated to the Bigness we see. Besides the Muscular Valves not being duly qualified for the Performance of their Office, the Blood recoiled into the Auricle, which it had distended in the like manner. This constant Regurgitation or Reflux of the Blood is besides sufficient of its self, to produce this extraordinary trembling or ἀλτήμος παρθενίως, as the Greeks call it.

IV. A ready Description and Quadrature of a Curve of the Third Order, resembling that commonly call'd the Foliate. Communicated by Mr. Abr. de Moivre, F. R. S.

I have look'd a little farther into that Curve which fell lately under my consideration. It is not the Foliate as I did at first imagine, but I believe it ought not to make a Species distinct from it. \( AEB(\text{Fig.1.}) \) is the Curve I thus describe. Let \( AB \) and \( BK \) be perpendicular to each other. From the point \( A \) draw \( AR \) cutting \( BK \) in \( R \), and make \( RE = BR \); the point \( E \) belongs to the Curve. Draw \( BC \) making an Angle of 45 grad. with \( AB \), this Line \( BC \) touches the Curve in \( B \); from the point \( E \) draw \( ED \) perpendicular to \( BC \), and calling \( BD, x; DE, y; AB, a; \) and making \( \sqrt[3]{8aa} = n \), the Equation belonging to that Curve is \( x^3 + xxy + xyy + y^3 = nxy \) or \( \frac{x^3}{x-y} = nxy \). Taking \( BG = AB \), and drawing \( GP \) perpendicular to \( BG \), \( PG \) is an Asymptote. In the Foliate the...
the Equation is \( x^3 + y^3 = \frac{1}{n} x y \), in which the two Terms \( x x y + x y y \) of the former Equation are wanting; and its Asymptote is distant from \( B \) by \( \frac{1}{2} B A \). Again draw \( EF \) perpendicular to \( AB \): let \( BF \) be called \( z \) and \( FE \) \( v \); the Equation belonging to the Curve \( AEB \) is \( vv = \frac{axz - z^3}{a + z} \). In the Foliate the Equation is \( vv = \frac{azz - z^3}{a + 3z} \).

From these two last Equations it seems that these Curves differ no more from one another than the Circle from the Ellipsis. I should be very glad to know your Opinion thereupon.

The Quadrature of the Curve here described has something of Simplicity with which I was well pleased. With the Radius \( BA \) and Center \( B \) describe a Circle \( AKG \), let the Square \( HPSI \) circumscribe it, so that \( HP \) be parallel to \( AG \): prolong \( FE \) till it meet the Circumference of the Circle in \( M \), and through \( M \) draw \( LMQ \) parallel to \( HP \). The Area \( BFE \) is equal to the Area \( KHLM \), comprehended by \( KH \), \( HL \), \( LM \) and the Arc \( KM \). And the Area \( Bfe \) is equal to the Area \( KMLH \) or \( KMPQ \). Therefore if \( BF \) and \( Bf \) are equal, the two Areas \( BFE \), \( Bfe \) taken together are equal to the Rectangle \( H2 \), and therefore the whole Space comprehended by \( BEXB \) is \( IGDZ \) (supposing \( I \) and \( Z \) to be at an infinite Distance) is equal to the circumscribed Square \( HS \).

N.B. This Quadrature is easily demonstrated from the Equation: for by it \( a + z :: a - z :: zz :: vv \), that is \( AF : EF :: MF : FB \), and so \( EF \) the Fluxion of \( AF \) to \( L \) is \( EF \) of \( MF \). Hence the Areola \( E \) \( F \) \( \phi \) of \( e \) will be always equal to the Areola \( M \) \( L \) \( \mu \), and therefore the Area \( AEF \) always equal to the Area \( MAL \).

Hence it appears that this Curve requires the Quadrature of the Circle to square it; whereas the Foliate is exactly quadrable, the whole Leaf thereof being but one third of the Square of \( AB \), which in this is above three sevenths of the same. Again in
in our Curve, the greatest Breadth is when the Point \( E \) divides the Line \( AB \) in extrem and mean Proportion: whereas in the Foliate it is when \( AB \) is triple in power to \( BF \). And the greatest \( EF \) or Ordinate in the Foliate is to that of our Curve nearly as 3 to 4, or exactly as \( \sqrt{3} \sqrt{3} - \frac{1}{3} \) to \( \sqrt{5} \sqrt{2} - 5 \frac{1}{4} \).

But still these Differences are not enough to make them two distinct Species, they being both defined by a like Equation, if the Asymptote \( SGP \) be taken for the Diameter. And they are both comprehended under the fortieth Kind of the Curves of the third Order, as they stand enumerated by Sir Isaac Newton, in his incomparable Treatise on that Subject.

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IV. An easy Mechanical Way to divide the Nautical Meridian Line in Mercator's Projection; with an Account of the Relation of the same Meridian Line to the Curva Catenaria. By J. Perks, M. A.

The most useful Projection of the Spheric Surface of Earth and Sea for Navigation, is that commonly call'd Mercator's; tho' its true Nature and Construction is said to be first demonstrated by our Countryman Mr. Wright, in his Correction of the Errors in Navigation. In this Projection the Meridians are all parallel Lines, not divided equally, as in the common plain Chart (which is therefore erroneous,) but the Minutes and Degrees (or strictly, the Fluxions of the Meridian,) at every several Latitude are proportional to their respective Secants. Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the Secant of the same Latitude is to Radius.
The Reason of which Enlargement of the Elements of Latitude is, to counterbalance the Inlargement of the Degrees of Longitude. For in this Projection, the Meridians being all parallel, a Degree of Longitude at (suppose) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globes Surface) but half as much, the Radius of the Parallel of 60 Deg (that is. its Cosine) being but half the Radius of the Equator. Therefore to proportion the Degrees of Latitude to those of Longitude, a Degree (or Elemental Particle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, viz. its Cosine.

In Fig. 3. let the Radius $CD$ represent half of the Equator, $DM$ an Arc of the Meridian; $MS$ its Sine, $CE$ its Secant; then is $CS$ equal to its Cosine: and $CS : CM :: CD (= CM) : CE$, that is, as Cosine : to Radius :: so is Radius : to Secant. The Cosines being then, in this Projection, suppos'd all equal to Radius, or (which comes to the same) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every point of Latitude, (by the precedent Analogy) is suppos'd equal to the Secant of Latitude; and consequently the Elements (Minutes, &c.) of the Meridian must be proportional to their respective Secants.

The Way Mr. Wright takes for making his Table of Meridional Parts, is by a continual Addition of Natural Secants, beginning at 1 Minute, and so proceeding to 89 Deg. Dr. Wallis (in Phil. Trans. No. 176.) finds the Meridional Part belonging to any Latitude by this Series, putting $S$ for its Natural Sine, viz. $S + \frac{1}{2}S^3 + \frac{1}{2}S^5 + \frac{1}{2}S^7 + \frac{1}{2}S^9 &c.$ which gives the Merid. part required: How to find the same Mechanically by means of an easily-constructed Curve Line, is what I shall now shew.
1. Prepare a Ruler $AB$ (Fig. 2.) of a convenient Length, in which let $B$ be equal to the Radius of the intended Projection. To the Point as a Center (on the narrower Edge of the Ruler) fasten a little Plate-Wheel $v^b$ and a little more than the thickness of the Ruler. Let $KR$ (Fig. 3.) represent another long Ruler, to which $AR$ is a perpendicular Line. Place the Ruler $AB$ upon the Line $AR$, with the Center of the Wheel at $A$. Then with one Hand holding fast the Ruler $KR$, with the other Hand slide the end $B$ of the Ruler $AB$ by the Edge of $KR$; so will the little Wheel $v^b$ describe on the Paper a Curve Line $ACB$, to be continued as far as is convenient.

2. Having drawn the Curve $ACB$, draw a straight Line $KR$ by the Edge of the Ruler $KR$, which Line is the Meridian to be divided, and also an Asymptote to the Curve $ACB$.

3. In this Meridian, (accounting $R$ to be the Point of its Intersection with the Equator,) the Point answering to any Degree of Latitude is thus found. In the perpendicular $AR$, make $RG$ equal to the Cosine of Latitude (Radius being $AR$) and from $G$ draw $GC$ parallel to $KR$, and intersecting the Curve in $C$. With Center $C$ and Radius $CM = AR$, strike an Arc cutting the Meridian at $M$; so is $M$ the Point desired.

4. In the Curve $AC$, let $c$ be a Point infinitely near to $C$, and $cm$ ($= CM$) a Tangent to the Curve at $c$, making the little Angle $MCm$, to which let the Angle $RAR$ be equal; so is $R = MA$ (a Perpendicular from $M$ to $cm$.) Draw $CD$ equal and parallel to $AR$, intersecting $KR$ in $S$. With Center $C$ and Radius $CD$ draw the Arc $DM$, and its Tangent $DE$ and Secant $CE$.

5. Because of the like Triangles $CDE$, $MDm$; $CD : CE : MD : MM$, that is, as Radius to Secant of the Arc $DM$, (whole Cosine is $CS = GR$,) $: : SO$ is $MD$
( = R \ r \ a \ Degree \ or \ Particle \ of \ the \ Equator:) \) to \ Mm \ the \ Fluxion \ or \ corresspondent \ Particle \ of \ the \ Meridian \ Line \ R \ M. \ Whence, \ and \ from \ what \ is \ premised \ concerning \ the \ Nature \ of \ this \ Nautical \ Projection, \ tis \ evident \ that \ R \ M is the meridional \ Part \ answering \ to \ the \ Latitude \ whose \ Cosme \ is \ G \ R. \ Or \ thus; \ With \ Center \ R \ and \ Radius \ A \ R \ describe \ the \ Quadrant \ A \ x \ a, \ in \ which \ let \ the \ Arc \ A \ x \ be equal \ to \ the \ given \ Lat. \ From \ x \ draw \ \ x \ C \ parallel \ to \ R \ R, \ and \ intersecting \ the \ Curve \ in \ C, \ so \ is \ C \ x \ the \ Meridional \ Part \ defir'd \ being \ equal \ to \ R \ M, \ as \ is \ easy \ to \ shew.

As \ to \ the \ other \ Properties \ of \ this \ Curve, \ tis \ evident, \ from \ its \ Construcion, \ that \ its \ Tangent \ (as \ C \ M) \ is \ a Constant \ Line \ every \ where \ equal \ to \ A \ R; \ the \ Curve \ being \ generated \ by \ the \ Motion \ of \ the \ Wheel \ at \ the \ End \ of \ the \ Rular \ which \ is \ its \ Tangent. \ And \ from \ hence \ the \ Curve \ A \ C \ B may, \ for \ distinction, \ be \ call'd \ the \ Equi-tangential \ Curve.

7. \ The \ Fluxion \ of \ the \ Area \ A \ R \ M \ c \ is \ the \ little \ Sector \ or \ Triangle \ M \ C \ d, \ which \ same \ is \ also \ the \ Fluxion \ of \ the \ Sector \ C \ D \ M: \ whence \ the \ Areas \ A \ R \ M \ c, \ C \ D \ M \ are \ equal, \ and \ the \ whole \ Area \ A \ C \ B \ &c. \ K \ M \ R \ being \ infinitely \ continued, \ is \ equal \ to \ the \ Quadrant \ A \ R \ a:

8. \ To \ find \ the \ Radius \ of \ Cutvature \ of \ any \ Particle, \ as \ C \ c, \ from \ C \ draw \ an \ indefinite \ Line \ C \ T \ perpendicular \ to \ C \ M, \ (on \ the \ concave \ side \ of \ the \ Curve) \ and \ from \ c \ another \ Line \ perpendicular \ to \ c \ m, \ which \ Lines, \ (because \ of \ the \ Inclination \ of \ C \ M \ to \ c \ m,) \ will \ somewhere \ meet \ as \ at \ T, \ making \ an \ Angle \ C \ T \ c = M \ c \ m. \ These \ Angles \ being \ equal, \ their \ Radii \ are \ proportional \ to \ their \ Arcs: \ therefore, \ M \ d : C \ c :: M \ C : C \ T. \ But \ C \ c = \ a \ m \ (because \ of \ C \ M = c \ m) \ so \ that \ M \ d : a \ m :: C \ D : D \ E; :: C \ M : C \ T. \ But \ C \ D = C \ M, \ therefore \ C \ T = D \ E = Tangent \ of \ the \ Arc \ D \ M.
9. So that supposing $AT$ a Curve Line in which are all the Centers of Curvature of the Particles of $ACB$, any point as $T$ being found as before, the Length $AT$ (by the nature of Evolution of Curves,) is everywhere equal to the Tangent of its correspondent Circular Arc $DM$. The Point $T$ is also found by making $MT$ perpendicular to $RM$, and equal to the Secant $CE$: for so is the Angle $CMT = MCD$, and the Triangle $MCT$ equal to the Triangle $CDE$.

10. Let $AH$ be an Equilater Hyperbola whose Semi-axis is $AR$ and Center $R$. In the Meridian let $RP$ be equal to the Tangent $DE$. Join $AP$, and draw $PH = AP$ and parallel to $AR$. Compleat the Parallelogram $HNRP$, so will the Point $H$ be in the Hyperbola, and its ordinate $HN (= RP - DE = CT)$ be equal to the Curve $AT$: From whence, and from Prop. 3 Coroll. 2. of Dr. Gregory's Catenaria (Phil. Trans. N. 231.) it appears that the Curve $AT$ is that call'd the Catenaria or Funicularia, viz. the Curve into whose Figure a slack Cord or Chain naturally disposes its self by the Gravity of its Particles.

11. Hence we have another Property of the Catenaria not hitherto taken notice of (that I know of,) viz. that supposing $AR (= a)$, the constant Line in Dr. Gregory) equal to the Radius of the Nautical Projection, and $RN$ the Secant of a given Latitude, then is $NT$ the Catenaria's Ordinate at $N$, equal to $RM$ the Meridional Part answering to the Latitude whose Secant is $RN$.

12. That $TA$ is the Catenaria is also demonstrable from Dr. Gregory's first Prop. Let $Tu$ be the the Fluxion of the Ordinate $NT$; and $tu (= Nn)$ the Fluxion of the Axe $AN$. Then because of like Triangles $TCM$, $Tut$, $CM : CT (= TA) :: Tu : ut$, that is, as $CM$ a constant-Line to $TA$ the Curve :: so is the Fluxion of the...

Ordi-
Ordinate, to that of the Axe \((y:x)\), according to Prop. 1. Catena.

13. From the Premises the Construction and several Properties of the Catena are easily deducible; one or two of which I'll set down.

1. The Area \(AM \times R\) is equal to \(AOP \times R\) a Rectangle contained by Radius \(AR\) and \(RP\) the Tangent answering to Secant \(HP = TM\). For because of the like Triangles \(CMm, CEe; CM : CE :: Mm : Ee\), that is, putting \(r, s, t, m\) for Radius, Secant, Tangent and Meridional part \(RM\). \(r : s :: m\); whence \(r = sm\), and all the \(r : s :: m\), that is \(AOPR = AM \times R\), which agrees with Dr. Gregory's Cor. 5. of Prop. 7.

14. Supposing the former Construction, let be added the Line \(RH\), including the Hyperbolic Sector \(ARH\). I say the same Sector is equal to half the Rectangle \(ARMQ\) contained by Radius \(AR\) and the Meridional Part \(RM\), \((\div r \times m\). For the Sector \(ARH = \) Triangle \(RNH\) wanting the Semileggment \(ANH\). The Fluxion of the

Triangle \(RNH\) is \(\frac{s + t}{2}\). The Fluxion of \(ANH\) is

\(\frac{t}{2}\). So the Fluxion of the Sector \(ARH\) is \(\frac{r + s}{2}\).

\(\frac{t}{2}\) \(\div \frac{s}{2}\) \(\div \frac{r}{2}\). 'Tis found before (Sec. 13.) that

\(r : s :: \frac{m}{s} \div \frac{t}{s}\). Whence \(s = \frac{m}{s}\). And because of the like Triangles \(CD, DE, EF\).

But \(EF = MM = m\), because both \(EF\) and \(MM\) are to \(MD\) in the same Reason, \(viz.\) as \(s\) to \(r\); therefore \(r : t :: \frac{t}{r} \div \frac{s}{r}\) \(\div \frac{r}{s}\). whence \(t : s = \frac{m}{s}\), and \(\frac{t}{s} = \frac{m}{s}\).
the Fluxion of the Hyperbolic Sector $ARH$, whose flowing Quantity is, therefore equal to $\frac{1}{r} m = \frac{1}{AR} MQ$. Q.E.D.

15. This shews another Property of the Catenaria, viz. that it squares the Hyperbola; for $RM$ is equal to $NT$ the Ordinate of the Catenaria.

16. In Fig. 4. Let $AR$ be Radius, $ACB$ the Equitangential Curve; $MRN$ its Asympotote, in which let $M$, $N$, be any two Points equally distant from $R$. Upon $M$ draw $ML$ parallel to $AR$ and equal to the Difference of the Secant and Tangent of that Latitude, whose Meridional Part is $RM$ (by § 13). Upon $N$ draw $NO$ parallel to $AR$, and equal to the Sum of the forefaid Secant and Tangent. Do thus from as many Points in the Asymptote as is convenient, and a Curve drawn equably through the Points $L$, $A$, $O$, &c. will be a Logarithmic Curve, whose Subtangent (being constant) is equal to Radius $AR$, to no small Extent of Figure. May it be noted.

17. Let $no$ be an Ordinate infinitely near and parallel to $NT$, the Fluxion of the Asymptote; $OT$ the Tangent, and $TN$ the Subtangent to the Logarithmic Curve in $O$. Then $oT:TO::ON:NT$. But $ON = s + t$; therefore $oT = s + t$. $pO = m$ (the Fluxion of the Meridian or Asymptote). So the Analogy is $s + t :: m :: s + t :: NT$. By Sect. 13, 14, $s :: m :: t :: t + s :: m :: t + s :: n$, wherefore is $NT$ (the Subtangent to $L A O$) equal to Radius $AR$, a constant Line; and consequently the Curve $L A O$ is the Logarithmic Curve, and its Subtangent known.

18. The same Demonstration serves for $LM$ (any Ordinate on the other Side of $AR$) only changing the Sine $+$ into $-$; and then it agrees with Mr. James Gregory's Prop. 3. pag. 17. of his Exercitatiuns, viz. That...
The Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their respective Tangents, Radius being Unity. So here $RM$ is the Logarithm of $ML$, the Difference of the Secants and Tangent of the Latitude whose Meridional part is $RM$. 

19. Supposing the precedent Construction, if through any point $C$ of the Curve $ACB$ be drawn a right Line $GCW$ parallel to $MR$, terminated with the Logarithmic Curve in $W$ and the Radius $AR$ in $C$, say that the same right Line $WG$ is equal to the intercepted part of the Curve Line $AC$.

20. Let $mg$ be a Line infinitely near and parallel to $WG$, and terminated by the same Lines; and $CS$, $MC$, perpendicular to the Meridian; $CS$ intersecting $mg$ in $x$, and $Wx$ in $y$. Let $CM$ be a Tangent to $AC$ in $C$; $Wx$ a Tangent to $AW$ in $W$; so is $CM = \sigma\tau$. Because of like Triangles $Cz\sigma$, $CSM$; and $Wx\omega$, $W\sigma\tau$; $CS:CM::Cz:Cs$; also $Wx:\sigma\tau::W\gamma:y\omega$. But $Wx = CS$; $\sigma\tau = CM$; $Cx = W\gamma$; therefore is $\gamma\omega$ the Fluxion of $GW$, equal to $Cc$ the Fluxion of the Curve $AC$. Consequently $GW = AC$. q.e.d.

It may be noted that this Equitangential Curve gives the Quadrature of a Figure of Tangents standing perpendicular on their Radius. In Fig. 1, let $AGF$ be a Curve whose Ordinates as $g\gamma$, $G\gamma$, are equal to the Tangents of their respective intercept Arcs $Ak$, $Ax$. Let $FG$ be produced to touch the Curve $AC$ in $C$: then is the Area $ATFG$ equal to the Rectangle contained by Radius $AR$ and $GC$ the produced part of the Ordinate; or $ATFG = AR \times GC$. The Demonstration of which, and of the following Section, I for Brevity omit.

21. If we suppose the Figure $ACB$ &c. $RR$ (Fig. 21, infinitely continued, to be turned about its Asymptote $RA$ as an Axe, the Solid so generated will be equal to ...
rectangule Cone whose Altitude is equal to $AR$. And its Curve Surface will be equal to half the Surface of a Globe whose Radius is $AR$. So, that if the Curve be continued both ways infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the same Radius $AR$.

The Description of the Rular and Wheel, Fig. 2, is sufficient for the Demonstration of the Properties of the Curve; but in order to an actual Construction for Use, I have added Fig. 5, where $AB$ is a brass Rular $m$ the little Wheel, which must be made to move freely and tightly upon its Axe (like a Watch Wheel) the Axe being exactly perpendicular to the Edge of the Rular. $s$ represents a little Screw-pin to set at several Distances for different Radii, and its under End is to slide by the Edge of the other fixed Rular $o$ is a Stud for convenient holding the Rular in its Motion.

Note, Most of these Properties of this Curve, by the Name of la Tranchise, are to be found in a Memoire of M. Bomie among those of the Royal Academy of Sciences for the Year 1712 but not published till 1715. Whereas this Paper of Mr. Perks was produced before the Royal Society in May 1714, as appears by their Journal.


WHEN I apply'd my self to consider thoroughly the Nature of the Method of Fluxions, which has justly been the Occasion of so much Glory to its great Inventor Sir Isaac Newton our most worthy President, I fell by degrees into the Method of Increments, which I have endeavour'd to explain in this Treatise. For it being the Foundation of the Method of Fluxions that the Fluxions
ions of Quantities are proportioned to the nascent Increment of those Quantities; in order to understand this Method thoroughly, I found it necessary to consider well the Properties of Increments in general: and from those Properties I saw it would be easy to draw a perfect Knowledge of the Method of Fluxions: for if in any case the Increments are supposed to vanish and to become equal to nothing, their Proportions become immediately the same with the Proportions of the Fluxions. In this Method I consider Quantities, as formed by a continual Addition of parts of a finite Magnitude, and those parts I call the Increments of the Quantities they belong to, because that by the Addition of them the Quantities are increased. These parts being considered as formed in the same manner by a continual Addition of other parts, thence follows the Consideration of second Increments, and so on to third, fourth, and other Increments of a higher kind. For Example, if \( x \) stands for any Number in the Series \( 1, 2, 3, 4, 5, 6, 7, \ldots \) in which the Numbers are formed by a continual Addition of the Numbers in the Series \( 1, 3, 5, 7, 10, 14, \ldots \) then the Numbers in the latter Series are called the Increments of the Numbers in the foregoing Series; thus, for Example, if to the third Number \( 4 \) in the first Series, I add the corresponding third Number \( 6 \) in the second Series, I shall produce the next, that is, the fourth Number \( 10 \) in the first Series, and so the rest. Any Number in the first Series being called \( x \), the corresponding Number (which is its Increment) in the second Series I express by \( x' \). And these Numbers \( x \) being formed in the same manner by the Numbers in the Series \( 1, 2, 3, 4, 5, \ldots \) I call these last Numbers \( x' \), they being the first Increments of the Numbers \( x \), and the second Increments of the Numbers \( x' \); and so on. Hence having given any Series of Numbers that are called by a general Character \( x \), their Increments are found by taking
their Differences; thus in the present Example, the first
Increments in the Series 1. 3. 6. 10. 15, &c. are found by
taking the Differences of the Numbers in the Series 1.
4. 10. 20. 35, &c. and the second Increments in the
Series 1. 2. 3. 4. 5, &c. are found in the like manner, by
taking the Differences of the Numbers, and so of the
third and other Increments. This Method consists of two
parts; One is concerned in shewing how to find the Re-
lations of the Increments of several variable Quantities,
having given the Relations of the Quantities themselves;
and the Other is concerned in finding the Relations of the
Integral Quantities themselves freed from the considera-
tion of their Increments, having given the Relations of
the Increments: either simply, or they being any how
compounded with their Integral Quantities. In the Me-
thod of Fluxions Quantities are not consider'd with their
parts, but with the Velocities of the Motions they are
supposed to be formed by; or to speak more accurately,
they are consider'd with the Quantities of the Motions by
which they are supposed to be generated; for the Fluxions
are proportional to the Velocities, only when the moving
Quantities, which produce the flowing Quantities con-
der'd, are equal. These Quantities of Motion, or Ve-
ocities when the moving Quantities are equal, are what
Sir Isaac Newton calls Fluxions. As in the Method of
Increments there are second, third, and other Increments;
so in the Method of Fluxions there are second, third, and
other Fluxions; the Fluxions themselves being consider'd
as Quantities that are formed by Motion, the Quantity
of which Motion is their Fluxions. As the Method of
Increments consists of two Parts; one being concern'd in
finding the Increments from the Integrals given, and the
other in finding the Integrals having given the Increments;
so does the Method of Fluxions consist of two Parts;
the one shewing how to find the Fluxions, having the

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the Fluents given; and the other shewing how to find the Fluents freed from Fluxions, having given the Relations of the Fluxions, whether compounded with their Fluents or otherwise. The Principles of this Method may all be drawn directly as a Corollary from the Principles of the Method of Increments. For Sir Isaac Newton having demonstrated (Phil. Nat. Princ. Math. Sect. 1. and in the Beginning of his Treatise De Quadratura Curvarum) that the Fluxions of Quantities are proportional to their nascent or evanescient Increments, if in any Proposition relating to Increments, you make the Increments to vanish and to become equal to nothing, and for their Proportion put the Fluxions, you will have a Proposition that will be true in the Method of Fluxions. This is but a Corollary to Sir Isaac Newton's Demonstration of the Fluxions being proportional to the nascent Increments. For this reason, to make the Method of Fluxions to be understood more thoroughly, I thought it proper to treat of these two Methods together, and I have handled them promiscuously as if they were but one Method. Some people, because that the Fluxions are proportional to the nascent Increments of Quantities, have thought that by the Method of Fluxions Sir Isaac Newton has introduced into Mathematicks the Consideration of infinitely little Quantities; as if there were any such thing as a real Quantity infinitely little. But in this they are mistaken, for Sir Isaac does only consider the first or last Ratio's of Quantities, when they begin to be, or when they vanish, not after they are become something, or just before they vanish; but in the very moment when they do so. In this case Quantities are not consider'd as infinitely little; but they are really nothing at the time that Sir Isaac takes the Proportions of their Fluxions; and the Truth of this Method is demonstrated from the Principles of the Method of Increments, in the same manner as the Ancients demonstrated.
Having premised thus much in general concerning the two Methods here treated of, to come to a particular Description of this Book: In the Preface I give a short Description of the Method of Increments, and an Account of Sir Isaac Newton's Notion of the Fluxions which I have already spoke of. The Book consists of two Parts, and contains 118 Pages in Quarto; the Propositions being number'd throughout from the Beginning. In the first Part I explain the Principles of both Methods: and in the second Part I shew the Usefulness of them in some particular Examples.

After having explain'd the Notation I make use of in the Introduction, in the first Proposition I explain the direct Method both of Increments and of Fluxions. The second Proposition shews how to transform an Equation wherein Integrals and their Increments, or wherein Fluents and their Fluxions are concern'd; so as in the Room of the Integrals or Fluents, to substitute their Compliments to a given Quantity with their Increments or their Fluxions, they increasing in a contrary Sense to the Quantities in the first Supposition. In the third Proposition I shew how to transform a fluxional Equation, so as to change the Characters of the Fluents, making that Quantity to flow uniformly which in the first Supposition flow'd unequally, having second, third and other Fluxions, and making that Quantity which in the first Supposition flow'd uniformly, now to flow unequally, so as to have second and third Fluxions, &c. This Proposition is of great use in the inverse Method, when you would invert the Expression of the Relation of the flowing Quantities; for Example, if in the Supposition \( z \) flows uniformly and \( x \) variably, by the inverse Method of Fluxions you will find \( x \) express by the Powers of \( z \); but if you would find \( z \) express
express by the Powers of \( x \); you must then transform the Equation by this Proposition. Sir Isaac Newton and Mr. de Moivre do this by the Reversion of Series; but I take this to be the more proper and more genuine Method of doing it directly. In the fourth and fifth Propositions are explain'd the Method of judging of the Nature and Number of the Conditions that may accompany an Incremental or a Fluxional Equation. This is a Circumstance that I don't find to have been explain'd by any one before, and the Propositions are something intricate; wherefore it will not be improper to explain this Matter a little more at large. Suppose then that \( z \) and \( x \) are two variable Quantities, and supposing \( z \) to increase uniformly by the continual Addition of its constant Increment \( z \), (according to the Notation I make use of in this Book) suppose \( x + z = x \). Then if it be proposed as a Problem to find the Value of \( x \), express'd by the Powers of \( z \), and quite freed from the Increments; by the fourth Proposition there may be three Conditions added to this Problem. The Demonstration of this is taken from the Formation of the Integrals by a continual Addition of their Increments. Suppose that all the Values of \( z \), \( x \), \( s \), \( x \), \( s \), \&c. were set down in order in so many Columns, and that at the Head of the Table, the corresponding Values of \( z \), \( x \), \( s \), \( x \), \( s \), were express'd by the Symbols \( a \), \( c \), \( e \), \( c \), \( e \). Then by means of the given Equation \( x + z = x \), or \( x = x - z \), will \( e = c - a \), whence the second corresponding Values of \( z \), \( x \), \( s \), \( x \), \( s \) will be \( a + z \), \( c + e \), \( c + e \), \( c + e - a ( = c + e ) \) and \( c + e - a - z \) (by the Eq.) Whence again the third Values are \( a + 2 z \), \( c + 2 e + e \), \( c + 2 e + e - a ( = c + 2 e + e ) \) \( c + 2 e - 2a + e - z \), and \( c + 2 e + e - a - 2 z \), and so you may proceed to find the fourth, fifth, and all the.
the other Values of $z, x, x, \ldots$. But by what is already set down, it is evident that all their Values will be express'd by $a$ and $z$, and the three Symbols $c, c, c$; and consequently all the Values of the rest of the Increments of $x$, viz. $z, x, \&c.$ will be express'd by the same Symbols. Whence it follows that to determine the Values of the Symbols $c, c, c$, there may be taken three Conditions relating to the Values of $x, x, x, \&c.$ promiscuously, as the fourth Proposition directs. The same Rule holds good in the Method of Fluxions. For Example, having given the Equation $a z^2 + x^2 x = 0$, if it be proposed to describe the Curve that it belongs to, by the fourth Proposition it may also be required as a Condition, that the Curve shall pass through two given Points; that it shall touch two given Lines; that it shall pass thro' a given Point, and when it cuts a given Line shall have a given Degree of Curvature; or that it shall have any other two Circumstances that depend upon the Values of the third, fourth or other Fluxions. These Conditions that attend Incremental or Fluxional Equations, I don't know to have been sufficiently taken notice of by any Body! but they ought well to be attended to in the Inverse Methods; the Solutions of particular Problems being never perfect, unless there be provision made for the satisfying of them, by the indetermined Coefficients in the Equation that contains the Solution of the Problem. Examples of this may be seen in Prop. 17 and 18, where I give the Solution of the Problems concerning the Isoperimenter, and the Catenaria.

The sixth Proposition contains the general Explanation of the Inverse Method both of Fluxions and of Increments, which consists in the Solution of this Problem. Having given the Relations of the Increments, or of the Fluxions of several Quantities, whether they be consid-
red with their proper Integrals or with their proper Fluents or not; to find the Relations of the Integrals or of the Fluents, freed from their Increments or from their Fluxions. The Direction I have given for finding the Solution in finite Terms is but tentative. And I must confess I know of no other Method that is general for all Cases. For I can find no certain Rule to judge in general, whether any proposed Equation, involving Increments or involving Fluxions, can be resolved in finite Terms. For this Reason we are obliged to seek the general Solution in infinite Series; which when they break off, or when they can any way be reduced to finite Terms, they then contain the Solutions which we always hope for. The Method of finding these Series is explain'd in the eighth Proposition, and that is by means of a Series that is demonstrated in the seventh Proposition. And this I take to be the only genuine and general Solution of the inverse Methods. For in this Solution you always have those indetermined Coefficients, which are necessary to adapt the Equation that is found to the Conditions of the Problem proposed. For want of this Circumstance all other Methods are imperfect; and particularly Sir Isaac Newton's Method of finding Series by a Ruler and Parallelograms labours under this Difficulty, because it brings no new Coefficients into the resulting Equation, which may afterwards be determined by the Conditions of the Problem. However because this Method is very ingenious and very elegant, I thought it proper to explain it in the following (viz. the 9th) Prop. The 10th, 11th, and 12th Propositions conclude the first Part, and in them I treat of the manner of finding the Integral or the Fluent, having given the Expression of a particular Increment, or of a particular Fluxion of it; without being involved with the Integrals, or with the Fluents, or with any other Increments, or with any other Fluxions of it.
This is a particular Case of the Inverse Method, but for its great usefulness I thought it deserved particularly to be taken notice of. This Problem is treated of in general in the 10th Proposition. The Method of solving it in finite Terms is only tentative; and when that does not succeed, recourse must necessarily be had to the Solution by a Series in the 8th Proposition. In the 11th and 12th Propositions I have shew’d how Series may be conveniently found, in some particular Cases when Fluxions are proposed.

In the second Part I have endeavour’d to shew the usefulness of these Methods in the Solution of several Problems; The 13th Proposition is much the same with Sir Isaac Newton’s Methodus Differentialis, when the Ordinates are at equal Distances: and in an Example at the End of this Proposition I have shew’d how easily Sir Isaac Newton’s Series for expressing the Dignity of a Binomial may be found by this Incremental Method. The 14th Proposition shews in some measure how this Method may be of use in summing up of Arithmetical Serieses. In the 15th Proposition I shew by some Examples how the Proportions of the Fluxions are to be found in Geometrical Figures; from whence immediately flows the Method of finding the Radiuses of their inosculating Circles, the Invention of the Points of contrary Flexure, and the Solution of other Problems of the like nature. In the 16th Proposition I shew how the Method of Fluxions is to be applied to the Quadrature of all sorts of Curves. In the following Proposition I give a general Solution of the Problem of the Isoperimeter, which has been treated of by the two famous Mathematical Brothers the Bernoulli's. In the 18th Proposition I give the Solution of the Problem about the Catenaria, not only when the Chain is of a given Thickness every where, but in general, when its Thickness alters according to any given Law. In the fol-
following Proposition I shew the Fornix or Arch which supports its own Weight to be the same with the Catena.
In the two next Propositions I shew how to find the Figures of pliable Surfaces which are charged with the Weight of a Fluid. In the 22d and 23d Propositions I treat of the Motion of a Musical String, and give the Solution of this Problem: To find the Number of Vibrations that a String will make in a certain time, having given its Length, its Weight, and the Weight that stretches it. This Problem I take to be entirely new, and in the Solution of it (in the last part of Prop. 23.) there is a remarkable Instance of the Usefulness of the Method of first and last Ratio's. The 24th Proposition gives the Invention of the Center of Oscillation of all Bodies; and in the 25th Proposition I have given the Invention of the Center of Percussion. It is known that this Problem is solved by the same Calculus as the foregoing; wherefore it is generally thought that these two Centers are the same. But that is a Mistake, because the Center of Oscillation can be but one Point; but the Center of Percussion may be any where in a certain Line, which this Proposition shews how to find. There is an Error in this Proposition, which I was not sensible of till after the Book was publish'd, wherefore I take this Opportunity of correcting of it. It does not affect the Reasoning by which I find the Distance of the Center of Percussion from the Axis of Rotation; but it is this, that I supposed the Center of Percussion to be in the Plane passing thro' the Center of Gravity, and perpendicular to the Axis of Rotation, which is a Mistake. It is corrected by the following Proposition.

PROP.
PROP. PROB.

To find the Distance of the Center of Percussion from the Plane passing thro’ the Center of Gravity and perpendicular to the Axis of Rotation.

SOLUTION.

Let the sixth Figure be supposed in the Plane passing thro’ the Axis of Rotation, and in which the Center of Percussion is sought.

Let $AB$ be the Axis of Rotation, $AGC$ be the Intersection of this Figure with the Plane passing thro’ the Center of Gravity, and perpendicular to the Axis of Rotation. $G$ be the Point whereon a Line, rais’d perpendicular to this Figure, will pass thro’ the Center of Gravity; $BE$ be a Line parallel to $AG$ wherein is the Center of Percussion. Then to find the Distance $AB$, let $p$ stand for an Element of the Body proposed standing perpendicularly on any point $D$. Draw $DC$ perpendicular to $AGC$, and $AB$ will be equal to the Summ of all the Quantities $p \times GC \times CD$ taken with their proper Signs, divided by the Body it self multiplied into the Distance $AG$.

Having thus found the Distance $AB$, suppose the Plane of the Figure in Prop. 25. to cut the present Figure at right Angles in the Line $BE$, and the Center of Percussion will be rightly determined by that Proposition.

The 26th Proposition shews how to determine the Density of the Air at any Distance from the Center of the Earth, supposing the Density always to be proportional to the compressing Force, and that the Power of Gravitation is reciprocally as the Distances from the Center of the Earth.
The last Proposition shews how to find the Refraction of a Ray of Light in its passage thro' the Atmosphere, upon the Supposition that Light is a Body, and that the Refraction of it is caused by the Attraction of the Bodies the Rays approach to. In this Proposition there is a remarkable Instance of the Usefulness of the Method of Increments in finding the Coefficients of a Series, which according to the Values of a certain Symbol, as \( n \), expresses both all the Fluents, and all the Fluxions of a certain Quantity.

II. Ludovici Ferdinandi Marsii DiSSERTATIO DE GENERATIONE FUNGORUM. Rom. 1714. 4to.

The Author tells us that he gave his youthful Inclinations to the Study of the Mathematicks and Observations of Nature, under the Tuition of the celebrated Malpighius, and Lelius Triumphettus Botanick Professor of Bononia: and amongst the various Productions of Nature, his chief Delight was in the Contemplation of the sudden Growth and various sorts of Mushrooms which both the Earth and Trees brought forth. Of the first Kind he observed the greatest Number to arise in Camps, produced from the Horses Dung, and are commonly called Prataioli.

In the Years 1699 and 1700, being then in Croatia and Transylvania; in the Armies there, he made a large Volume of Designs of Fungi, which he sent to Triumphetti to put in order; who added a great Number to them of such as he found about Bononia; yet after all the most diligent Search, he could never find them to produce any Seed either in their Gills or other Parts.

The Origin and Generation of Mushrooms he says is not easy to demonstrate, since both the Antients and Moderns
disagree very much about it. The late Botanists seem to bet of three different Sentiments concerning their Produce. Mr. Ray, Dr. Sherrard, Mr. Doody, Beccone and Mentzelius—having observed some Mushrooms to have had Seed, were of opinion others might have the same Original. Clusius and John Baptista Porta had in some also observed their Seed: Others, viz. Sharrock and the accurate Mulpigius, who could not find any Seed in them, altho' with the Assistance of Microscopes, did suppose they might be produced by Pieces of themselves, carried by the Winds from place to place, as other Plants are by Slips and Offsetts. The third Opinion, which he says most agree in, is that they arise from Putrefaction, or a Mixture of certain Salts, Sulphur and Earth impregnated with the Dung of Beasts.

The Fungus seminifer Campaniformis Mentzeli, &c. being the Mushroom which first gave the occasion of the Opinion of their having Seed, this nice Author has accurately figured and observed, and supposes with others that these seedlike Bodies may be the Ovaria of some Insects; and the rather because they are so very large in proportion to the smallness of the Mushroom: and that they had often been sowed by Dr. Amadoes a curious Botanist, without any Success towards raising them. From whence he concludes these Bodies ought to have another Denomination than Seed; neither is he of the Opinion that they are produced by parts of themselves.

In his Division of Mushrooms he first treats of the Truffles and their Increase, Situation and Soil, Colour, Taste and Consistence. He next proceeds to soft Mushrooms, such as he observed in his own Garden; which having in the Spring been meliorated with Horse-dung, about the middle of June there sprung up divers of that sort which the Italians call Pratinioli, amongst a Bed of Lettice. These continued till near the midst of August before they went off.
off. Of these and some other Kinds he accurately figures the first Shootings and Fibres.

His next Tribe are such as grow from Wood, but yet are themselves soft. Of these he observes three Kinds; the first a large one in his Window, out of a piece of Firwood which it had often rained on; with two smaller sorts from some rotten Boards in his Garden. All these he figures both in their natural and divided States, as also Microscopically.

Treating of hard woody Mushrooms (of which he also gives you some accurate Figures) he observes they rarely appear on the Trees, in Germany and Croatia, before they are twenty or thirty Years old; but most commonly when forty or fifty: and the Original of them he attributes chiefly to the Rottenness of the Wood, and says they generally break out in the Spring, when the Leaves begin to shoot. And that usually they grow below the middle of the Trees, and are cause of so much Decay in them, that they often die in three or four Years.

It may not here be amiss to subjoin what Dr. Lantillus communicates to our Author, concerning the Lapis Fun-garius, viz. that altho' this Mushroom-producer has the Name of a Stone, it ought not to be reckoned of that Genus, it being really no other than a Mass or Congeries of Roots, Seeds and Juices coagulated with Earth into, as it were, a stony Substance. Upon which pouring Water and setting it in a warm Place, it loosens its hardned Substance; and by mollifying its Fibres and moistening its concrete Juices, out of the Cliffs and Chinks thereof the Mushrooms spring, as they do in other places from simple Dung and loose Earth. And it is also farther to be noted, that when this stony Mass has thus yielded these its Offspring, the Remains grows light, porous and decay'd, its nutritive Juices being then exhausted.

FINIS.
For the Months of November and December, 1715.

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I. A short History of the several New-Stars that have appear'd within these 150 Years; with an Account of the Return of that in Collo Cygni, and of its Continuance observed this Year 1715.

Wether it be owing to the greater Diligence of the Moderns, or that in reality no such thing has happen'd for many Ages past, I will not undertake to determine; but this is certain that, within the Space of the last 150 Years, more Discoveries have been made of Changes among the Fixt Stars, than in all Antiquity before. And tho' it be said that Hipparchus, on occasion of a New Star that appeared in his Time, was induced to number the Stars, and make the first Catalogue of them, which was, in the Opinion of Pliny, Res vel Deo improba; yet neither he or any of the Ancients have left us the Place of that New Star, to compare with those lately seen, one of which might perhaps be the same with it, re-appearing after a long Period of Years. Now though several Authors have severally described those that have been seen nearer to our Times, it may not perhaps be amiss here to give a short Recapitulation of what was principally remarkable in each of them, with the Times of their first Appearance, as far as can be collected.

And first, That in the Chair of Cassiopeia, was not seen by Cornelius Gemma on the Eighth of November 1572, who says, he that Night considered that Part of Heaven in a very serene Sky, and saw it not: but that the next Night, Novemb. 9°, it appeared with a Splendour surpassing all the fixt Stars and scarce less bright than Venus. This was not seen by Tycho Brahe before the 11th of the same Month, but from thence He assures us that it gradually decreased and died away, so as in March 1574,
1574, after sixteen Months, to be no longer visible; and at this Day no Signs of it remain. The Place thereof in the Sphere of Fixt-Stars, by the accurate Observations of the same Tycho, was os. 9°. 17'. à 1°. to, with 53°. 45'. North Latitude.

Such another Star was seen and observed by the Scholars of Kepler, to begin to appear on Sept. 30°. ft. vet. anno 1604, which was not to be seen the Day before: but it broke out at once with a Lustre surpassing that of Jupiter; and like the former it died away gradually, and in much about the same time disappear'd totally, there remaining no Footsteps thereof in January 1606. This was near the Ecliptick, following the Right-Leg of Serpentarius; and by the Observations of Kepler and others, was in 7°. 20°. 00' à 1°. 30', with North Latitude 1°. 56'. These two seem to be of a distinct Species from the rest, and nothing like them has appear'd since.

But between them, viz. in the Year 1596, we have the first Account of the wonderful Star in Collo Ceti, seen by David Fabricius on the third of August, st. vet. as bright as a Star of the third Magnitude, which has been since found to appear and disappear periodically: its Period being precisely enough 7 Revolutions in Six Years, tho' it return not always with the same Lustre. Nor is it ever totally extinguish'd, but may at all times be seen with a Six-Foot Tube. This was singular in its Kind, till that in Collo Cygni was discovered. It precedes the first Star of Aries 1°. 40', with 15°. 57' South Latitude.

Another New Star was first observed by Will. Fansonius in the Year 1600, in Pectore or rather in edultione Colli Cygni, which exceeded not the third Magnitude. This having continued some Years, became at length so small as to be thought by some to disappear entirely: but in the Years 1657, 58 and 59, it again arose to the third Magnitude, tho' soon after it decay'd by degrees to the
fifth or sixth Magnitude, and at this Day is to be seen as such in 9°. 18°. 38'. a 1\textsuperscript{ma} \&. \gamma, with 55°. 29'. North Lat.

A fifth New Star was first seen and observed by Hevelius in the Year 1670, on July 15. 5°. vet. as a Star of the third Magnitude, but by the Beginning of October was scarce to be perceived by the naked Eye. In April following it was again as bright as before, or rather greater than of the third Magnitude, yet wholly disappeared about the middle of August. The next Year, in March 1672, it was seen again, but not exceeding the sixth Magnitude: since when it has been no further visible, though we have frequently sought for its Return; its place is 9°. 3°. 17'. a 1\textsuperscript{ma} \&. \gamma, and has Lat. North. 47°. 28'.

The Sixth and last is that which we described from the Acta Berolinensia, in No. 343 of these Transactions; discovered by Mr. G. Kirch in the Year 1686, and its Period determined to be of 404 + Days: and though it rarely exceed the fifth Magnitude, yet is it very regular in its Returns, as we found in the Year 1714. Since then we have watched, as the Absence of the Moon and the Clearness of Weather would permit, to catch the first beginning of its Appearance in a Six-Foot Tube, that bearing a very great Apperture discovers most minute Stars. And on June 15. last, it was first perceived like one of the very least Telescopical Stars; but in the rest of that Month and July it gradually increased, so as to become in August visible to the naked Eye; and so it continued all the Month of September. After that it again died away by degrees, and on the 8th of December at Night was scarce discernible by the Tube, and as near as could be guessed, equal to what it was at its first Appearance on June 15th: so that this Year it has been seen in all near Six Months, which is but little less than half its Period: And the middle, and consequently the greatest Brightness, falls about the 10th of September. Those that please to seek for it, may expect its first Appearance in July next, and find it in 9°. 6°. 30'. circiter a 1\textsuperscript{ma} \&. \gamma, with Lat. Bor. 52°. 40'.

11.
II. Continued from No. 345. By James Petiver, F. R. S.

S E C T. II.

Indian Herbs and Trees.

94. Antego Green Blite:

Blitum Antegoanum viride, caule & pediculis rubris.

This rises about a Yard high, its Leaves smooth, veined and oval, standing on long slender Pedicles, red, as are the large Stalks, from whence grow Tufts of green Pânciles, which Flower and Seed from July till Autumn.

I received Plants and Seed of this and many others from my worthy Friend Mr. John Douglass Surgeon, and Brother to his Excellency the Governor of Antego.

95. Climbing Virginia Eupatorium.


Clematitis novum genus, Cucumerinis foliis Virginianum Pluk. Tab. 163.fig. 3.


This elegant Plant is accurately Figured in Dr. Pluknet's Phytographia, and Mr. Banister's Herbarium Virginianum which is now in the Press from his own Original Designs. It is the only Virginia Climber of this Tribe that has as yet come to our Knowledge, and never raised in any European Garden before.

K k k
Coma Caponis coronopii folio.
Bellis Afric. lutea minor, fl. nudo cernuo Alm. Bot. 66.
Bellis Afric. Coronopii folio. fl. nudo Crassii.
Bellis Afric. capitulo aphylo luteo, Coronopii folio, cauliculus procumbentibus Herm. Hi. Leyd. 86.
Bellis annua, capitulo aphylo luteo H. Ox. Sect. 6. Tab. 7.
fig. ult. opt.
Chrysanthemum exoticum minus, capitulo aphylo, Chamaemeli nudi facie Brey. Cent. 156. c. 76. fig.
Chrysanthemum Tingianum minimum procumbens, foliis versus inum dentatis, flosculo nudo cernuo Moris.
Its Leaves, naked Flowers, and Way of growing distinguish it from others.
97. Trifid Cape Gold Tufes, Ray 3. p. 179. t.
It Flowers in September, and the Seed is ripe about Christmas.
98. Small Headed American Live ever.
Helichrysum annuum majus, creatum Virginiamum H. Ox. Sect. 7. Tab. x Ser. 2. pl. r.
Helichrysum five Chrysoleone Gnaphaloides Virginiana annua, folis obtusobus, capitulis argentis conglobatis Bob. Oxon. 88. pl. 2 r.
Gnaphalium Lythi comis argenteis Pluk. Tab. 31. fig. 5. extent. F. Bobari.
This rises near a yard high, is somewhat Woody, with many Branches ending in Tufts of small straw coloured Heads, which rarely or never fully open. The Indians rub their Head with this Plant, and assert its good for the Eye-sight, as my late curious Friend Mr. John Lawson informed me. By another Person I am told they drink a Tea of it in Feavers.

Eichrysum fl. pallecentibus minimis, Spise foliis.

Its Stalks are thick set, with whitish pointed Leaves; the Flowers grow in Tufts of a pale Yellow and very small. It Flowered in July, &c. in Mr. Thomas Fairchild's Garden at Hoxton, the only Place I have yet seen it in.

100. Woolly Live-ever with red thready Flowers.
Eichrysum Lychnidis Coronarie folio.

This elegant Plant and the next, I have with Pleasure seen in the Bishop of London's Gardens at Fulham, under the Care of Mr. Millward.

Eichrysum Capense, Perfoliate folio.

These Leaves are round, somewhat bristle-edged, grow alternate and saddle the Stalk, like our Perfoliata or Thorow Wax. My late industrious Friend Mr. William Brown, Surgeon, brought me from the Cape of Good Hope the only Specimens of this singular Plant I have since seen.

102. Welted Antego Spike-Cudweed.
Gnaphaloides Antegoana Spicata, caule alato.


This elegant Plant was raised in Chelsey-Garden from Seed sent me by my kind Friend Mr. John Douglas at Antego.

Conyza Afric. incana arb odore Salvia & Rorifmarini Flor. Pruss. 149. n° 3. occurrit.

It Flowers in March and April in our Stows, and at the same time at the Cape of Good Hope.

Elichryso affinis Peruviana frutescens Herz. H. Leyd. 666. Pluk. 27. fig. 1.

This by some has been erroneously shewn in our Gardens for the true Cortex Peru.

Bobartia lutea hirsuta, caule Echii.


Its lower Leaves somewhat like Plantain, lightly notched, rough and hairy, the Stalks speckled with red, and rough as Pipers Bugloss, its Flowers composed of 13 yellow Petala set in a double Row of narrow green hairy pointed Leaves. What is remarkable in this Tribe, is a large purple Umbo or Disk, rising in the midst of the Flower like a Button. A very peculiar sort, of this Family, I first saw many Years since with Mr. Jacob Bobart in the Physick-Garden at Oxford; for which reason I have presumed to di-

(356)
friuing it by his Name, that it may be the easier known from *Chryanthemum*, *Dracunculus* or *Ptarmica* to which others have ranked it. — This *Flowers* most part of the Tear.

106. *Fairchild’s* broad *Bobart*.

*Bobartia Virginiana*, fol. lato scabro, basi alato.

Its Flower-stalk swells gradually towards the Head, which is composed of many regular broad pointed blackish green Scales, the Rim consists of 15 or more yellow Petala, out of its purple Disk come many small yellow *Floesculi*, which I have not observed in the other. Mr. *Thomas Fairchild* raised this Plant from Seed he received, with many others, from that curious Botanist Mr. *Mark Catesby* of Virginia. It flowered at Hoxton about the middle of October.

107. Cape *Uvedale* with a *Poplar Leaf*, Ray. 339. i.

*Uvedalia Capensis Populi folio.*


an *Chryanthemum lanatum*, crenatis foliis *Æthiopicum* Pluk. Tab. 274. f. 5.

This differs from the *Chryanthemums* in having a *baciferous* Rim.


*Nodiflora Jamaic. Scrophulariae folio.*

*Sideritis* *spicata* *Scrophulariae* folio, fl. albo, spicis brevibus habitioribus rotundis, pediculis insidentibus *Sloan*. Cat. Jam. 65. Hist. Tab. 109. fig. 2.

That curious *Naturalist* Dr. *Hans Sloane*, has well described this Plant, and given a very accurate *Figure* of it, which truly agrees with a fair Specimen my hearty Friend Capt. *Thomas Walduck* sent me from Barbadoes, from the Seed of which this Plant was raised. I take the *Nodiflora*
Luzon. Sideritidis folio Gazoph. Naturæ Tab. 69. fig. 6. to be a sort of this with narrower Leaves.

Clinopodium Virgin. angustifol. flor. amplis luteis, punctis purpureis Pluk. Tab. 24. fig. 1.

Origanum flor. amplis luteis purpureo maculatis, cujus caulis sub quavis verticillo 10 vel 12 foliis est circumcinctus Banist. Ray 1927.

I have seen this in Flower with Mr. Fairchild, raised from Seed which Mr. Catesby sent him.


Horminum haftatis amplioribus foliis f. Ari modo alatis, caulibus & pediculis araneosâ lanugine villosis ex Insula Gomera Pluk Tab 301. fig. 2.
Mustazi Insulanis & Salvia arborea vulgar Alm. Bot. 185. pl. 5.


This Flowers in divers of our curious Gardens from June till Autumn.


Stachys Canarienfs frutesc. Verbasco folio Instit. 186.


Stachys

It holds its Leaves all the Winter in our Stoves.


Clematis Passionalis triphylllos f. luteo Ox. Mor. 7. 3. Tab. 2. Ser. 1. f. 3.

Clematis Passiflora f. luteo Munt. Prax. 422. fig. opt.
Cucumis Flos Passionis dictus, Hederaceo folio, f. ex luteo viridanti, Herm. H. Leyd. 205.

Clematis f. Flos Passionis Americana luteo flore Vallot.

53.

an Clematititis Indica f. minimo pallido Plum. 73. Tab. 88.

Balsamina altera Indica repens, Hedera arboreae foliis, f. subvirendi Ambros phyt. 91.


Granadilla pumila, f. parvo luteo D. Alex. Balam. H. Patav. à Turre 55.

Flos Passionis minor, folio in 3 lacinias non ferratas minus profundas diviso Cat. Jam. 104.


This Plant is a Native of Virginia from whence Mr. Catesby has lately sent it.

113. The Old Trefoil Maracoc Ray 649. 1.

Balsamina Indica repens, Pomifera Homurph. 5.

Clematis Passionalis triphylllos, f. Roseo Mor. Ox. 7. 2. Tab. 1. fig. 9.
Clematis trifolia, f. Roseo clavato C B. 30r. xi.
Granadilla Hispanis, Flos Passionis Italits Col. in Hern: 887. & 90. fig. opt.
Granadilla triphylla, f. Roseo clavato Fl. Nor. 199.
Maracot fives Clematis Virginiana Park. Gard. 393. 10. fig. 395. 7.
Maracot Indicum Aldini Horr. Farnes. 50.
Passiflora foliis crenatis tripartito divisị Alm. Bot. 281. pl. 3.

This beautiful Old Plant has been an Ornament in our English Gardens for near a Hundred Years, and was first brought to us from Virginia.

114. Common Fingered Maracoc.

Clematis Passiflora pentaphylla, f. caeruleo punctato Munt. 421. fig.
Clematis pentaphylla, f. Roseo clavato Mor. Ox. 6. 8. Tab. 1. fig. 8.
Clematis quinquefolia Americana f. Flos Passionis Roberti Icon.

Cucumis Flos Passionis dictus, pentaphyllos, f. clavato Herm. H. Leyd. 205.

an Flos Passionis major pentaphyllos Cat. Jam. 104. pl. 1.

This elegant Climber is found in divers Gardens of the Curious, but I could never yet observe the Fiddle-like Dents, in the middle Leaf, as Munting, Morison, &c. have Figured it.

115. Ever-green Fingered Maracoc.

Passiflora pentaphylla sempervirens.
This chiefly differs in being ever-green. I have observed it both in Flower and Fruit, in Mr. Thomas Fairchild's Garden at Hoxton.

Argemone Mexicana El. Bot. 204. Inst. 239.
**Papaver spinosum** C B. 171. 17. prodr. 92. fig. phyt. 311
93. fig. 1. Imperat. 662. fig. 2. Jonst. 371. f. 4.

**Papaver campestre** spinosum Chabr. 459. x. Ic.

**Papaver spinosum Americanum** Park. 366. f. 5. opt.

**Papaver spinosum luteum,** foliis albis venis notatis Moris.

Ox. 277.

**Papaver spinosum,** fl. luteo simplici Munt. p. c. 555.

**Carduus Chrysanthemeus Peruanus** Ger. 993. f. 2. Jonst.

Chicallotl Mexic. seu Spina Hernand. 215. c. 60. fig.

**Ficus Infernalis** Italorum an Glaucium Diosc. Ray 856. 8.

The Purging Thistle, i. e. Carduus Purgans nostratibus dicta

This singular Plant is a Native of both the East and
West Indies.

It Flowers and Seeds in our Gardens in July, August, &c.


**Papaver Oriental. hisfortissimum,** fl. magn. T. Coral. 17 Co-

mel. Plant. rar. 34. fig.

My hearty and very worthy Friend Mr. George London
was the first who shew'd me this in Flower; since which I
have observed it in our Physick Garden at Chelsey.

118. Spanish Potatoes Ray 728. 22.

**Batata Hispanor.** Camotes fl. Amotes & Ignanes. Lob. Ic.

647. Obs. 369. f.

**Batatas Clus. Exot. 341. & Inhame Lufitan. Hist. 78. fig.**

**Batatas Occid. Indie & Inhame Orient. Lufit. Park. 1383.**

f. 3.

**Battades, Ignames Adv. 317. fig. radicum.**

**Convolvulus Indicus** Batatas dictus Ray H. Pl. 728. pl 22.

**Convolvulus Ind. Orient. Inhame seu Batatas Mor. Ox. x1:**

4. Tab. 3. fig. 4.

**Convolvulus Ind. Or rad. tuberosa eduli,cortice rubro & al-
bo Battatas dictus Par. Bat. prodr. 326.**

L 111

Convol-
Convolvulus rad. tuberosa esculenta, Spinacia folio, fl. albo fundo purpureo, femine post singulos Flores singul.-lo Fam. Cat. 53. Hist. 150.

I have gathered this in the Duchess of Beaufort's Garden at Chelsea.

119. White Virginia Bindweed, with a blackish bottomed Flower.

Convolvulus Virg. fol. cordato, fl. albo minore, fundo nigricante.

an Convolvulus Scamoni folio subrotundo, fl. albo, umbone nigro Curassavicus Herm. Par. Bat Cat. 6.

Its Leaves cordated, small and very pointed, the Flowers also small, slender and white, but blackish at the Bottom. We owe the Discovery of this to the inquisitive Mr. Catesby.

120. Broad-leaved Cape Goat-Cranesbill.

Geranium Tragodes Capense, folio maximo.


Its Flowers are generally composed out of two small flesh coloured Petals, with a Blot of deep scarlet in the Middle, and a Streak of white running from thence down to the Bottom. I first observed this with the Bishop of London at Fulham, and since in our Physick Garden at Chelsea. It Flowers in April and May.

121 Cape Codling-Cranesbill with bright Flowers.

Geranium Capense frutesc. folio rotundo, Pomaceo odore, fl. vivido.


I have only seen this at the Bishop of London's Garden at Fulham; it differs from Botan. Hortense No. 2. pl. 103. in having a much livelier red Flower.
122. Small Smooth curl-leaved Cape Cranesbill, Geranium Capense folio Betonica crispo, fl. minimo.
   Its lower Leaves are smooth and more or less round and broad, the upper Leaves are deep cut, with two or three small purple Flowers standing on very fine slender Footstalks.

123. Dr. Uvedale's Spot-Flowred Cape Cranesbill R. 3 p. 510. pl. 2.
   Geranium Capense fol. Betonica molli, fl. maculato.
   Dr. Pluknet, that copious Botanist, first observed this in Dr. Uvedale's curious Garden at Enfield. It's Stalks and Leaves are soft and hoary, the Flowers small and deeply spotted like the Lychnis hirta minor.

124. Spotted American Arum, with whitish Veins.
   Arum Amer. majus maculatum, venis eleganter albis.
   I have seen this beautiful Arum in Flower in the Bishop of London's curious Gardens at Fulham.

125. Small Virginia Trefoil Arum.
   Arum humile Virgin. triphyllum.
   Arum Zeylan. trilobato folio, humilis & minus Par. Bat. 79. fig.
   Mr. Thomas Fairchild gave me the only Specimen of this which I saw growing in his Garden at Hoxton.

   Sumach Capensf. fol. rotundo splendido majore.
   Rhus Afric. trifoliatum majus, splendente folio rotundo integro. Ray Dendr. 58. 12.
   This and the next I have seen in the Duchess of Beaufort's Gardens at Badminton.

   Sumach Capens. fol. rotundo splendido minore.
   Lentiscus humilis, trifolia Africana P. Bat. pr.
Rhus Afric. trifoliatum minus glabrum, splendente foli subrotundulo integro Pluk. Tab. 129. fig. 9. & Ray Dendr. 58. xi
This chiefly differs from the last in being less, particularly in its Leaves.
Rhus sef Sumach Afric. trifoliatum hirsutum & crenatum Herm. Ray Dendr. 56. 1.
Rhus Afric. trifol majus, fol. obtusis & incisis hirsutis pubescentibus Pluk. 129. f. 7. & Ray Dendr. 56. 1.
These Leaves are deeply dented and hairy, by which its known from the rest.
129. Narrow white Cape Sumac. R. Dendr. 57. 8.
Rhus Afric. trifol majus, fol. subtus argenteis acutis & margine incisis Pluk. Tab. 129. fig. 6. & Ray Dendr. 5. p. 57. 8.
Rhus Afric. trifol. angustissimo folio subtus incano, leu Agni Casti lobis Herm. R. Dendr. 56. 2.
This differs from the next in having narrow Leaves somewhat jagged.
All these Trifoliata Sumachs grow spontaneously about the fertile Cape of good Hope, and from thence brought into the Gardens of the most Curious, where I have seen them both in England and Holland. III.
III. An Extract of a Letter from Dr. Helvetius at Paris, to Monsieur Duyvenvoorde Embassador Extraordinary from the States-General, and by him communicated to the Royal-Society.

I am extremly pleased, Sir, that you have applied your self to me, for my Advice about the Use of the Pareira Brava which has been recommended to you, because I can give you a very good Account of it, having been one of the first that introduced it in France; I have made abundance of lucky Experiments about it, which have made this Medicine very well known to me: wherefore I assure you, you can do nothing better than to make tryal of it.

The Pareira Brava is a Root which comes to us from Brazil by the way of Lisbon, but which the War has rendered pretty scarce; however it is to be found among the good Druggists, and is sold at Paris for 40 Livres the Pound: 'tis call'd in Brazil the Universal Medicine, and is made use of there in all kind of Distempers. A Capuchin Monk, who came from thence, told me that he could not give it a greater Character than by assuring me, that in all their Voyages they carried the Gospel in one Pocket and the Pareira Brava in the other.

'Twill be very easy to convince you, Sir, that the Pareira Brava is perfectly good in your Distemper. The Business with you is to restore the Digestions, to the end, that in the first Passages there may not be form'd so much Phlegm and acid Crudities; and it is also necessary to hinder the Serosity of the Blood from spreading it self too much upon the Parts. Now as Experience shows us that
that the *Pareira Brava* does abundantly provoke Urine; it will follow from thence, that it will discharge by the Kidneys the corrosive Acidity of the Mass of Blood; it is also good to break and thin the pituitous and viscous Humors; and it cures the Suppressions of Urine occasion'd by Obstructions in the Kidneys.

One may conclude from hence, that the Salts of the *Pareira Brava*, which are moderately Volatile, are proper to dissolve or separate the too thick and too close Texture of the Sulphur of the Lympha. Finally this Medicine has a light or gentle Bitterness which corrects the Acids of the Stomach, and renders them more pure and fine.

Hence the Chyle becomes better digested and more Balsamic, and fitted to assimilate it with the Blood, and to preserve therein that degree of Division and Fluidity which is necessary for it.

The Method of using this Root with success, is to reduce it to an impalpable Powder, and to infuse thereof the weight of a Demi-gros in a Pint of boiling Water, and let it lye in it all Night, and next Morning boil it one moment. Then pour the Liquor off gently from the Powder, and take of it a Demi-farier in two Cups with a little Sugar as hot as Tea, putting into each Cup 5 Grains of the said Root reduced to an impalpable Powder, which you must stir with a Spoon, that none of it may remain at the Bottom. You may repeat the same Dose about 4 Hours after Dinner, but you must not eat any thing within an Hour after you have taken it.

This Medicine does not oblige you to alter the ordinary Course of your living; and you may continue the use of it several Months together, in which time also you may discontinue it two or three Days together at a time, if you please; but you should take some gentle Purge every Fortnight or Three Weeks during the use of the said Medicine.
The Preparation of the Pareira Brava, as Monsieur Duyvenvoorde uses it.

Take Eleven Grains of this Root, and put it into a Pewter Tea-pot fill'd with boiling Water, and so let it infuse all Night over warm Ashes, or a very small fire; and in the Morning boyl it again, but very gently, till you use it. You must drink it just as you do Tea, and the Liquor which comes from this Infusion must not exceed the Quantity of 5 small Dishes of Tea.

IV. A Letter of Mr. Francis Nevile to the Right Reverend St. George Lord Bishop of Clogher, R. S. S. Giving an Account of some large Teeth lately dugg up in the North of Ireland, and by his Lordship communicated to the Royal-Society.

Belturbet, July the 29th. 1755.

My Lord,

The Curiosity I here send your Lordship, is so far beyond any thing that I have had the honour to communicate to your Lordship, or that I have ever met with, that I presume your Lordship will think it fit to communicate to the Royal-Society; I have sent the Draught, after the best manner I could draw it, enclosed; it is the Draught of two Teeth lately found within Eight Miles of this Town at a place call'd Maghery, in part of the Bishop of Killmore's Lands, sinkin the Foundation for a Mill near the Side of a small Brook that parts the Countys of Cavan and Monaghan.
There are in all four Teeth, two of a larger and two of a smaller sort, the largest is the farthest Tooth in the under Jaw, the other is like it and belongs to the opposite Side; the lesser Tooth I take to be the third or fourth Tooth from it, and has its Fellow: these are all that were found, and one of them in a Piece of the Jawbone, which fell to dirt as soon as taken out of the Earth; there was part of the Scull found also of a very large Size and Thickness, but as soon as exposed to the Air that mouldered away as the Jaw had done.

The Account I had led me last Week to the Place, where I was resolved to make the nicest Search I could; but the Water-wall of the Mill being built, and the Ground all incumbered with the Earth that was thrown up, I could have little Opportunity of doing any thing, but to enquire of the Workmen the manner of finding the Teeth, and where and how they lay. There were some few Peices of Bones found, but none entire, yet by those Bits that were found, one might guess that they were Parts of those that were of a larger Size.

The Place where this Monster lay was thus prepared, which makes me believe it had been buried, or that it had lain there since the Deluge. It was about four Foot under Ground, with a little Rising above the Superficies of the Earth, which was a Plain under the Foot of a Hill, and about 30 Yards from the Brook or thereabout. The Bed whereon it lay had been laid with Fern, with that sort of Rushes here call'd Sprits, and with Bushes intermixed. Under this was a stiff blew Clay on which the Teeth and Bones were found: Above this was first a Mixture of yellow Clay and Sand much of the same Colour; under that a fine white sandy Clay which was next to the Bedd: the Bedd was for the most part a Foot thick, and in some Places thicker, with a Moisture clear through it; it lay fad and close and cut much like Turfe, and would
would divide into Flakes, thicker or thinner as you would; and in every Layer the Seed of the Rushes was as fresh as if new pull'd, so that it was in the Height of Seed-time that those Bones were lay'd there. The Branches of the Fern, in every Lay as we open'd them, were very distinguishable, as were the Seeds of the Rushes and the Tops of Boughs. The whole Matter smelt very fower as it was dug, and tracing it I found it 34 Foot long, and about 20 or 22 Foot broad.

It will be well worth consideration what sort of a Creature this might be, whether Human or Animal; if Human, there was some reason for the Interrment, and for that Preparation of the Bed it was laid on; if Animal, it was not worth the Trouble: if Human, it must be larger than any Giant we read of; if Animal, it could be no other than an Elephant, and we do not find that those Creatures were ever the Product of this Climate. And considering how long this must have lain here, I do not believe the Inhabitants then had any Curiosity or Conveniency to bring such into this Kingdom; for I suppose the best of their Ships could not carry one. Then if an Elephant, or some other Beast which must have proportion to the Teeth, it must have lain there ever since the Flood; and if so, then the Bed on which it lay must be of its own making: whence it will follow that the Flood coming on him while he lay in his Den, he was there drown'd, and covered with Slime or Mud, which since is turn'd into the Substance of the Earth before-mention'd. I forgot to mention that there was a great many Nutshells found about the Bed, perhaps those might have been on the Bushes which composed part of the Bed.

The two large Teeth are of equal Weight, two Pound three Quarters each; the two little Teeth are six Ounces each;
each; but there are some of them wasted, and some of
Holders that go into the Jaw broken off.

I am,

My LORD,

Your Lordship's most dutiful
and obedient Servant.

FRANCIS NEVILLE.

V. Remarks upon the aforesaid Letter and Teeth, by
Thomas Molyneux, M. D. and R. S. S. Phys-
sician to the State in Ireland: Address'd to his
Grace the Lord Archbishop of Dublin.

MY LORD,

WHEN your Grace was pleased to communicate
to me a Letter you received some while since,
containing an Account of an extraordinary Natural Cu-
riosity, lately discover'd in the North of Ireland, in the
County of Cavan, you desired I would give you my
Thoughts concerning it, and the Purport of the Letter:
but truly when first your Grace gave me the Opportunity
of perusing this Account, and I consider'd the imperfect
Sketches of the Teeth annex'd to it, I was not a little con-
cern'd, that upon the making so surprizing a Discovery,
I could not command a Sight of the Originals themselves,
from whence the Draughts were taken; or that so great
a Curiosity should be express'd by the Hand of an Artift
that shew'd so little Skill: however, by the best Judgment
I could make from so imperfect an Information, I told your Grace then, I was pretty well convinced they must have been the Grinding Teeth of an Elephant: Yet I engaged, if hereafter I might be so lucky as to procure a View of the Teeth themselves, I would be more positive in my Opinion, and give the Reasons on which I grounded my Conjecture; as likewise I would have the Shape of the Teeth express'd in their full Dimensions, by more true and exact Figures.

Since that, the four Teeth, with some of the Fragments of the Bones that were found with them, have been brought here to Dublin, where, by the Favour and Assistance of my ingenious Friend Sir Thomas Southwell, I procured the Loan of them, so long as to examine them particularly, make some Remarks, and take the following correct Sketches, that express their Form truly, just as big as the Life; and your Lordship seem'd well satisfied with the Performance of the Artist, when at the same time I produced the Draughts and the Originals from whence they were copied, that we might compare them both together.

Upon the whole, I am now fully convinced, and I can upon sure Grounds affirm to your Lordship, that they must certainly have been the Four Grinding Teeth in the lower Jaw of an Elephant: and that the many loose Fragments of those large Bones that were found with them, must have been Remains of the same Animal. This I take to be one of the greatest Rarities that has been yet discovered in this Country.

In order to clear this Matter 'twill be first requisite to have recourse to, and explain the annexed Figures

Figure the 1st. A A is the large Grinder of the under Jaw on the right Side, weighing two Pounds and three Quarters of a Pound.
b. b. b. b. b. b. are white, rough, indented Borders, Seven in Number, of an irregular Shape, rising about the tenth of an Inch higher than the hard black shining Surface of the Tooth; this rough raised Work serves for the bruising and grinding the Animal's Food, the tough Grains of Rize, Leaves, Fruits and the Boughs of Trees; and is made of so extream an hard Texture, that it resembles large knotted Threads of white Glass, laid on and closely fastned to the dark Superficies of the Tooth: and answers that glaffy Surface wherewith Nature has armed the Outside of the Teeth of most Animals, to prevent their wearing from the constant Attrition in Chewing of their Foods.

c. c. c. c. c. is that part of the Tooth which rises above the Gumms, and continues even now distinguiſh'd from the rest of the Bone, by having its Colour of a different Shade.

d. d. d. d. d. d. d. are many ſtrong Tangs or Roots, seemingly united altogether, by which the Tooth received its Sense and Nourishment, and tho' it was so large and ponderous, by these it kept firmly fixt into the Jaw.

For the Mechanism Nature shews it ſelf to have followed in framing the Teeth of this Animal, is no more than this: whereas in other Creatures, she has divided that bony Subſtance wherewith they chew their Food, each having its peculiar Roots to secure its Articulation in the Jawbone: she has in this of so great Bulk (As Pliny the Naturalift ſtiles it Terreſtrium maximum Elephas,) for the greater Strength, Štabiliment, and Duration of it's Teeth, and the better to provide for a compleat Attrition of the Aliment, in order to perfect the Digestion so thoroughly, as to ſustain the Life of the Animal for two or three hundred Years, (as it is a common received Opinion in the East) She has, I say, contrived to make the Subſtance of the Teeth in their Roots below, and in their upper...
per parts above the Gumms, closely unite together; and coalescing thus, form a few large massy Teeth instead of many small ones.

As for instance, in Man's Body, that is of so much a less size, the Number of the Teeth, (when the whole sett is compleat) reckons to thirty two, whereas in the large Elephant, the Teeth of both the Jaws amount in all but to Eight, besides it's two great Tusks, which rather serve as Horns for its defence than Teeth to prepare it's Food, and therefore I think not so very properly call'd Teeth.

Figure the 2d. E. E. is the smaller Grinding Tooth of the under Jaw on the same side: it's Surface covered over with the same white indented Work, as before describ'd for grinding of the Food.

f.f.f. are three large Roots that kept it firmly fixt in the Jaw Bone.

This smaller Tooth weighed full six Ounces.

Figure the 3d G. G. is the large Grinder of the under Jaw on the left side, much of the Size and Shape and Weight with it's fellow Tooth describ'd Figure the 2d. It shews its Roots and all its parts, with the rough protuberant white Work on its upper Surface made after the same Contrivance, and formed after the same strong Model as the former.

And truly if one considers it, 'tis plain that were not the Teeth of this Creature made of so large a Size, and withal of so massy and firm a Substance, it were absolutely impossible they could resist the Force, and bear all that Pressure wherewith those vast Muscles exert themselves, that move the lower Jaw in Mastication in this so strong an Animal.

Figure the 4th. H. H. is the smaller Grinding Tooth of the under Jaw on the same side; it is less compleat than the small Tooth describ'd before in Figure 2d. for some of the Root is wanting, and part of its outward grinding Surface
Surface is broke off at $k.k.$ so that it weighs somewhat less; yet what remains exactly shews the same kind of Work and Shape of the other Tooth, that answer'd it on the right Side.

These *Four Teeth* here describ'd, fully compleat the Sett of the Teeth, wherewith Nature has furnished the lower Jaw of the *Elephant*; and are answered by just as many more, formed after the same manner in the upper Jaw, as Dr. *Moulin* informs us, who dissected the *Elephant* that was burnt here at *Dublin* in 1681. In it's *Anatomy* p. 40. Speaking of the Teeth he assures, there were besides the Tusks only four Teeth in each Jaw, two in every side: and that these eight Teeth were all *Molares*, so that he had no *Incisores*.

But notwithstanding this, perhaps it will be said, we may not hastily conclude from hence, that our *Great Teeth* dug up in *Ireland* must certainly have been the *Four Grinders* of an *Elephant*, since they might as well belong to some other large kind of *Terrestrial* or *Marine Animal*. As for the Hint of their being *human* or *gigantick*, 'tis so groundless a Thought, and so contradictory to *comparative Anatomy* and all *Natural History*, it does not deserve our Consideration.

To obviate this, I shall take notice first in general, that the differing Kinds of living Creatures, wherewith Nature has stock'd the World, are not more distinguish'd by the Make of any part of their Bodies from one another than by the various Shape and Disposition of their *Teeth*: and hence it is, we shall not find any two distinct *Classes* of *Animals* that do exactly agree in the same Make and Ranging of their *Teeth*.

But yet to be more particular, and make this Point so plain, I hope, as that it may admit of no Controversy, I shall here set down at length, as I find them, the Words
Words of two late Authors, that purposely have described the Teeth of the Elephant.

The first I shall mention is Mr. Patrick Blair, who has publish'd a Treatise he calls Osteographia Elephantina, or a Description of the Bones and other Parts of an Elephant, that died and was dissected near Dundee in Scotland, anno 1706. in the London Philosophical Transactions for April, May, June, July, August and September, 1710. Numb. 326. and 327. Here giving us a Description of the Teeth of this Animal pag. 110. he says, Dr. Moulins well observes that they are all Molars, being two Inches broad in that part of them wherewith they grind, and six Inches and a half long on the Right Side, and five Inches and a half on the Left. Their Surface, tho' flat, is yet very unequal, for they have alternately placed, running from the Right to the Left Side, an Hollowness and then an Eminence; and this Eminence is surrounded by a rough protuberant Border. There are Nine of these Hollownesses and as many Eminences, undulated as they paint Sea Waves,

'Tis remarkable how very exactly all this agrees with our Figures; 'tis true those Hollownesses and Eminences which he mentions to be Nine, do not so nicely hit with the Number of those in our Teeth: but this Difference proceeds from hence, that he describes here the Grinders of the upper, whereas ours are the Teeth of the lower Jaw; tho' such a Distinction as this, I am apt to think, may very well arise even in those of the same Jaw, in various Animals, from some peculiar Disposition in one from another, nay and perhaps in the same Animal, at differing times, according as it happens to be older or younger, but this by the bye.

A little farther pag. 114. and 115. where he gives an Account of those of the under Jaw, he says

The hind Tooth of the Right Side is four Inches, and that on the Left five: the one half of their Surface, where they begin
gin to appear above the Gumms, is semicircular, with the forementioned Ridges and Sulci running transversely, four on the Right Side and five on the Left; the other half (or Tooth I suppose he means) has five of these Eminences where it grinds on the Right, and four on the Left: each of the four Teeth is six Inches long, and has six or seven of the forementioned Eminences and as many Depressions: these Teeth are the most firm, solid and weighty Bones of any Animal yet known.

So much from Mr. Blair.

The other Author I shall produce for the further Illustration of this Matter, is the laborious and accurate Naturalist Mr. Ray, who, in his Synopsis Animalium Quadrupedum, when he comes to give us the Description of the Elephant, has the following Words. *Os pro mole Bellox parvum, quatuor in utrque maxilla Dentibus molaribus seu Dentium molarium Malleis instrutum; si quidem plurimi Dentes in Os solidum & durum ita infixi sunt, ut cum eo & inter se unum & continuum Corpus efficiant. Dentes hi lineas parallelas undulatas octo vel novem in superficie malle efficiunt; suntque reliquo osse candidiores: Malleae integrae, Dentium singularium modo, per Gomphosis maxillis inferuntur. Incisoribus omnino caret.*

Thus Mr. Ray in very proper and expressive Terms describes the Teeth of this Animal: and truly if your Grace will but compare Mr. Blair's Words with his, and the Particulars of both Accounts with the Description and Figures we have before given of the Teeth dug up in Ireland, and observe how they all agree exactly, even so as one may say they tally together, I think it will amount to nothing less than Demonstration, and that all our Ideas have been taken from one and the same Natural Object; and as they, so we, must certainly have described no other Teeth but those of the Elephant.

But then perhaps it will be ask'd what is become of all the rest of the Teeth that were in the upper Jaw, which being
being as firm and solid Bones as those that are here preserved, might for the same reason have still remained intire. But since we find it otherwise, 'tis obvious to imagine a probable Conjecture how this might come about. From what Mr. Novius mentions in his Letter, 'tis plain that the Bed where all these Bones were found, must once have been the outward surface of the Earth, the Green-Sod, producing Rushes, Fern and Nuts: and when the heavy Beast first fell dead upon this Spot, the Scull, with all the Bones and Teeth of the upper Jaw, being the highest Parts of the Animal, might likely fall in such a Posture, as to be exposed some while above the Earth; tho' those of the under Jaw first coming to the Ground, might make themselves a Bed, and being covered with the Mould remain preserv'd, whilst the upper Teeth, and most of the other Bones, lying exposed to the Injuries of the Air and Weather, before they got a Covering, might rot and quickly moulder all away.

But tho' this be allowed, yet still a greater Difficulty remains unsolv'd; how this large Body'd Animal, a Native of the remote warm Climates of the World, should be deposited in this wild Northern Island, (where Greeks or Romans never had a footing,) so many Miles from Sea, and distant from those Places of the Isle where People might most probably resort.

And still to make the Difficulty yet greater, we must consider, not only from the dark black Colour of the Teeth, contracted by their lying long under Ground, and the remarkable Alteration wrought on their bony Substance, which (by the mineral Streams and Exhalations it has imbib'd whilst it was in the Earth) is now become more solid, hard, and ponderous, than it was naturally at first, (nay in some Parts we find it plainly petrif'ied) but also from the perishing of all the other Bones of the Animal's Body, and from the considerable Depth of Earth.
Earth that covered those that were found: we must conclude, I say, from hence, that they have lain in this place for many Centuries; I won't say with Mr. Neville ever since the Flood, because I can't suppose that the slight texture of vegetable Substances, Nuts and the Seeds of Rushes, could possibly have been preserved so long: But this, at least, may safely be affirmed, that these remains must be Contemporaries with some of the remote Ages of the World; which carries us so far back into the earliest Times, that we can ne'er imagine the rude Inhabitants of Ireland, or any of their neighbouring Countries, were Masters of so much Art, in those Days of Ignorance and Darkness, as to make Carriages by Sea strong and capable or of Curiosity and Politeness enough, to transport a Beast of this large size from those far distant Countries where 'twas bred; which they that now attempt do find a Work of vast Care, Trouble and Expense, even in this Age wherein Navigation is brought to such perfection.

These Considerations, my Lord, grounded on other Instances of the like kind, make me inclined to think this Elephant we are speaking of, might not be brought hither by any Care or Industry of Man: but the Surface of this Terraqueous Globe might, in the earliest Ages of the World, after the Deluge, but before all Records of our oldest Histories, differ widely from its present Geography, as to the Distribution of the Ocean and Dry-land, its Islands, Continents and Shores; so as to allow this Beast, and others of its Kind, for ought I know, that may by some such Accident hereafter be luckily discovered, a free and open Passage into this Country from the Continent.

For otherwise, how can we otherwise explain that that vast large stately Animal the Moose-Deer, little inferior to the Elephant itself, could have been brought to Ireland, (where elsewhere I have shewn it formerly was common) from distant North America, even long before that Quarter
ter of the World was known, and is the only Region I can hear, where this great Beast is found at present.

And can we well imagine that Foxes, Otters, Badgers, Tigers, Wolves, with Linxes and such ravenous Animals as we have been told, have lately been discovered by the great Snows that fell this present Winter in the Island of Sardinia and other Places, should ever be imported (being useless noxious Beasts of Prey) by the Industry of Man, to propagate in Islands, that they might destroy Men's Food and Flocks, and make their Lives not only uneasy but unsafe?

Nay how can we suppose that Birds of shortest Flight, the various Sorts of poisonous Serpents, and of offensive Creeping Vermin, with all the various Tribes of Smaller Insects, could possibly be found in Islands, unless they had been flock'd with those Inhabitants when the Intercourse between them and the Continent was free and open.

But in whatever manner this Elephant (to return to our Subject) might first have made its way for Ireland; this is beyond dispute, that the Bones of Elephants have been discovered deep under Ground, in other Places as well as this Kingdom: and those too out of the way, far distant from the Native Countries of this Animal.

For not many Years ago, in a Hill near Erfurt, a Town of the Upper Saxony in Germany, several Parts of the Skeleton of an Elephant were Dug up: on which Occasion Wilhelmi Ernesti Tentzelius Historiographer to the Duke of Saxony, Writ a Letter to the very learned Antonio Magliabechi, Library Keeper to the great Duke of Florence: This Treatise is published, but I have not been so lucky as to procure a Sight of it, and know no more but just the Title-page Wilhelmi Ernesti Tentzelii Historiographi Ducalis Saxoniae Epistola, de Sceleto Elephantino Tonne nuper effso, ad Antonium Magliabechium, Magni Ducis Heururiae Bibliothecarium.

And
And I am well persuaded, by the best construction I can make of those imperfect and obscure accounts, we have in Evert Isbrand Ildes curious Travels from Muscovy to China over Land; Chap. the 6th, (which he confesses he only gathered from the barbarous Officaks Inhabitants of that Country) concerning the vast Teeth and Bones and Limbs of Mamimuths as he calls them, frequently found (and diligently sought after to make profit of them) in the Hills, and Banks of several Rivers in Siberia, the Keta, Jenize, Trugan, Montgamseca and Lena; that they are nothing else but the Remains and Skeletons of Elephants buried there, and accidentally discovered by the Earth's opening, and falling down on the sudden Thaws, after severe long Frosts. But of this, please to consult the Author, whose Words are too prolix to be inserted here.

But to bring this Matter still nearer home to our selves, Mr. Camden in his Britannia is of opinion, that those great monstrous Teeth and Bones, which he takes notice to have been at several times dug up in many parts of Great Britain, must have been the Remains of Elephants; but then he thinks, they must be of those that Dion Cassius the Historian tells us, the Roman Emperor Claudius brought over, when he made his Expedition into that Island. But that this truly is so, I own is but Surmise as yet, and has not been so fairly proved by him or any other, as that we can rely upon't with satisfaction.

What Mr. William Somner the learned Antiquary has published in his Discourse of Chartham News is more remarkable; (this is reprinted lately in the Philosophical Transactions for July 1701. No. 272.) where he informs us, that in the Year 1668 in the Village of Chartham near Canterbury in England, digging within 12 Rods of a River, they found a Parcel of strange monstrous Bones, some whole, some broken, together with four Teeth perfect and sound, each weighing something above half a Pound, and some of them almost
most as big as a Man's Fist. They are all Cheek-Teeth or Grinders; the Earth in which they lay being like a Sea Earth, or Fulling Earth with not a Stone in it.

'Tis observable how this Account in many of its Circumstances, agrees with that of Mr. Nevil in his Letter to your Grace: as that the Teeth were all Grinders, Four in number, found with other large broken Bones near a Brook, and in a Clayey Earth, without a Stone: but then the weight and Magnitude of our largest Teeth, so far surpass'd those that were found in England, that these did not come up to a fifth Part of those, which shows they could not be the Teeth of the same Animal. I must confess the Author does not so much as suspect they were Elephant's Teeth, but on the contrary is of opinion that they belong'd to another Species, the Hippopotamus or River-Horse, a Beast that's yet a greater Stranger in these Parts of the World, than the Elephant itself; and therefore it's Passage hither can never be accounted for, but by some such like Supposition as we have made.

However Mr. John Luffkins in his Letter, wherein he designs to have reference to that Discourse; and which is inserted in the Philosophical Transactions for Sept. 1701. No. 274. differs in his Judgments from Mr. Somners about these Teeth, which he thinks must have been Elephant's Teeth; as he is positive those large Bones he describes in the same Letter, and found near Harwich in Essex, certainly must have been.

Not having seen, much less examined, any of the Bones or Teeth concern'd in this Controversy; either those that were found in Kent, or those in Essex; I cannot well take upon me to determine any thing in this matter; tho' those dug up at Chatham, as I understand, may still be perused by the Curious among the Natural Rarities of the Royal Society in their Repository at London. But this at present I can safely say, that if the Figures of the Teeth given
given us by Mr. Somner, and represented in the Plate of
the foremention'd Transaction No. 272. be genuine and
well express'd (as I have no reason to doubt, as coming
from one so skilful and so accurate) they no way seem to
agree either in Shape or Make, or in that particular and
Characteristic Work on the grinding Superficies, with the
Teeth of the Elephant; or with the Description and Figures
we have given, which I am sure are both correct and na-
tural.

I should now, my Lord, make some Apology for de-
taining your Grace so long upon what may seem so light
and trivial a Subject, a Piece of mere Curiosity: but I
am so vain as to hope, whatever others may fancy, it may
not appear so inconsiderable altogether to your Lordship's
more discerning Judgment.

For I am inclined to think, (even from these Imperfect
Hints) that if we had more correct Histories and Obser-
vations of this kind, made in distant Countries, and skil-
fully registered, with all their instructive Circumstances,
they might lead us into great and momentous Truths re-
ting to the Deluge; to the wise Methods of Providence,
in replenishing all Regions of the World with Animal Beings
soon after the Flood; and to the Knowledge of several im-
portant Changes that may have happen'd on the Surface
of this our Terraqueous Globe: Inquiries that are truly
worthy the utmost Application of the most learned Divine
and the most sagacious Philosopher.

But I shall stop here, and only beg leave to subscribe
my self, with the utmost Respect,

My LORD,

Your Graces most devoted
faithful and humble Servant.

T. Molyneux.

This
This Letter of Mr. Nevile with Dr. Molineux's curious Draughts of the Teeth, and his learned Remarks upon them, having been produced and read before the Royal Society, they ordered that what Teeth they had of like sort should be look'd out and laid before them; to which Sir Hans Sloane was pleased to furnish a yet greater Variety, out of his incomparable Collection of Natural Rarities. And to obviate all Doubts, there being at this time in Westminster the entire Skull of a large Elephant with the Teeth in it. That was likewise ordered to be viewed and compared with the Figures: which done, it appeared that the Teeth in question could be no other than those of an Elephant.

By this Enquiry we were likewise satisfied, that the Number of Teeth found, being but four, was no Objection: it appearing that the Number of Molares in this Animal is not certain. Pliny Lib. XI. Cap. 37. says expressly Dentes Elephanti in usum mandendum quatuor, præter eos qui prominent. And in the Remains of that mighty Elephant described by Tenzelius, Phil. Trans. No. 236. there were no more than four Teeth found. In that at Westminster there are Six, viz. One in each lower Jaw, and Two in each of the Upper, whereof the inner Tooth is about three times as long as the other, and both together longer than those of the under Jaw by about an Inch; the upper small Teeth being much worn by grinding. These we have thought fit to represent by Fig. 5. shewing the rough grinding Surface of the left under Tooth, being considerably Concave; and by Fig. 6. the same Roughness on the upper Teeth is shewn, having a Convexity tallying with the Concavity of the under, which is a Circumstance not observed by any of those that have described them.

And altho', by the Observation of Mr. Du Verney, Dr. Moulins, and Mr. Blaire, who dissected three different Elephants, it appear that each of them had eight Molares: yet from them it is also evident that in the division of them Nature observes no Rule. For Dr. Moulins found the two Teeth in each of the upper Jaws of that he dissected, to be divided
ded after a different manner; so that the inner Tooth on the one side, and the outer on the other, was bigger than its adjoining Fellow, yet not so as to be very unequal: and Mr. Du Verney and Mr. Blaire had on both sides the much greater Tooth outwards: whereas the Westminister-Skull, on the contrary, has only a small one outwards, and the much greater Grinder within. All which considered, we may with Assurance conclude, that this Elephant found in Ireland had but four Teeth in his Head when he died; and that the two Greater were those of the upper Jaws, and the other two those of the Under.

Again, by the Size of the grinding Part, we may conclude these to be the Teeth of a very young and small Elephant; since they are not much above half the Length of those that are to be seen at Westminister, which belonged to a Beast of not more than between 10 and 11 Foot high; nor much above one Third of the Length of a fossile Elephant's Grinder in the Royal Society's Repository, the which is here represented by Fig. 7. (all the Figures being drawn to the Scale of half their true Dimensions). Hence it is not to be marvilled that the Bones of so young an Animal, having not acquired their Firmity, as being in a growing State, should be dissolved by long lying in the Earth, as also the Roots of the Teeth.

On this Occasion, perhaps it may not be amiss to quote a Passage out of Mathew Paris his History, who assures us, that in his Time Louis IX. (afterwards St. Louis) King of France, made a Present of an Elephant to his Cotemporary Henry III. of England; and that in the Tear 1255, after the English had been fourscore Ters Masters of Ireland. Of this says Mathew, Nec credimus quod unquam aliquis Elephas vi-fus est in Anglia präter illum.

The Author, having some 1 Years since publish'd a Comment on Julius Vitalis his Epitaph, which, (together with his Monument) is to be seen at Bath; does now present the publick with another Volume of Belgic Antiquities; intending hereby, to illustrate part of a Statue, which was found likewise near that City, and is at this time immured near the Monument aforesaid, at the eastern End of the Abby-Church, looking toward the Grove.

This Fragment of an Equestrian Statue, is in Basse Relief: The Rider has in his right Hand a Hafta pura, and a Parma in his left; as in Fig. 1. of the Book. It appears from 2 Dio that Caius and Lucius, Cesars, (the Nephews, and adopted Sons of Augustus) had each of them a Parma and an Hafta given him: and there being no Instance of this Honour paid to any of an inferior Rank among the Romans, but only to such as were of very great Quality; if not to Cesars only; we are from hence be allowed to think, That this Statue represented some Person of that Quality.

But to discover the particular Person, (if it might be done) the Author compared a very good Draught he had procured of this Horseman, with such Roman Coyns,

1 Ilæ Dunmon: MDCCXI,
2 Lib. LV.
as He could meet with. This Comparison shewed a great Resemblance between the Face in the Statue, and that in two of Geta's Coyns.

This Argument, drawn from the Similitude of Faces (of great force to determine the Reader's Judgment in favour of Geta) is farther confirmed by the Horse; a Creature of which Geta was very fond; insomuch, as that He affected to be represented under the Figure of Castor, (as the Roman Emperors often were under the Figures of their Gods) of whom it is said, Castor gaudet Equis; — Of this Figure there is in 3 Oisellus, a Coyn of Geta's, very much to this purpose; represented Tab. iv. fig. 5. of this Book.

These things bring to mind, the Authority which Geta had in South-Britain: where (as *Herodian affirms) all matters were under his Administration, during the Stay which Severus and Caracalla made in the North; which was a Year, or more. In this time, Geta had it in his Power, to do many things, in favour of Cities and Countreys, here in the South. The great Generosity of his Mind prompted Him to publick Works; such as are, to this day, attested by *Inscriptions, with his Name in them: and it is highly probable, [That this Statue was erected to Geta on some such account.]

If this be granted, (as from the concurrence of so much, and so good Testimony, it seems highly probable) here is a large and pleasant View opened into Antiquity; not of late taken notice of by any Writer: It shews, that Geta was a great Benefactor to old Bath; either by laying, in a perfect Morafs, the Foundation of that Town; or by preserving the Hot-Springs entire, from the Influx of

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*Tab. XLII. 1.
*Lib. i. Sec. XLVIII.
*Guertii Thef. Incriptionum, pag. CLVII. 2, 3, 4, 5.
other Waters; or both: Works of great Munificence, and becoming Geta's Spirit. By these, or some such Ways, it is probable, this People was obliged to Geta; but no one is more probable, than that of preserving the Aqeæ Calida; which were in those days so famous, as to give a Denomination to the place. It is well known, That Rome had her Thermae Severiane and Antoniniane, so called from their respective Founders; the former being built by Severus, the Father; the latter by Antoninus, the Brother, of Geta: so that to take care of Baths, was a sort of Greatness, that Family seemed to delight in; and Geta may reasonably be supposed, to have his share of this Delight.

From the great probability of this Opinion, the Author has, out of Love to his native Country, and the Honour due to Geta, collected and put together, what He can meet with relating to that Emperor. He has made a new Edition of Geta's Life, from the Historia Augustæ Scriptores; relating it to its true Author, Julius Capitolinus; and explaining it, with the Notes of Casaubon, Gruter, and Salmasius; to which he has added some of his own. He has reprinted all the Inscriptions he can meet with, of Geta's, and many of his Coys; with short Notes on both.

After all this, He is not so far engaged in this Opinion, but that if, (by any Inscription on the Basis of this Statue, or any other testimony) it shall hereafter appear, that this Fragment deserves another Explication, he shall readily comply with any such clearer Testimony: being no way disposed, to give farther credit to this broken Monument, than shall answer the imperfect Condition it is now in.

To this Disertation, de Geta Britannico, He has added the Chronology of his illustrious House; shewing, how his Father, Severus, from a private Gentleman in Africa, came
came by degrees to be Emperor of Rome; and indeed one of the Greatest, that ever Rome had: How He, with his two Sons, Bassianus, and Geta, (three Roman Emperors) resided, at one and the same time, here in Britain; and from hence sent their Imperial Edicts, Orders, and Dispatches, into all parts of the Empire: and after an amazing Greatness of about twenty four Years, and a Course of almost all Virtues and Vices, at length tumbled down; submitting to the Accidents and Fate of other Men; and were all buried at Rome, in the Septizodium built by Severus.

To these Memoirs of Geta, the Author has subjoyned a Discourse, concerning that curious Cimelium, which was, some Years since, found at Athelney in Somerset. It did belong to K. Ælfred, and is now in the Possession of Col. Palmer of Fairfield, in that County. Beside the critical use made of it, by the learned Dr. Hickes, our Author writes of it, as an undeniable Instance of the use of Images, coming from the Heathens into the Christian Church.

The Book is adorned with several Cuts, of the Broken Statue at Bath, of two of Geta's Silver-Coins, of the Septizodium Severi, (out of Perac) and three sides of the ææMihw Ælfred.

* In Thesauro Ling. Septentrionalium non ita pridem edito.

FINIS.

LONDON: Printed for W. INNYS at the Prince's Arms in St. Paul's Church-yard. MDCCXVI.
PHILOSOPHICAL TRANSACTIONS

For the Months of Jan. Febr. and March 1716.

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I. An Account of several Nebulæ or lucid Spots like Clouds, lately discovered among the Fixt Stars by help of the Telescope.

In our last we gave a short Account of the several New-Stars that have appeared in the Heavens, within the last 150 Years, some of which afford very surprizing Phænomena. But not less wonderful are certain luminous Spots or Patches, which discover themselves only by the Telescope, and appear to the naked Eye like small Fixt Stars; but in reality are nothing else but the Light coming from an extraordinary great Space in the Ether; through which a lucid Medium is diffused, that shines with its own proper Lustre. This seems fully to reconcile that Dificulty which some have moved against the Description Moses gives of the Creation, alledging that Light could not be created without the Sun. But in the following Inftances the contrary is manifest; for some of these bright Spots discover no sign of a Star in the middle of them; and the irregular Form of those that have, shews them not to proceed from the Illumination of a Central Body. These are, as the aforesaid New Stars, Six in Number, all which we will describe in the order of time, as they were discovered; giving their Places in the Sphere of Fixt Stars, to enable the Curious, who are furnish'd with good Telescopes, to take the Satisfaction of contemplating them.

The first and most considerable is that in the Middle of Orion's Sword, marked with θ by Bayer in his Uranometria, as a single Star of the third Magnitude; and is so accounted by Ptolemy, Tycho Brabe and Hevelius: but is in reality
reality two very contiguous Stars environed with a very large transparent bright Spot, through which they appear with several others. These are curiously described by Hugenius in his Systema Saturnium pag. 8, who there calls this brightness, *Portentum*, cui certe simile aliud nusquam apud reliquas Fixas potuit animadvertere: affirming that he found it by chance in the Year 1656. The Middle of this is at present in $\pi 19^\circ.00$, with South Lat. $28^\circ.\frac{1}{2}$.

About the Year 1661 another of this sort was discovered (if I mistake not) by Bullialdus, in Cingulo Andromedae. This is neither in Tycho nor Bayer, having been omitted, as are many others because of its smallness: But it is inserted into the Catalogue of *Hvelin*, who has improperly call'd it *Nebulos* instead of *Nebula*; it has no sign of a Star in it, but appears like a pale Cloud, and seems to emit a radiant Beam into the North East, as that in *Orion* does into the South East. It precedes in Right Ascension the Northern in the Girdle, or $\nu$ Bayer, about a Degree and three Quarters, and has Longitude at this time $\nu.24^\circ.00$, with Lat. North $33^\circ.\frac{1}{2}$.

The Third is near the Ecliptick between the *Head* and *Bow* of *Sagittary*, not far from the Point of the Winter Solstice. This it seems was found in the Year 1665, by a German Gentleman *M. J. Abraham Ihle*, whilst he attended the Motion of Saturn then near his *Aphelion*. This is small but very luminous, and emits a Ray like the former. Its Place at this time is $\nu 4^\circ.\frac{1}{2}$ with about half a Degree South Lat.

A fourth was found by *M. Edm. Halley* in the Year 1677, when he was making the Catalogue of the Southern Stars. It is in the *Centaur*, that which Ptolemy calls $\delta$ *Theri* $\tau$ *vota* $\epsilon k$ *phos*, which he names *in dorfo Equino Nebula* and is *Bayer* $\omega$; It is in appearance between the fourth and fifth Magnitude, and emits but a small Light for its Breadth,
and is without a radiant Beam; this never rises in England, but at this time its Place is $m^ 5\frac{3}{4}$ with $35^\circ\frac{1}{2}$ South Lat.

A Fifth was discovered by Mr. G. Kirch in the Year 1681, preceding the Right Foot of Antinous: It is of its self but a small obscure Spot, but has a Star that shines through it, which makes it the more luminous. The Longitude of this is at present $n. 9^\circ. circiter$, with $17^\circ\frac{1}{2}$ North Latitude.

The Sixth and last was accidentally hit upon by M. Edm. Halley in the Constellation of Hercules, in the Year 1714. It is nearly in a Right Line with $\zeta$ and $n$ of Bayer, somewhat nearer to $\zeta$ than $n$: and by comparing its Situation among the Stars, its Place is sufficiently near in $m^ 26\frac{1}{2}$ with $57^\circ. 00$. North. Lat. This is but a little Patch, but it shews it self to the naked Eye, when the Sky is serene and the Moon absent.

There are undoubtedly more of these which have not yet come to our Knowledge, and some perhaps bigger but though all these Spots are in Appearance but little, and most of them but of few Minutes in Diameter; yet since they are among the Fixt Srars, that is, since they have no Annual Parallax, they cannot fail to occupy Spaces immensely great, and perhaps not less than our whole Solar System. In all these so vast Spaces it should seem that there is a perpetual uninterrupted Day, which may furnish Matter of Speculation, as well to the curious Naturalist as to the Astronomer.
II. Nova & tuta Variolas excitandi per Transplantationem Methodus, nuper inventa & in usum tracta:

Per Jacobum Pylarinum, Venetum, M. D. & Reipublicae Venetae apud Smyrnenses nuper Confulum.

Operationem Medicam inventu non minus quam eventu mirandam Orbi literario pandimus; Non a Physicæ cultoribus, aut à doctis in Apollinea arte viris, sed à plebea rudique gente in humani generis adjumentum, in sævissimi morbi solamen detectam, vetustis Scholarum Lucubrationibus, sedulis recentiorum investigationibus ignotam; sed quœ ex innocentioris & apollineorum familias penu defluxit. Verum ignoratur ejus adinventor: in Græcia tamen, præcisè in Thessalia, primó invaluisset certissimum est; hinc in propinqua successive ferpendo loca & civitates, in Byzantinam tandem irrepit urbem; ubi latuit quidem initiò per aliquot annos, rarò quoque, & inter humiliores dumtaxat recepta: Immaniter autem graffante nuper Variolarum epidemia, latius innotescere cœpit; Numquam tamen sublimiores aulae est ingredi aulas; donec Nobilis quidam nec obscurus inter praæstantiores Græcos, & ex antiquo Caryophyllorum stipite clarus, mihi verò intimiori amicitiae titulo notus, anno salutis 1701, sub hyemis finem, serio mè quidnam de hac infectione sentirem, consuluit; & an ad eandem in quatuor propriis filiis celebrandam praæstarem assistam: Nam tum temporis lethaliter totam ferè civitatem morbus hic invaserat; qui summum ei de natorum salute metum incuriendo, anxium valde reddiderat. Verùm quid ipse super ignota re decernerem, nulla praecedente novissimæ hujusce methodi notitia, penitus ignorare me dixi; ac simul Operatoris conveniendi copiam petii. Triduo praæcto, cum ad amicum denuò accessissem, & de eadem mater-
materia fermo iteratò inter nos esset initus; Ecce paulò post mulier Græca ad decentiam satis composita cubiculum intravit; quæ totam operationis seriem, modum, locum, tempus, cæteraque circumstantias, ut ego deinceps aperiam, clærè satis latèque nobis exposuit; quamquam ipsa veram ex initiatione excitationis Variolarum causam haud intellexerit: His experimenta & causus innumeròs tuò semper & ad salutarem eventum perductos annexit; è quibus aliquot (omnes enim quis potuisset in urbe amplissima exquirere?) verissimos èst ex assertoribus sude dignis deprehendi: Idciròre bene penitata, rationi & naturæ haud absenam omninò compéri : præsertim autem causibus jam dictis permutus, amico jam servidius post aliquot dies consilium iterum expetenti, mè haud alienum, sub levi tamen hæsitantia, præbui; Quà; tamquam dato assensu, arreptrâ, & deservando regimine toto ægrotationis tempore satis edoctus, insitionem per mulierem Græcam in quatuor filiis audacter instituit: quorum tres natu minores (quinquennes, & vix septennes) leviter ægrotarunt; paucisque apparentibus pustulis post hebdomadam, febre penitus & periculo evaferunt: Ætate vero major, octavum super decimum agens annum, graviter decubuit: Nam continentis febre & malignante correptus, superveniente difficilium symptomatum syndrome, plasculïque quamquam non copiosis exanthematibus obturus, vix post decimum quartum diem morbum elusit: Quod ego atrabilari ejus temperamentó, succisque pravis, ut & ne gleæ prius (contrà datam admonitionem) corporis expiationi tribuendum velit. Felix operationis eventus mirum quam multas nobiliorum familias ad imitationem traxit; Ut hodie sine hæsitantia, præter timidiusculos aliquos, unusquisque transplantationis emolumentum sentire velit. Soli Turcæ, utpote Fati decredis addixti minusque dociles, hanc neglexerunt hucusque.

Naturalis est penitus hæc Operatio, nulloque obducta superstitionis fuco; quamvis ipsum Transplantationis No-
men prima facie haœsitantiam pariat. Differt toto coelo à Curationibus Sympatheticis; cò magis à Magnetismo-illo transplantatorio, per quem Morbi ex uno subjecto (mediante imaginaria quadam, gratis effecta, & imperceptibili Mumia) in aliud traduci dicuntur; qua de re Tenzelius, Bartholinus, Maxuellus, Etmullerus, aliique ex Recentioribus, dobi alias Vīri, agunt sedulō; qui verustas Antiquorum in Medicina quisquilias expurgandas esse cum jactent; inter quisquilias ipsi quandoque se voluntar, novissima vanitatis amurca venerandam haœ- nus purissimamque Scientiam deturpantes. Quare, ut verum fatear, quemadmodum Operationes hæ Magneticæ, vel Sympatheticæ superstitionis vanitas suspicione non carent (ut in Unguento Armario, Pulvere sympathetico, & similibus), utpōr extra activitatis sphæram in distans agentes; ita Variolarum Transplantatio, vera, meram, pura Physica est; quia puris mediis Physicis, & ad oculum parentibus, nec non ipso sensibili contactu compleetur; ut è mœx dicendis clariœs elucefcer.

Hujusmodi igitur Variolarum excitatio sit per Metaphorice sic dictam Infitionem sive Transplantationem; quæ nihil aliud est, quam fermenti morbifici seu puris ex Variolis extraœti in corpus sánum, per vulnuscula ad hoc facta, insertio.

Physicus excitationis modus hoc pacto succedit. Intrœum in vulnuscula pus veri fermenti suscepit indolem; Hinc beneficio circulationis per vasæ & canales proprios ad sanguinis massam delatum, dispositas ad hoc particulas, & delitescente vitio turgidas statim aggregatur, inficit, venenumque communicando latitans in illis seminarii fermentativum excitat, agitat, actuat, inque motum ciet; Unde ebullitio universalis, seu fermentatio oritur; Vi cujus impuriores & heterogeneæ partes separeœ critice protruduntur ad cutem; natura interim placidè virtute talis operationis totum opus moderante.

Qq q 2

Sed
Sed ad Transplantationem ipsam, & ejus celebrandae formam properandum; modum quo eadem Mulier Transplantatrix tutissime operabatur fideliter describendo, ordinem, et aliterque omnia; ex quibus regula quædam in situ poterit pro operatione hac obeunda. Cæterum quamvis de omnibus oculatus Teflis, ut ingenuè fater, esse non possim; ex Operatrixem tamen ore multa, plura ex fidei relatione tranfumpsi; plurima & potiora ipfem met obseravi. Omitto quæ ad majorem confirmationem universalis fama canit. Quibus omnibus praèstandam esse fidem candidè ac strenue affevero.

Ergo primò tempus ad celebrandam institionem opportunum feligendum est; Quod secundum Operatricem hibernum desideratur; & non nisi tali tempore ipsa institionem instituebat: Venum ego pariter idoneum crediderim, propter clementiorem aeris temperiem.

Secundò selectissimum adhibet fermentum; Pus fecilicet inferendum non ex quolibet subiecto recipit; sed Variolis epidemicè graffiantibus, è pustulis jam maturis decumbentis alicujus pueruli alias uosáps, iisque benignis, punctione illud extrahit exprimitque; & in conchulam aliquam vel vitreum vasculum mundissimum, nec nimis actu frigidum reponit reconditque; quod Vasculum bene fartum rectum in pedissequi sinum fovendum intrudit; mox sine mora ad operationem properat: Pus ex institiis rejicit, ut inefficax. Quod tamen ego benignoris indolis, nec minoris interim energiae fecerim: qua in re experientia confulenda.

Tertio, temperatissimum vult patientis inhabitandum cubiculum quoad aeris modificationes.

Quartò, ad operationem jam celebrandam accedens mulier, frontem in continio capillorum & quidem medio in loco; mentum & utralisque genas acu ferrea vel aurea pungit; non rectà, sed obliquè impingendo, cutoque acuta cuspide à subiecta carne aliquantillimum separando.
hinc eadem acu pus jam praeparatum in vulnusculum in-
stillat et vasculo, intruditque; superinducta per fasciam
ligatura: Manus item ambas in metacarpis, pedes in me-
tatarcis eodem modo ferit, pulsque inferit, fasciaque le-
ninger stringit; serio imponendo patienti, ne partes il-
las scalpat madefaciave. Potius carnosifora pungere
loca, quatenus inflammationibus dolorique minus ob-
oxia, nec tendinibus intertexta.

Præter hanc operandi formam cæteri omnes rejiciun-
tur modi, utpote absoni, inuñitiati, malè suceedentes, in-
felicisque exitus.

Interim lectulo moderatè manendum, neque plus quàm
opus fuerit jacendum.

Qvintò bonum in sex rebus non naturalibus regimen,
præcipue in viætù, injungit; Non solum enim vino &
carnibus, verum etiam carumdem jusculo rigorosè pa-
tentes ad quadragesimum diem interdicti: Pluribusque
monitum non curantibus sæpius male suceñit; nam ad
oculos, poena erroris, novas erupisse pustulas, aliaque
non parvi discriminis symptomata supervenisse, quis
est.

Sic rite peracta transplantatione, non omnibus eodem
temporis intervallo suscitari solent variolarum symptom-
ta; Variè enim fermentum agit; serius vel citius, pro-
urn unicuique proprium favet temperamentum, ætas, ro-
brur: Quamquam Variolae ipsæ, in septimo ferè semper
apparere incipiant, qui dies vere criticus est. Nec defu-
ere, quod rarò contingere soler, quibus statim primo die
efloruerint.

Symptomata ægrotantibus evenientia variant secundum
temperamentorum diversitatem, successum in massa san-
guineæ habitudinem, & particularam in singulis naturæ
dispositionem: rémissiora némpe vel intensiora ingruunt;
At commune ter graffantibus non dissimilia, quamvis ele-
mentiori ut plurimum facie: Plures vix alterationem la-
esionemve sentiunt aliquam.

Exc-
Excitatae Variolæ fere semper sunt de genere Distinctarum; nec numero multæ; Decem, ut plurimum viginti, triginta, rarò ad centum, rarissimè ad ducentas erumpunt. Notandum primò nonnullis, unico dumtaxat vulnificulo ad brachium infìcto contentos, excita6e variolas; paucisque apparentibus pustulis præservatos tamen imposte-rum suisse à contagio.

Notandum secundò accidisse interdum, ut ex insitutione nullæ penitus excitatae fuerint Variolæ, vel ob non praèxistentem ullam prorsùs variolicam in corpore dispositionem, vel ob enervatum infra6tumque fermenti contagium: At postea grañlante populariter morbo, corretta sunt promiscuè ejusmodi corpora jam insitionem passa, communi cæterorum sorte.

Tertio, Inscriptionis loca seu vulnificula in pustulas semper evadere solent: Quibusdam verò excre6ent in purulenta tubercula, nullis interim apparentibus pustulis; Nonnullis in apostemata quidem majora degenerant magnam puris copiam effundentia: Non semel eadem loca, in pedibus præcipue manibusque, summo cum dolore intumescent; pureque effuso subsident, iterumque in tumorem attolluntur. Quibusdam, rarissimè tamen, ad glandulosas partes & emunctoria, post aliquod tempus, abscessus emergunt, ac suppurantur paulatim: ludente sic in diversiformi corporum crasi natura.

Postremò, nunquam sèrè ex transplantatione hac funeste quid accidisse observatum fuit hacenus; etiamsi in quocumque sexu, temperamentò, òtate celebrata fuerit; quinimò, rite rexèque tractata, & in corporibus per peritum Medicum aptè præparatis, certissimam promittit salutem. Variolæ enim hoc modo excitatae benignoris sunt indolis, quam sunt illæ quæ populariter grañlantur; Utpotè ex fermento, seu contagio, omni malignitate car-rente promotae: Ebullitio, per quam massa sanguinis agitatur ac totum opus perfitur, blandè non violenter,
moderante natura, conamina sua molitur; Sed præter hæc, tempus ad operationem atque anni tempestatas magis idonea pro transplantatione ad libitum eligi, ut & corpus insitioni subjiciendum congruis adminiculis ad recipien-
dam illam ex arte præparari disponique poterit; Quod re-
vera maximi ad salutarem faustumque morbi succedum
momenti censeri debet.

De hæc re vide etiam Philos. Trans. No. 339.

III. Problematis olim in Actis Eruditorum Lipsæ
propositi Solutio Generalis.

IN Actis Eruditorum pro mensë Octobri Anni 1698.
pag. 471. D. Johannes Bernoullius hæc scripsit:
"Methodum quam optaveram generalem secan
di [Curvas] ordinatim positione datas, sine algebraicas sine
transcendentales, in angulo recto sine obliquo, invari-
bili sine data lege variabili, tandem ex voto erui: cui,
"Leibnitio approbatore, ne quid addi posset ad ulteri-
orem perfectionem, & vel ideo tantum quod perpetuo
ad æquationem deducat: in qua si interdum indetermi-
natæ sunt inséparables, methodus non ideo imperfecti-
or est, non enim hujus sed aliæ est methodi indetermi-
natas separare. Rogamus igitur fratrem ut velit suas
quoque vires exercere in re tanti momenti. Sucepti
laboris non pœnitebit, si felix succedus fructu jucundo
compensaverit. Scio reliquiturum suum quem nunc dovet
modum, qui in paucissimis tantum exemplis adhiberi
poteft.

Hi tres Viri celeberrimi fese, jam ab annis quatuor vel
quinque circiter, in solvendis hujusmodi Problematibus ex-
ercerant. Absque spiritu divinandi eandem solutionem
cum Bernoulliana tradere difficile fuerit. Sufficit quod Solu-
tio sequens sit generalis, & ad æquationem semper
deducat.
**Problema.**

Quæritur Methodus generalis inveniendi Seriem Curvarum, quæ Curvas in serie alia quacumque data constitutas, ad angulum vel datum vel data lege variabilem secabunt.

**Solutio.**

Natura Curvarum secundarum dat Tangentes earundem ad intersectionum puncta quacumque; & anguli intersectionum dant perpendicula Curvarum secantium; & perpendicula duo coeuntia, per concursum suum ultimum, dant centrum Curvaminis Curvæ secantis ad punctum intersectionis cujuscumque. Ducatur Abscissa in situ quacumque commodo, & sit ejus Fluxio Unitas; & positione perpendiculi dabit Fluxionem primam Ordinatae ad Curvam quæsitam pertinentis; & Curvamen hujus Curvæ dabit Fluxionem secundam ejusdem Ordinatae. Et sic Problema semper deducetur ad æquationes. Quod erat faciendum.

**Scholium.**

Non hujus sed alius est methodi æquationes reducere, & indeterminatas separare, absolutè si fieri possit, sin minus per Series infinitas. Problema hocce, cum nullius fere sit usus, in Actis Eruditorum annos plures neglectum & insolutum manse. Et eadem de causa solutionem ejus non ulteriorius prosequor.
IV. Some late curious Astronomical Observations communicated by the Reverend and Learned Mr. James Pound, Rector of Wanstead, and R. Soc. Soc.

The Occultation of Jupiter by the Moon observed at Wanstead the 14th of July in the Morning, 1715.

Having after Midnight carefully corrected the Clock by no less than ten Observations of the Altitude of the Lucida Arietis, the Error thereof was found 5' 13" too fast, the extremes not differing above 6": And in the morning about 7h, by as many Altitudes of the Sun, with a like Agreement, the same Error was found 5'. 14", to be deducted from the Times shewn by the Clock.

<table>
<thead>
<tr>
<th>Julian 13°. P. M. N.</th>
<th>Time by the Clock</th>
<th>Time corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Third Satellite of Jupiter was hid by the Moon</td>
<td>13 27 33</td>
<td>13 22 20</td>
</tr>
<tr>
<td>The first Satellite was hid</td>
<td>13 32 35</td>
<td>13 27 22</td>
</tr>
<tr>
<td>The Second Satellite was hid</td>
<td>13 34 25</td>
<td>13 29 11</td>
</tr>
<tr>
<td>The first Contact of the Limbs of X. and ο</td>
<td>13 34 54</td>
<td>13 29 41</td>
</tr>
<tr>
<td>Jupiter wholly hid</td>
<td>13 36 23</td>
<td>13 35 10</td>
</tr>
<tr>
<td>The Third Satellite came out from behind the dark side of the Moon</td>
<td>14 7 25</td>
<td>14 2 12</td>
</tr>
<tr>
<td>The first Satellite</td>
<td>14 12 25</td>
<td>14 7 12</td>
</tr>
<tr>
<td>The Second Satellite</td>
<td>14 14 38</td>
<td>14 9 25</td>
</tr>
<tr>
<td>The first Limb of Jupiter came out</td>
<td>14 14 45</td>
<td>14 9 32</td>
</tr>
<tr>
<td>The following Limb of Jupiter, or last Contact</td>
<td>14 16 15</td>
<td>14 11 2</td>
</tr>
<tr>
<td>The fourth Satellite emerged</td>
<td>14 18 49</td>
<td>14 13 36</td>
</tr>
</tbody>
</table>

Jupiter
Jupiter and the Satellites were to the Northward of the visible Way of the Moon's Center.

This Occultation was observed through a Telescope, in which the Focal Length of the Object Glass was 14½ Feet, and of the Eye Glass 2½ Inches. And the Aperture of the Object Glass was 1½ Inch.

I could perceive no Colours on Jupiter's Limb, either at his Immerfion or Emersion, when the Axis of the Tube was directed to him.

<table>
<thead>
<tr>
<th>Time</th>
<th>Apparent</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>09 00</td>
<td>The Eclipse had been for some time begun</td>
</tr>
<tr>
<td>17</td>
<td>00</td>
<td>The Moon's Diameter measured by a Micrometer was 34 04</td>
</tr>
<tr>
<td>22</td>
<td>25</td>
<td>The Chord connecting the Horns 30 28</td>
</tr>
<tr>
<td>35</td>
<td>45</td>
<td>The illuminated Part of the Diameter continued to the Chord between the Horns 19 58</td>
</tr>
<tr>
<td>43</td>
<td>24</td>
<td>The illuminated Part of the Diameter 13 52</td>
</tr>
<tr>
<td>49</td>
<td>50</td>
<td>The same repeated 12 02</td>
</tr>
<tr>
<td>52</td>
<td>43</td>
<td>The same repeated 11 44</td>
</tr>
<tr>
<td>56</td>
<td>31</td>
<td>The illuminated part of the Diameter continued to the Chord between the Horns 15 22</td>
</tr>
<tr>
<td>59</td>
<td>27</td>
<td>The illuminated Part of the Diameter 10 35</td>
</tr>
<tr>
<td>16</td>
<td>04 04</td>
<td>The same repeated 9 48</td>
</tr>
<tr>
<td>18</td>
<td>34</td>
<td>The same again repeated 9 02</td>
</tr>
<tr>
<td>23</td>
<td>45</td>
<td>The Chord between the Horns 32 35</td>
</tr>
<tr>
<td>26</td>
<td>30</td>
<td>The same repeated 13 33 07</td>
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<tr>
<td>h.</td>
<td>14</td>
<td>16</td>
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<td>17</td>
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<td>18</td>
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<td>20</td>
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<td>22</td>
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<td>50</td>
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<tr>
<td>41</td>
<td>17</td>
<td>35</td>
</tr>
</tbody>
</table>

The fame again
At which time also the Shade passed thro' the middle of Schikardus.

The Chord between the Horns, agreeing with the D's Diameter

The inlightned part of the Diameter

The fame produced to the Chord between the Horns

The fame repeated

The inlightned part of the Diameter

The fame

The Edge of the Shadow passed thro' the Middle of Gaffendus.

The inlightned part produced to the Chord between the Horns

The Chord between the Horns

The fame repeated

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again

The fame again
At 17h. 39. the Eclipse was thought to be ended; and was visibly so at 17h. 41'. But by comparing the last Observations of the Chords between the Horns, it follows that the true End of the Eclipse was at 17h. 38. 20". At 17h. 43 the Moon's Diameter was 33'. 40".

The Middle cannot be supposed to be very accurately determined by these Observations, which were not sufficiently distant from the time of the greatest Obscurcation. However by comparing several of them together, the Middle will be obtained, viz.

| By Obs. 3. compared with Obs. 24. at | 16 15 21 |
| By Obs. 4. compared with Obs. 22. at | 16 15 58 |
| By Obs. 5. compared with 19. and 20 at | 16 16 00 |
| By Obs. 6. and 7. compared with 16. at | 16 15 48 |

By reason of Clouds I could not see the Beginning of the Eclipse, nor make such Observations of the Moon's immersing into the Shadow as I did of her emerging out of it.

By Observation 21. compared with Observation 15. the Digits Eclipsed were 8½.

The Angles were measured by a Micrometer in a 15 Foot Telescope. I have not considered how far they are consistent with one another; they being set down here exactly as they were first taken.

This Eclipse is the more considerable, as happening very near the Moon's Perigee, and therefore useful to verify her Anomaly; as also to limit the greatest Diameter of the Shadow of the Earth, and consequently the Parallax of the Moon. This may very properly be compared with that of the 19th of October, 1697, whose middle was at 7h. 41'. P. M. at London; and Quantity the same as now.
The Times by the Clock were 17.45" sooner than the apparent time, as was found by the following Observations of *Cor Leonis* and *Aréturus*, which through the Clouds were but just discernible.

<table>
<thead>
<tr>
<th>Apparent Zenith Distance</th>
<th>Time by the Clock</th>
<th>Apparent Time by Calculat.</th>
<th>The Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>of Cor o6 16 1/6</td>
<td>13 32 43</td>
<td>13 50 35</td>
<td>17 52</td>
</tr>
<tr>
<td>69 38</td>
<td>36 50</td>
<td>54 44</td>
<td>17 54</td>
</tr>
<tr>
<td>69 09</td>
<td>40 06</td>
<td>57 51</td>
<td>17 45</td>
</tr>
<tr>
<td>68 40</td>
<td>43 09</td>
<td>14 00 59</td>
<td>17 50</td>
</tr>
<tr>
<td>68 08</td>
<td>46 37</td>
<td>04 26</td>
<td>17 59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>of Arétur.</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>65 19</td>
<td>17 37 40</td>
<td>17 55 24</td>
<td>17 44</td>
</tr>
<tr>
<td>65 06</td>
<td>39 12</td>
<td>56 48</td>
<td>17 36</td>
</tr>
<tr>
<td>64 41</td>
<td>41 49</td>
<td>59 29</td>
<td>17 40</td>
</tr>
<tr>
<td>63 47</td>
<td>47 40</td>
<td>18 05 17</td>
<td>17 37</td>
</tr>
</tbody>
</table>

Clock too slow 17 45

The Latitude of Wanstead is 51° 34'. Its Longitude is 8" in time Eastward from the Observatory at Greenwich.

The Account given of this Eclipse by the Reverend Mr. William Derham, who observ'd it at Upminster, is agreeable to this, as far as Clouds would permit him to observe.
V. An Account of the late surprizing Appearance of the Lights seen in the Air, on the sixth of March last; with an Attempt to explain the Principal Phenomena thereof; As it was laid before the Royal Society by Edmund Halley, J. V. D. Savilian Professor of Geom. Oxon, and Reg. Soc. Secr.

The Royal Society, having received Accounts from very many Parts of Great Britain, of the unusual Lights which have of late appeared in the Heavens; were pleased to signify their Desires to me, that I should draw up a general Relation of the Fact, and explain more at large some Conceptions of mine I had proposed to them about it, as seeming to some of them to render a tolerable Solution of the very strange and surprizing Phenomena thereof. The Desires of the Society having with me the force of Commands, I shall not decline the Task: only premising that if, in delivering the Etiology of a Matter so uncommon, never before seen by my self, nor fully described by any either of the Ancients or Moderns, I fail to answer their Expectation or my own Desires; yet 'tis hoped a good History of the Fact, deduced partly from our own Observations, and partly collected from the uniform Relations of credible Persons, or from the Letters of such, may not be unacceptable to the Curious; and may perhaps excite the Genius of some more able Meteorologist to a more satisfactory Enquiry. The Account of this Appearance take as follows.
On Tuesday the sixth of March, f. vet. in the current Year 1716, (the Afternoon having been very serene and calm, and somewhat warmer than ordinary) about the Time it began to grow dark, that is much about 7 of the Clock, not only in London, but in all Parts of England, where the Beginning of this wonderful Sight was seen; out of what seemed a dusky Cloud, in the N. E. parts of the Heaven and scarce ten Degrees high, the Edges whereof were tinged with a reddish Yellow like as if the Moon had been hid behind it, there arose very long, luminous Rays or Streaks perpendicular to the Horizon, some of which seem'd nearly to ascend to the Zenith. Presently after, that reddish Cloud was swiftly propagated along the Northern Horizon, into the N. W. and still farther Westerly; and immediately sent forth its Rays after the same manner from all Parts, now here, now there, they observing no Rule or Order in their rising. Many of these Rays seeming to concur near the Zenith, formed there a Corona, or Image which drew the Attention of all Spectators, who according to their several Conceptions made very differing Resemblances thereof; but by which compared together, those that saw it not, may well comprehend after what manner it appeared. Some likened it to that Representation of Glory wherewith our Painters in Churches surround the Holy Name of God. Others to those radiating Sterrs wherewith the Breasts of the Knights of the most Noble Order of the Garter are adorned. Many compared it to the Concave of the great Cupola of St. Paul's Church, distinguisht with Streaks alternately Light and obscure, and having in the middle a Space less bright than the rest, resembling the Lantern. Whilst others, to express as well the Motion as Figure thereof, would have it to be like the Flame in an Oven reverberated and rolling against the arched Roof thereof: And some there were that thought it liker to that tremulous Light: 
Light which is cast against a Ceiling by the Beams of
the Sun, reflected from the Surface of Water in a Basin
that's a little shaken; whose reciprocal vibrating Motion
it very much imitated. But all agree that this Spectrum
lasted only a few Minutes, and shou'd it self variously
tinged with Colours, Yellow, Red and a dusky Green:
Nor did it keep in the same Place; for when first it began
to appear, it was seen a little to the Northwards of the
Zenith, but by degrees declining towards the South, the
long Stria of Light, which arose from all Parts of the
Northern Semicircle of the Horizon, seemed to meet to-
gether, not much above the Head of Castor or the
Northern Twin, and there soon disappeared.

After the first Impetus of this ascending Vapour was
over, the Corona we have been describing appeared no
more; but still, without any order as to Time, or Place,
or Size, luminous Radii like the former continued to arise
perpendicularly, now oftner and again seldomer, now
here, now there, now longer, now shorter. Nor did they
proceed at first out of a Cloud, but oftner would
emerge at once out of the pure Sky, which was at that
time more than ordinary serene and still. Nor were they
all of the same Form. Most of them seemed to end
in a Point upwards, like erect Cones; others like trun-
cate Cones or Cylinders, so much resembled the long
Tails of Comets, that at first sight they might well be ta-
ken for such. Again, some of these Rays would continue
visible for several Minutes; when others, and those the
much greater part, just shew'd themselves and died away.
Some seem'd to have little Motion, and to stand as it
were fix'd among the Stars, whilst other with a very per-
ceptible Translation moved from East to West under the
Pole, contrary to the Motion of the Heavens; by which
means they would sometimes seem to run together, and
at
at other times to fly one another; affording thereby a
surprizing Spectacle to the Beholders.

After this Sight had continued about an Hour and a
Half, those Beams began to rise much fewer in Number
and not near so high, and by degrees that diffused Light,
which had illustrated the Northern Parts of the Hemis-
phere, seemed to subside, and settling on the Horizon
formed the Resemblance of a very bright Crepusculum:
That this was the State of this Phenomenon, in the first
Hours, is abundantly confirmed by the unanimous Con-
sent and concurring Testimony of several very worthy
Persons no ways inclined to deceive. For by the Letters
we have received from almost all the extream Parts of the
Kingdom, there is found very little Difference in the De-
scription from what appeared at London and Oxford; un-
less that in the North of England, and in Scotland, the Light
seemed somewhat stronger and brighter.

Hitherto I am forced to relate the Observations of o-
thers, wherein I fear many very material Circumstances
may be omitted: and assuredly I am not a little concern'd
that I had no Notice of this Matter, till between Nine and
Ten of the Clock, being at that Time at a Friend's
House, and no ways suspecting what past without Doors.
But upon the first Information of the thing, we imme-
diately ran to the Windows, which hapned to regard
the South and South-West Quarter; and soon perceived,
that though the Sky was very clear, yet it was tinged
with a strange sort of Light; so that the smaller Stars
were scarce to be seen, and much as it is when the Moon
of four Days old appears after Twilight. And whilst
we regarded the Heavens with attention, we perceived
a very thin Vapour to pass before us, which arose from
the precile East part of the Horizon, ascending oblique-
ly, so as to leave the Zenith about 15 or 20 Degrees to
the Northward. But the swiftness wherewith it procee-
ded was scarce to be believed, seeming not inferior to that of Lightning; and exhibiting, as it past on, a sort of momentaneous Nubecula, which discovered it self by a very diluted and faint Whiteness; and was no sooner formed, but before the Eye could well take it, it was gone, and left no Signs behind it. Nor was this a single appearance; but for several Minutes that we regarded it, about six or seven times in a Minute, the same was again and again repeated; these Waves of Vapour (if I may be allowed to use the Word) regularly succeeding one another, and, as we guess, at intervals very nearly equal; all of them in their Ascent producing a like transient Nubecula.

By this particular we were first assured, that the Vapour we saw, whatever it were, became conspicuous by its own proper Light, without help of the Sun's Beams: for these Nubecula did not discover themselves in any other part of their passage, but only between the South-East, and South, where being opposite to the Sun they were deepest immerst in the Cone of the Earth's Shadow; nor were they visible before or after. Whereas the contrary must have happened, had they borrowed their Light from the Sun.

We then made all the haste we could to a place where there is a free Prospect of the Northern Horizon. Being come there, not much past Ten of the Clock, we found, on the Western Side, viz. between the W. and N.W. the Representation of a very bright Twilight, contiguous to the Horizon; out of which there arose very long Beams of Light, not exactly erect toward the Vertex, but something declining to the South, which ascending by a quick and undulating Motion to a considerable Height, vanished in a little time, whilst others, tho' at uncertain Intervals, supply'd their Place. But at the same time, through all the rest of the Northern Horizon, viz. from the North-West
Wefl to the true East, there did not appear any sign of
Light to arise from, or joyn to, the Horizon; but on the
contrary, what appeared to be an exceeding black and dis-
mal Cloud seem'd to hang over all that part of it. Yet
was it no Cloud, but only the serene Sky more than or-
dinary pure and limpid, so that the bright Stars shone
clearly in it, and particularly Cauda Cygni then very low
in the North; the great Blackness manifestly proceeding
from the Neighbourhood of the Light which was collected
above it. For the Light had now put on a Form quite
different from all that we have hitherto described, and
had fashioned it self into the Shape of two Lamina or
Streaks, lying in a Position parallel to the Horizon, whose
Edges were but ill terminated. They extended them-
selves from the N. by E. to the North East, and were each
about a Degree broad; the undermost about eight or nine
Degrees high, and the other about four or five Degrees
over it; these kept their Places for a long time, and made
the Sky so light, that I believe a Man might easily have
read an ordinary Print by the Help thereof.

Whilst we stood astonish'd at this surprizing Sight,
and expecting what was further to come, the Northern
End of the upper Lamina by degrees bent downwards,
and at length closed with the End of the other that was
under it, so as to shut up on the North Side an interme-
diate Space, which still continued open to the East. Not
long after this, in the said included Space, we saw a great
Number of small Columns or whitish Streaks to appear sud-
denly, erect to the Horizon, and reaching from the one
Lamina to the other; which instantly disappearing were
too quick for the Eye, so that we could not judge whe-
ther they arose from the Under or fell from the Upper,
but by their sudden Alterations they made such an Ap-
pearance, as might well be taken to resemble the Conflicts
of Men in Battle.
And much about the same time, to encrease our Wonder, there began on a sudden to appear, low under the Pole and very near due North, three or four lucid Areas like Clouds, discovering themselves, in the pure but very black Sky, by their yellowish Light. These, as they broke out at once, so after they had continued a few Minutes, disappeared as quick as if a Curtain had been drawn over them: Nor were they of any determined Figure, but both in Shape and Size might properly be compared to small Clouds illuminated by the full Moon, but brighter.

Not long after this, from above the aforesaid two Laminae, there arose a very great Pyramidal Figure, like a Spear, sharp at the Top, whose Sides were inclined to each other with an Angle of about four or five Degrees, and which seemed to reach up to the Zenith or beyond it. This was carried with an equable and not very flow Motion, from the N. E. where it arose, into the N. W. where it disappeared, still keeping in a perpendicular Situation, or very near it; and passing successively over all the Stars of the Little Bear, did not efface the smaller ones in the Tail, which are but of the Fifth Magnitude; such was the extream Rarity and Perspicuity of the Matter whereof it consisted.

This single Beam was so far remarkable above all those that for a great while before had proceeded it, or that followed it, that if the Situation thereof among the Circumpolar Stars had at the fame Instant been accurately noted, for Example, at London and Oxford, whose Difference of Longitude is well known, we might be enabled thereby with some certainty to pronounce, by its diversitas Aspectus, concerning the Distance and Height thereof; which were undoubtedly very great, tho' as yet we can no ways determine them. But as this Phænomenon found all those that are skill'd in the Observation of the Heavens unprepared, and unacquainted with what was.
was to be expected; so it left all of them surprized and astonished at the Novelty thereof. When therefore for the future any such thing shall happen, all those that are curious in Astronomical Matters, are hereby admonished and entreated to set their Clocks to the apparent Time at London, for Example, by allowing so many Minutes as is the Difference of Meridians; and then to note at the End of every half Hour precisely, the exact Situation of what at that time appears remarkable in the Sky; and particularly the Azimuths of those very tall Pyramids so eminent above the rest, and therefore likely to be seen furthest: to the intent that by comparing those Observations taken in the same Moment in distant Places, the Difference of their Azimuths may serve to determine how far those Pyramids are from us.

It being now past Eleven of the Clock, and nothing new offering itself to our View, but repeated Phases of the same Spectacle; we thought it no longer worth while to bear the Chill of the night-Air sub dio. Wherefore being returned to my House, I made haste to my upper Windows, which conveniently enough regard the N. E. Parts of Heaven, and soon found that the two Lamina or Streaks parallel to the Horizon, of which we have been speaking, had now wholly disappeared; and the whole Spectacle reduced itself to the Resemblance of a very bright Crepusculum settling on the Northern Horizon, so as to be brightest and highest under the Pole itself; from whence it spread both Ways, into the N. E. and N. W. Under this, in the middle thereof, there appeared a very black Space, as it were the Segment of a lesser Circle of the Sphere cut off by the Horizon. It seemed to the Eye like a dark Cloud, but was not so; for by the Telescope the small Stars appeared through it more clearly than usual, considering how low they were: and upon this as a Basis our Lumen Auroriforme rested, which
which was no other than a Segment of a Ring or Zone of the Sphere, intercepted between two parallel lesser Circles, cut off likewise by the Horizon; or, if you please, the Segment of a very broad Iris, but of one uniform Colour; viz. a Flame-colour inclining to yellow, the Center thereof being about forty Degrees below the Horizon. And above this there were seen some Rudiments of a much larger Segment, with an Interval of dark Sky between, but this was so exceeding faint and uncertain that I could make no proper estimate thereof.

I was very desirous to have seen how this Phenomenon would end, and attended it till near Three in the Morning, and the rising of the Moon: but for above two Hours together it had no manner of Change in its Appearance, nor Diminution nor Encrease of light; only sometimes for very short Intervals, as if new Fuel had been cast on a Fire, the Light seem'd to undulate and sparkle, not unlike the rising of vaporous Smoak out of a great Blaze when agitated: But one thing I assur'd myself of by this Attendance and Watching, viz. that this Iris-like Figure did by no means owe its Origine to the Sun's Beams: for that about Three in the Morning, the Sun being in the Middle between the North and East, our Aurora had not follow'd him, but ended in that very Point where he then was: whereas in the true North, which the Sun had long past, the Light remained unchanged and in its full Lustre.

Hitherto I have endeavoured by Words to represent what I saw, but being sensible how insufficient such a verbal Description of a thing so extraordinary and unknown may be to most Readers, I have thought fit to annex a Figure exhibiting that particular Appearance of the two Lamina, which I saw at London between the Hours of Ten and Eleven: more especially, because I do not find, among the many Relations I have seen, any one
one that has taken notice of it. In this Figure $AB$ is the under $Lamina$, somewhat broader and brighter than the upper $CD$: it had near its under Edge the $Lucida Lyre$, and below its Northern Extremity, on the Left-hand, $Cauda Cygni$: and as well above and below these, as in the intermediate Space between them, and indeed all round about that Part of the Heavens, the Sky was so unusually dark and black, as if all that exotic Light that had shew'd it self before, had been then collected into those two Streaks. Only at $Q$ between the West and Northwest and no where else, out of a Brightness adjoining to the Horizon, there arose conical Beams as $M, L, N$, after the same manner as at first.

Whilst we stood looking on, the Streak $CD$ at its Northern End bent downward, and joyned with the Under $AB$ at $E$, and included the Space $DC EAB$, which still kept open at the other End towards the East. And in the mean time, out of the very clear Sky, some luminous Spots, situated and figured as in the Scheme at $G, G, G, G$, presented themselves to the Eye, in Colour much like the $Lamina$. These did not shew themselves all together, but came successively, yet so as two or three of them were seen at a time; and as their coming was instantaneous, so they went away in a Moment. At the same time likewise, the several little white Columns marked $F, F, F, F$, occupied that Part of the Space between the two Streaks next to $E$, and by their sudden and very irregular Motion, and the vanishing of some whilst others at the same time emerged, gave occasion to the Conception of those that fancy'd Battles fought in the Air. Lastly from about the middle of $CD$, there arose suddenly a Cone or Obelisk of a pale whitish Light, greater than any we had yet seen, as $H$; which moving from East to West, with a Motion sufficiently regular, was translated to $K$, in the North West, and there disappeared.

That
That we might by the same Scheme shew the Appearance of the last Hours, after Midnight; the Reader is desired to take notice that we have made the Light at 2, much bigger than what appeared in the West about Ten of the Clock; so as to represent truly that other. In this Case the Point 2 must, by the Imagination, be supposed transferred to the Intersection of the Horizon and Meridian under the Pole. And that we might the better be understood in what follows, we have made this short Recapitulation as annex'd to, and explicative of, the Scheme, which could by no means be contrived to answer the wonderful Variety this Phænomenon afforded; since even the Eye of no one single Observer, was sufficient to follow it in the Suddenness and Frequency of its Alterations.

Thus far I have attempted to describe what was seen, and am heartily sorry I can say no more as to the first and most surprizing Part thereof, which however frightful and amazing it might seem to the vulgar Beholder, would have been to me a most agreeable and wish'd for Spectacle; for I then should have contemplated proprium oculis all the several Sorts of Meteors I remember to have hitherto heard or read of. This was the only one I had not as yet seen, and of which I began to despair, since it is certain it hath not happen'd to any remarkable Degree in this Part of England since I was born; nor is the like recorded in the English Annals since the Year of our Lord 1574, that is above One Hundred and Forty Years ago, in the Reign of Queen Elizabeth. Then, as we are told by the Historians of those times, Camden and Stow, Eye-Witnesses of sufficient credit, for two Nights successively, viz. on the 14th and 15th of November that Year, much the same wonderful Phænomena were seen, with almost all the same Circumstances as now.

Nor
Nor indeed, during the Reign of that glorious Princess, was this so rare a Sight as it has been since. For we find in a Book entituled a Description of Meteors, reprinted at London in the Year 1654, whose Author writes himself W.F. D.D. that the same thing, which he there calls Burning Spears, was seen at London on January 30. 1560; and again by the Testimony of Stow, on the 7th of October 1564. And from foreign Authors we learn, that in the Year 1575, the same was twice repeated in Brabant, viz. on the 13th of February and 28th of September; and seen and described by Cornelius Gemma, Professor of Medicine in the University of Louvain, and Son of Gemma Frisius the Mathematician. He, in a Discourse he wrote of the Prodigies of those Times, after several ill-boding Prognosticks, thus very properly describes the Cupola and Corona that he saw in the Chasma (as he calls it) of February Paulo post undecunque surgentibus Hastra & flammas novis, flagrarar caelum à Borea parte utque ad verticem videbatur: ac denique ne nihil qua contigerunt hactenus prafiguratum antea videbatur, conversa est Caeli facies, per borne spatium, in Fritilli aleatorii speciem peregrinam; alternantibus sese caruleo & candido, non minore vertigine motusque celeritate, quam solares radii solent, quoties ab objecto speculo regeruntur. Here it is not a little remarkable, that all these four already mentioned fell exactly upon the same Age of the Moon, viz. about two Days after the Change.

As to the other of September in the same Year 1575, these are the Words of Gemma. Minus quidem horrendum, sed varia tamen magisque confulta nobis apparuit alterius Chasmatis forma, quarto Calendas Octobreis subsecuti, statim ab occasu Solis. Nam in illo vis sunt arcus illustres plurimi, ex quibus Hasta sensim eminentes, Urbetque turrita & Acies militares. Erant hinc radiorum excursus quaquaversum, & nubiurn fluctus & pralia: insecutabantur invicem & fugiebant, facta in orbem conversione mirabili. From hence tis manife
nifert that this Phenomenon appeared in our Neighbourhood three several Times, and that with considerable Intervals, within the Campaigns of one Year; though our English Historians have not recorded the two latter; nor did Gemma see that of November 1574, as 'tis most likely by reason of Clouds. After this, in the Year 1580, we have the Authority of Michael Maäslin, * (himself a good Astronomer, and still more famous for having had the honour to be the great Kepler's Tutor in the Sciences) that at Bakenang in the Country of Wirtemburg in Germany, these Chasmata, as he likewise styles them, were seen by himself no less than seven times within the Space of twelve Months. The first of these, and most considerable, fell out on the very same Day of the Month with ours, viz. on Sunday the sixth of March, and was attended with much the same Circumstances, which, for Brevity's sake, I omit. And again the same things were seen in a very extraordinary manner on the 9th of April and 10th of September following: but in a less degree, on the 6th of April, 21st of September, 26th of December and 16th of February, 1581: the last of which, and that of the 21st of September must needs have been more considerable than they then appeared, because the Moon being near the Full, necessarily effaced all the fainter Lights. Of all these however no one is mentioned in our Annals to have been seen in England, nor in any other place that I can find; such was the neglect of curious matters in those Days.

The next in order that we hear of was that of the Year 1621, on September the 2d. st. ves. seen all over France, and well described by Cassendus in his Physicks, who gives it the Name of Aurora Borealis. This, tho' little inferior to what we lately saw, and appearing to the Northwards both of Rouen and Paris, is no where said to have been observed in England, over which the Light seemed to lie. And since then for above 80 Years, we have no Account

* M. Maäslin. lib. de Cometa 1580.
of any such Sight either from home or abroad; notwithstanding that for above half that time, these Philosophical Transactions have been a constant Register of all such extraordinary Occurrences. The first we find on our Books, was one of small Continuance seen in Ireland by Mr. Neve on the 16th of November 1707, of which see Phil. Trans. No. 320. And in the Miscellanea Berolinesia published in 1710, we learn that in the same Year 1707, both on the 24th of January and 18th of February, st. vet. something of this kind was seen by M. Olaus Römer at Copenhagen: and again on the 23d of February, the same excellent Astronomer observed there such another Appearance, but much more considerable; of which yet he only saw the Beginning, Clouds interposing. But the same was seen that Night by Mr. Gotfried Kirch, at Berlin above 200 Miles from Copenhagen, and lasted there till past Ten at Night. To these add another small one of short Duration, seen near London, a little before Midnight between the Ninth and Tenth of August 1708, by the Right Reverend Philip Lord Bishop of Hereford, and by his Lordship communicated to the Royal Society: so that, it seems, in little more than eighteen Months this sort of Light has been seen in the Sky, no less than five times; in the Years 1707 and 1708.

Hence we may reasonably conclude that the Air, or Earth, or both, are sometimes, though but seldom and with great Intervals, disposed to produce this Phenomenon: for though it be probable that many times, when it happens, it may not be observed, as falling out in the Daytime, or in cloudy Weather, or bright Moon-shine: yet that it should be so very often seen at some times and so seldom at others, is what cannot well be that way accounted for. Wherefore calling about and considering what might be most probably the Material Cause of these Appearances; what first occurred was the Vapour of Water rarified.
rarified exceedingly by subterraneous Fire, and tinged with fulfureous Steams; which Vapour is now generally taken by our Naturalists to be the Cause of Earthquakes. And as Earthquakes happen with great Uncertainty, and have been sometimes frequent in Places, where for many Years before and after they have not been felt; so These, which we might be allowed to suppose produced by the Eruption of the pent Vapour through the Pores of the Earth, when it is not in sufficient Quantity, nor sudden enough to shake its Surface, or to open it self a Passage by rending it. And as these Vapours are suddenly produced by the Fall of Water upon the nitro-sulphurous Fires under Ground, they might well be thought to get from thence a Tincture which might dispose them to shine in the Night, and a Tendency contrary to that of Gravity; as we find the Vapours of Gun-powder, when heated in Vacuo, to shine in the Dark, and ascend to the Top of the Receiver though exhausted: the Experiment of which I saw very neatly performed by Mr. J. Whiteside Keeper of Ashmole's Museum in Oxford.

Nor should I seek for any other Cause than this, if in some of these Instances, and particularly this whereof we treat, the Appearance had not been seen over a much greater Part of the Earth's surface that can be thus accounted for. It having in this last been visible from the West Side of Ireland to the Confines of Russia and Poland on the East (nor do we yet know its Limits on that Side) extending over at least thirty Degrees of Longitude; and in Latitude, from about fifty Degrees over almost all the North of Europe; and in all Places exhibiting at the same time the same wonderous Circumstances, as we are informed by the Publick News. Now this is a Space much too wide to be shaken at any one Time by the greatest of Earthquakes, or to be affected by the Perspiration of that Vapour, which being included and wanting vent, might
might have occasioned the Earth to tremble. Nor can we this Way account for that remarkable Particular attending these Lights, of being always seen on the Northside of the Horizon, and never to the South.

Wherefore laying aside all hopes of being able to explain these Things by the ordinary Vapours or Exhalations of the Earth or Waters, we are forced to have recourse to other sorts of Effluvia of a much more subtile Nature, and which perhaps may seem more adapted to bring about those wonderful and surprizingly quick Motions we have seen. Such are the Magnetic Effluvia, whose Atoms freely permeate the Pores of the most solid Bodies, meeting with no Obstacle from the Interposition of Glass or Marble or even Gold itself. These by a perpetual Efflux do, some of them, arise from the Parts near the Poles of the Magnet, whilst others of the like Kind of Atoms, but with a contrary Tendency, enter in at the same Parts of the Stone, through which they freely pass; and by a kind of Circulation surround it on all Sides, as with an Atmosphere, to the Distance of some Diameters of the Body. This thing des Cartes has endeavoured to explain (Princip. Philosoph. Lib IV.) by the Hypothesis of the Circulation of certain skrewed or striate Particles, adapted to the Pores they are to enter.

But without enquiring how sufficient the Cartesian Hypothesis may be for answering the several Phænomena of the Magnet: that the Fact may be the better comprehended we shall endeavour to exhibit the manner of the Circulation of the Atoms concerned therein, as they are exposed to view, by placing the Poles of a Terrella or Spherical Magnet on a Plane, as the Globe on the Horizon of a Right Sphere: Then strewing fine Steel dust or Filings very thin on the Plain all round it, the Particles of Steel, upon a continued gentle knocking on the underside of the Plain, will by degrees conform themselves to
to the Figures in which the Circulation is performed. Thus in Fig. II. Let $A B: C D$ be a Terrella, and its Poles $A$ the South, and $B$ the North; and by doing as prescribed, it will be found that the Filings will lie in a Right Line perpendicular to the Surface of the Ball, when in the Line of the Magnetical Axis continued. But for about forty five Degrees on either side, from $B$ to $G$ or $I$, and from $A$ to $H$ or $K$, they will form themselves into Curves, more and more crooked as they are remoter from the Poles; and withall more and more oblique to the Surface of the Stone: as our Figure truly represents, and as may readily be shewn by the Terrella and Apparatus for that Purpose in the Repository of the Royal Society. Hence it may appear how this exceeding subtile Matter revolves; and particularly how it permeates the Magnet with more force and in greater Quantity in the circumpolar Parts, entring into it on the one side, and emerging from it on the other, under the same oblique Angles: whilst in the middle Zone about $C$ and $D$, near the Magnet's Equator (if I may use the Word) very few if any of these Particles do impinge, and those very obliquely.

Now by many and very evident Arguments it appears that our Globe of Earth is no other than one great Magnet, or (if I may be allowed to alledge an Invention of my own) rather two; the one including the other as the Shell includes the Kernel (for so and not otherwise we may explain the changes of the Variation of the Magnetical Needle) but to our present Purpose the Result is the same. It suffices that we may suppose the same sort of Circulation of such an exceeding fine Matter to be perpetually performed in the Earth, as we observe in the Terrella; which subtile Matter freely pervading the Pores of the Earth, and entring into it near its Southern Pole, may pass out again into the Ether, at the same Distance from the Northern, and with a like Force; its Direction being still
still more and more oblique, as the Distance from the Poles is greater. To this we beg leave to suppose, that this subtile Matter, no otherways discovering it self but by its Effects on the Magnetick Needle, wholly imperceptible and at other times invisible, may now and then, by the Concourse of several Causes very rarely coincident, and to us as yet unknown, be capable of producing a small Degree of Light; perhaps from the greater Density of the Matter, or the greater Velocity of its Motion: after the same manner as we see the Effluvia of Electrick Bodies by a strong and quick Friction emit Light in the Dark: to which sort of Light this seems to have a great Affinity.

This being allowed me, I think we may readily assign a Cause for many of the strange Appearances we have been treating of, and for some of the most difficult to account for otherwise; as why these Lights are rarely seen any where else but in the North, and never, that we hear of, near the Equator: as also why they are more frequently seen in Iceland and Groenland, than in Norway, though nearer the Pole of the World. For the Magnetical Poles, in this Age, are to the Westward of our Meridian, and more so of that of Norway, and not far from Groenland; as appears by the Variation of the Needle this Year observed, full twelve Degrees at London to the West.

The erect Position of the luminous Beams or Strie so often repeated that Night, was occasioned by the rising of the Vapour or lucid Matter nearly perpendicular to the Earth's Surface. For that any Line erected perpendicularly upon the Surface of the Globe, will appear erect to the Horizon of an Eye placed any where in the same spherical Superficies; as Euclid demonstrates in a Plain, that any Line erected at Right Angles to it, will appear to be perpendicular to that Plain from any Point thereof. That it should be so in the Sphere is a very pretty Proposition,
not very obvious, but demonstrated from Prop. 5. Lib. I. Theodosi Spharic. For by it all Lines erect on the Surface pass through the Center, where meeting with those from the Eye, they form the Plains of Vertical Circles thereto. And by the Converse hereof it is evident, that this luminous Matter arose nearly perpendicular to the Earth's Surface, because it appeared in this erect Position. And whereas in this Appearance (and perhaps in all others of the Kind) those Beams which arose near the East and West, as $L$, $M$, $N$, were furthest from the Perpendicular, on both sides inclining towards the South, whilst those in the North were directly upright; the cause thereof may well be explained by the Obliquity of the Magnetical Curves, making still obtuser Angles with the Meridians of the Terrella, as they are further from its Poles.

Hence also it is manifest how that wonderful Corona that was seen to the Southwards of the Vertex, in the Beginning of the Night, and so very remarkable for its tremulous and vibrating Light, was produced; to wit, by the Concurs of many of those Beams arising very high out of the circumjacent Regions, and meeting near the Zenith: the Effluvia whereof they consisted mixing and interfering one with another, and thereby occasioning a much stronger but uncertain wavering Light. And since it is agreed by all our Accounts that this Corona was tinged with various Colours, 'tis more than probable that these Vapours were carried up to such a Height, as to emerge out of the Shadow of the Earth, and to be illustrated by the direct Beams of the Sun: whence it might come to pass that this first Corona was seen coloured and much brighter than what appeared afterwards in some Places, where the Sight thereof was more than once repeated, after the Sun was gone down much lower under the Horizon. Hence also it will be easily understood that this Corona was not one and the same in all Places, but was
different in every differing Horizon; exactly after the same manner as the Rainbow seen in the same Cloud is not the same Bow, but different to every several Eye.

Nor is it to be doubted, but the Pyramidal Figure of these Ascending Beams is Optically: since according to all likelyhood they are parallel-sided, or rather tapering the otherway. But by the Rules of Perspective, their Sides ought to converge to a Point, as we see in Pictures the Parallel Borders of straight Walks, and all other Lines parallel to the Axis of Vision, meet as in a Center. Wherefore those Rays which arose highest above the Earth and were nearest the Eye, seemed to terminate in Culsps sufficiently acute, and have been for that reason supposed by the Vulgar to represent Spears. Others seen from afar, and perhaps not rising so high as the former, would terminate as if cut off with Plains parallel to the Horizon, like truncate Cones or Cylinders: these have been taken to look like the Battlements and Towers on the Walls of Cities fortified after the ancient manner: Whilst others yet further off, by reason of their great Distance, good part of them being intercepted by the Interposition of the Convexity of the Earth, would only shew their pointed Tops, and because of their Shortness have gotten the Name of Swords.

Next the Motion of these Beams, furnishes us with a new and, as it seems to me, most evident Argument to prove the diurnal Rotation of the Earth: (though that be a matter which, at present, is generally taken by the Learned to be past dispute.) For those Beams which rose up to a Point, and did not presently disappear, but continued for some time, had most of them a sensible Motion from East to West, contrary to that of the Heavens; the biggest and tallest of them, as being nearest, swiftest; and the more remote and shorter, slower. By which means, the one overtaking the other, they would some-
times seem to meet and jostle; and at other times to separate, and fly one another. But this Motion was only Optical, and occasioned by the Eye of the Spectator being carried away with the Earth into the East; whilst the exceeding rare Vapour of which those Beams did consist, being, as I take it, raised far above the Atmosphere, was either wholly left behind, or else followed with but part of its Velocity, and therefore could not but seem to recede and move the contrary Way. And after the same manner as the Stars that go near the Zenith, pass over those Vertical Circles which border on the Meridian, much swifter than those Stars which are more distant therefrom; so these luminous Rays would seem to recede faster from East to West, as their Bases were nearer the Eye of the Spectator; and vice versa.

Nor are we to think it strange, if after so great a Quantity of luminous Vapour had been carried up into the Ether out of the Pores of the Earth, the Cause of its Effervescence at length abating, or perhaps the Matter thereof consumed; these Effluvia should at length subside, and form those two bright Lamina which we have described, and whose Edges being turn'd to us were capable to emit so much Light that we might read by them. I choose to call them Lamina, because, without doubt, though they were but thin, they spread Horizontally over a large Tract of the Earth Surface. And whilst this luminous Matter dropt down from the upper Plate to the under, the many little white Columns were formed between them by its Descent, only visible for the Moment of their Fall. These by the Swiftness with which they vanished and their great Number, shewing themselves and disappearing without any order, exhibited a very odd Appearance; those on the Right seeming sometimes to drive and push those on the left, and vice versa.
I have been obliged to omit several particulars of less moment: But there are the principal phenomena; of whose causes I should have more willingly and with more certainty given my thoughts, if I had had the good luck to have seen the whole from beginning to end; and to have added my own remarks to the relations of others: and especially if we could by any means have come at the distances thereof. If it shall by any be thought a hard supposition that I assume the effluvia of the magnetic matter for this purpose, which in certain cases may themselves become luminous, or rather may sometimes carry with them out of the bowels of the earth a sort of atoms proper to produce light in the ether. I answer that we are not as yet informed of any other kinds of effluvia of terrestrial matter which may serve for our purpose, than those we have here considered, viz. the magnetic atoms, and those of water highly rarified into vapour. Nor do we find anything like it in what we see of the celestial bodies, unless it be the effluvia projected out of the bodies of comets to a vast height, and which seem by a vis centrifuga to fly with an incredible swiftness the centers both of the sun and comet, and to go off into tails of a scarce conceivable length. What may be the constitution of these cometical vapours, we inhabitants of the earth can know but little, and only that they are evidently excited by the heat of the sun; whereas this meteor, if I may so call it, seldom is seen but in the polar regions of the world, and that most commonly in the winter months. But whatever may be the cause thereof, if this be not, I have followed the old axiom of the schools. Entia non esse te mere necque absque necessitate multiplicanda.

Lastly I beg leave on this occasion to mention what, near 25 years since, I publish'd in No. 195. of these transactions, viz. That supposing the earth to be concave,
with a lesser Globe included, in order to make that inner Globe capable of being inhabited, there might not improbably be contained some luminous Medium between the Balls, so as to make a perpetual Day below. That very great Tracts of the Ethereal Space are occupied by such a shining Medium is evident from the Instances given in the first Paper of this Transaction: And if such a Medium should be thus inclosed within us; what should hinder but we may be allowed to suppose that some parts of this lucid Substance may, on very rare and extraordinary Occasions, tranfude through and penetrate the Cortex of our Earth, and being got loose may afford the Matter whereof this our Meteor consists. This seems favoured by one considerable Circumstance, viz. that the Earth, because of its diurnal Rotation, being necessarily of the Figure of a Flat Spheroid, the thickness of the Cortex, in the Polor Parts of the Globe, is considerably less than towards the Equator; and therefore more likely to give Passage to these Vapours; whence a reason may be given why these Lights are always seen in the North. But I desire to lay no more Stress upon this Conception than it will bear.

It having been noted that in the Years 1575 and 1580, wherein this Appearance was frequent, that it was seen not far from the Times of the two Equinoxes; it may be worth while for the Curious, to bestow some Attention on the Heavens in the Months of September and October next; and in case it should again happen, to endeavour to observe, by the Method I have here laid down, what may determine, with some degree of Exactness, the Distance and Height thereof; without which we can scarce come to any just Conclusion.

\[ F \quad I \quad N \quad I \quad S. \]

Errata. No. 346. p. 383. l. 18. read 234. p. 408. l. 20. read, proceed, as at first.
PHILOSOPHICAL
TRANSACTIONS.

For the Months of April, May and June, 1716.

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IV. An Account of what appeared on opening the big-belly'd Woman near Haman in Shropshire, who was supposed to have continued many Tears with Child. Communicated by Dr. Hollings, M.D. from Shrewsbury.

I: A description of the Phenomenon of March 6. last, as it was seen on the Ocean, near the Coast of Spain. With an Account of the return of the same sort of Appearance, on March 31, and April 1, and 2. following.

In our last, we endeavour'd to give the Publick as good an Account of the late surprizing Meteor, seen in the Heavens, on the sixth of March last, as could be gathered from the several Relations of very distant Spectators, which had then come to the Royal Society's Notice. And since then, we can only add thereto, that at Paris, the Light was so inconsiderable, that it was not regarded: But from a Letter to Mr. Alexander Geekie, Surgeon, dated on Board a Ship in Nevis Road, in America, April 19. 1716. we have copied the following Passage. "On the sixth of March, at 9 a Clock in the Evening, we, being then in the Latitude of 45° 36' (off of the N.W. Coast of Spain). A clear Cloud appeared East of us, not far distant from our Zenith, which afterwards darted itself forth into a number of Rays of Light, every way like the Tail of a Comet, of such a great Length, that it reach'd within a short way of the Horizon. There likewise appeared a Body of Light, N. N. E. of us, and continued as Light almost as Day, till after 12 a Clock. It appear'd at a good distance from us, and darkned on a sudden.

Hence it should seem, that the Vapour which caused this Appearance, arose indifferently out of the deep Ocean Sea, as well as from the Land; by which we may conclude the great Subtlety of the Matter thereof; since it could permeate so great a Quantity of Water, and yet retain its Velocity; which is a Circumstance deserving the further consideration of the Curious.

But since this, most of the same Phenomena have been repeated three several Nights successively, viz. On the last of March, and first and second of April. The best and fullest Description
tion of the two first, is, from a Letter of Dr. Brook Taylor, LL.D. and Secretary to the Royal Society, dated April 2, from Cotterstock, near Oundle in Northamptonshire, who thus describes them. "On Saturday Night last, and last Night, I saw Appearances of the same kind, with those of March 6, but not to compare for Extent and Strength to the other. They both began soon after Sun set, and continued till after 12, but much longer I cannot tell. They were both about 10 or 15 degrees to the Westward of the North, and took up about 80 degrees of the Horizon; and the Aurora rose about 30 gr. high, with a dark Bottom, like what was seen in the First; and from whence there sprung out several Bodies of Light, which immediately run into Streams, ascending about 30, or at most 40 gr. high. There was no flashing nor waving Light, but in all other respects, these Lights were of the same kind with what we saw at London. Indeed in that last Night, there was one Phenomenon like the flashing Light, for a Body of Light about 15 or 20 degrees long, parallel to the Horizon, rose till it came about 6 degrees above the black Basis, and then sent up two strong Streams of Light about 40 gr. high, which at top dash'd against one another, and disappeared."

At London, the first Night, March 31. It did not begin to radiate, till towards Mid-night, and was seen but by few curious Persons; the Beams not rising very high, and scarce appearing over the Houses, were little taken notice of; but by the Relation of those that saw it, it was much more considerable than the next Night following Easter-day, for it then sent out but few and very short Beams, mostly terminating in a sharp Point, and presently disappearing. Only it beginning to stream so soon as it became Dusky, it was very observable, that those Rays which arose out of the West end of the Luminous Arch, next the Sun, were enlightened by its Beams, and shew'd themselves much brighter than those which sprung up under the Pole, or to the Eastward thereof. And after Nine, till Mid-night, no more Beams arose; and the Luminous Arch with its black Basis, settled down very low in the Northern Horizon.

The same two Nights, by the Observation of Mr. William Lingen, the like Appearance was seen at Dublin, about the hours.
hours of Nine or Ten; at which time, in the former Night, it was near as Light as in a Moon-light Night. And from France we have an Account, that both those Nights, the same was seen at Paris, with much the same Circumstances as at Dublin. So that it seems this Meteor, though no ways comparable to that of the 6th of March, was seen not less than 150 Leagues, and probably much further.

The following Night, April 2. When it began to be dark, a Luminous Arch appear'd in the North, with a very narrow black Bottom under it, very low, and depress'd to the Horizon; nor was it seen at, or about London, to project any pointed Rays as the former.

But what was most remarkable that Evening, was, what was seen at London, by that ingenious Gentleman Martin Foulks, Esq; R. S. S. about Nine that Night. He being in the open Air at that time, saw in an Instant, a bright Ray of very white Light, appear in the East, out of the pure Sky, then very serene and still; it very much resembled the Tail of a Comet, and was about 20 gr. inclined from the Perpendicular to the Right, beginning about γ of Bayer in the Corona Borea, and terminating about the Informis by some call'd Cor Caroli. This having appear'd but a very short time, disappear'd at once, as in a Moment. When on a sudden, such another Beam was instantly produced, not exactly in the same place, but in the same Situation. Its lower end being about 20 gr. high, was terminated exactly between ξ and η, in the Right Hand and Arm of Hercules, and the Middle of it past over σ and ε in the Girdle of Bootes, and thence proceeded Westwards, leaving Cor Caroli four or five degrees to the Northwards. After it had continued in this posture near 10 Minutes immovable among the Stars, it began to move slowly towards the North: and the lower end passing over the Northern Edge of the Crown, and the Ray it self over Cor Caroli, it grew fainter, and vanished, having continued in all about 20 Minutes. This latter with some Interruptions was extended between Castor and Pollux, very far into the West. And about that time, the same, or such another Beam, was seen at St. Asaph, by Doctor Stanley, the Reverend Dean of that Church.
II. An Account of some Experiments of Light and Colours, formerly made by Sir Isaac Newton, and mention'd in his Opticks, lately repeated before the Royal Society by J. T. Desaguliers, F. R. S.

The manner of separating the primitive Colours of Light to such a Degree, that if any one of the separated Lights be taken apart, its Colour shall be found unchangeable, was not published before Sir I. Newton's Opticks came abroad. For want of knowing how this was to be done, some Gentlemen of the English College at Liege, and Monsieur Mariotte in France, and some others took those for primitive Colours, which are made by immittting a Beam of the Sun's Light into a dark Room through a small round Hole, and refracting the Beam by a triangular Prism of Glass placed at the Hole. And by trying the Experiment in this manner, they found that the Colours thus made were capable of change, and thereupon reported that the Experiment did not succeed. And lately the Editor of the Acta Eruditorum for October 1713, pag. 447, desired that Sir I. Newton would remove this Difficulty. Objectiones, inquit, quas Viri docti tum in Gallia tum in Anglia contra illam [de Coloribus] Theoriam fecere, felicissime diluit Vir perspicacissimus Newtonus, quemadmodum ex Transactionibus Anglicanis N. 84, 85, 88, 96, 97, 121, 123, 128 abunde constat. Unde multi optant ut mentem suam aperire dignetur, de difficutate ab ingeniosissimo Mariotto, rerum naturalium (dum vive-ret) scrutatore indefesso nec infelici, in Tractatu de Coloribus p. 207. & seq. contra eam mota. In distantia scientiae, sic licet 25 circiter aut 30 pedum, charta exceptit radium soli-dumper exiguum foramen in cameram obscuram immissum, Y y y &
& per Prisma vitreum trigonum transmissum, coloremque violaceum spatium majus quam trium linearum occupantem per crenam duarum linearum trajectum, alio Prisma-
tate exceptis admodum oblique oppoeeo: quo facto quam-
dam ejus partem in Rubrum & Flavum transmutari obser-
vavit. Similiter luminis rubri partem in coeruleum & vi-
olaceum transmutari expertus est. Hac vero transmuta-
tione admissa corruere Theoriam Newtonianam, ex Actis
A. 1706. p. 60. & seq. manifestum est. Assumptam autem
Mariottus distantiam 30 pedum, ne quis excepert in mi-
nori distantia nondum factam esse pleniam radiorum heterogeneorum separationem. Nobis experimentum
Mariotti tum demum videretur decisivum, si lumen co-
eruleum integrum in alium mutatum fuisse. Thus far the
Editor of the Acta. In answer to which it is to be observed
that the Red and Yellow which came out of the Violet, and the
Blew and Violet which came out of the Red, might proceed from
the very bright Light of the Sky next encompassing the Sun,
and that several sorts of Rays which come from several Parts of
the Sun's Body are intermixt in all parts of the coloured Spec-
trum which falls upon a Paper at any Distance from the Prism.
In this manner of trial, for making the Experiment succeed,
the Light of the bright Clouds, immediately surrounding the
Sun, should be intercepted by an opaque Skreen placed in the open
Air without, at the distance of Ten or Twenty Foot from the
Hole through which the Sun shines into the dark Room. And in
the Skreen there should be a small Hole for the Sun to shine
through. The Hole may be either round or oblong, and not
above on.e eighth or one tenth Part of an Inch broad; so that
the Skreen may intercept not only the bright Light of the
Clouds next encompassing the Sun's Body, but also the greatest:
Part of the Sun's Light. For thereby the Colours will become
less mixed. The Beam of Light which passes through this Hole
must afterwards pafs through the other Hole into the dark Room,
and the Prism must be placed parallel to the oblong Hole in the
Skreen
Sheen, and the refracting Angle thereof be sixty Degrees or above. In this manner the Experiment may be tried with Success, but the Trial will be less troublesome if it be made in such a manner as is described in the fourth Proposition of the first Book of Sir I. Newton's Opticks.

Sir Isaac Newton therefore, upon reading what has been cited out of the Ada Eruditorum, desired Mr. Desaguliers to try the Experiment in the manner described in the said Proposition; and he tried it accordingly with Success before several Gentlemen of the Royal Society, and afterwards before Monseur Monmort and others of the Royal Academy of Sciences; and still shews it to those who desire to see it. How this and other concomitant Experiments were tried and succeeded, is described as follows.

**EXPERIMENT I.**

Having tied together end-wise two Pieces of Ribbon four Inches long each, the one blew and the other red, whole common Breadth was \( \frac{1}{4} \) of an Inch; I caus'd it to be held in such manner, that the Light which fell from the Clouds thro' the Window was so reflected, that the Angle made by the Rays of Light, which came in at the Middle of the Window, with the Plane of the Ribbon produced, was equal to the Angle made by a Line drawn from the Ribbon to my Eye and the said Plain of the Ribbon. My Eye was plac'd as far behind the Ribbon as the Window was before it, the Distance from which to me was about 12 Feet. Then looking thro'a Prism at the Ribbon, it appear'd broken asunder in the Place where the blew and red Half joint'd. If the Prism was held with the refracting Angle downwards (or laid with one of its Planes flat upon the Nose) the blew Half of the Ribbon appear'd to be carried down lower than the red, as at B, R in Fig. 1, but if the refracting Angle of the Prism was turn'd up

\[ Y y y z \]
wards (as when the Prism has one of its Planes laid flat to the Forehead) then the blew Half of the Ribbon was lifted up, as at $\mathcal{E}$.

The Prism was of white Glass, having every Angle of 60 Degrees: but when instead of it, one of a greenish sort of Glass, such as Object Glasses of Telescopes are made of, was used, having the refracting Angle which I look'd thro' of about 48 Degrees; the fame Phænomenon was more distinct, this Glass having no Veins, but the Red and Blue were nearer to a straight Line: in such manner that if $A$ represent the Ribbon seen through the first Prism, $B$ will represent the Ribbon seen thro' the second Prism, Fig. 2. If the refracting Angle of the last Prism had been as great as that of the first, the Light being transmitted thro' too great a Body of greenish Glass, the Phænomenon would not have succeeded so well.

The blue Ribbon being somewhat too pale, and the red a little dull; I repeated the Experiment with a Sken of blue, and one of red Worsted join'd together in the Middle as the Ribbons were before; and, the Colours of both being very intense, the Experiment succeeded better with both Prisms. All that were present trying the Experiment found it to succeed, and that every Circumstance answer'd to the Account given in Prop. 1. Theor. 1. Book 1. of Sir Isaac Newton's Optics, as far as the Directions there given were followed. So that it appear'd that the Blue being carried lower than the Red in the first case, and lifted higher in the second, was owing to the greater Refraction of the blue Ray: for tho' each Part of the Ribbon or Worsted reflected all manner of Rays, yet the Phænomenon was very apparent; as also that the blue Ribbon or Worsted reflected the blue Rays more copiously than the red Rays, and that the red Ribbon or Worsted reflected the red Rays more than the blue ones, because the Red of the blue Half seen thro' the Prism was less intense than
than that of the red Half, and the Blue or Purple of the red Half seen throu' the Prism was less intense than that of the blue Half.

N. B. If the Ribbon or Worsted is laid upon any enlightened Body, the Phænomenon will not succeed so well; the Colours of the Body seen throu' the Prism mixing with those of the Ribbon or Worsted. Even a black Body will not do, if Light falls upon it: but there must be a black Cloth behind, in such manner that no Light falling upon it can be reflected so as to disturb the Phænomenon. And if a short-sighted Person looks through the Prism, a concave Lens between his Eye and the Prism will render the Phænomenon more distinct than it wou'd otherwise be.

EXPERIMENT II.

Some Days after, the Sun shining, I made two Holes $H, h$, in the Window Shut $S$, of a darkned Room; thro' which letting the Sun's Beams pass, by means of two Prisms $A, B$, (one near each Hole) I open'd the Rays coming from the Sun into the two colour'd Spectra $\alpha, \beta$, where the following Colours were very distinct, viz. Red, Orange, Yellow, Green, Blue, Purple and Violet. Now the Reason of their being more distinct than ordinary, was, that the Prisms which I made use of were made of the greenish Glass mentioned before, which is very free from those Veins by which the Colours are too much thrown into one another, by the best white Prisms of the common sort.

The foremention'd colour'd Spectra being thrown into the Room, to the Distance of about 20 Feet from the Window where the Sun's Light came in, I caus'd a Piece of white Paper $r, \frac{1}{4}$ Inch broad and 5 Inches long, to be held within the refracted Rays, (at the Distance of 10 Feet from the Windows,) which produc'd these Colours in such manner, that by turning the Prisms round their Axes,
Axes, I cou'd make the red Ray of the Spectrum, made by the one Prism fall upon one half of the Paper, and the purple Ray of the Spectrum made by the other Prism fall upon the other Half; for the Spectra were both vertical, the Lines which terminated the long Sides of them towards each other just touching, as appears in Fig. 3. Then at the Distance of 9 Foot, looking thro' the Prism C at the Paper thus colour'd, the red Half appear'd very much separated from the Purple, the one seeming lifted up from the other; the Red or the Purple appearing the highest, according as the refracting Angle of the Prism was either held upwards or downwards. The Phenomenon is much more distinct this way than any other; for the Paper not only seems divided into two, when it is coloured by a red and a purple Ray, but also by a Red and Blue, (Fig. 4.) by a red and a green Ray, (Fig. 5.) or indeed by any two Colours that are different, how near soever their Places in the Spectrum be to each other. The Halves of the Paper appear, when view'd thro' the Prism, to be farther from each other, when the Paper is ting'd with such Colours as are farther from each other in the Series of Colours in the Spectrum: and nearest, tho' still divided, when neighbouring Colours fall upon the Paper, as Yellow and Green, or a light and a deep Green. But the Paper appears no way divided, when colour'd with the Red of the two Spectra, (Fig. 6.) if those Reds are equally intense: and so of the other Colours.

EXPERIMENT III.

I held a Lens of about 3 Foot Radius at the Distance of Six Feet from the oblong Paper (on which a red and purple Ray falling, made it look half Red and half Purple) and I projected the Image of the said colour'd Paper at the Distance of about Six Foot on the other Side of the Lens, on
on a white Sheet of Paper; where it was observeable, that when the red Half was distinctly painted on the white Paper (which was known by the Edges of the Image being regularly terminated) then the blue Half of the Image was confus'd: but if the white Paper was brought about two Inches nearer to the Lens, the Image of the blue Half became distinct, and that of the red Half confus'd.

I try'd the Experiment with a Paper colour'd half red and half blue, the red with Carmine and the blue with Smalt, making the Candle to enlighten the Paper (the Room being otherwise dark) and the Experiment succeed-ed in the same manner. The Experiment thus made is the same that Sir Isaac Newton gives an Account of, Book i. Part. i. Theor. 1. of his Optics. Only it is to be observ'd that when the oblong Paper is coloured with red and blue from the Prisms, the focal place, where the red part of the Image is distinct, is more distant from the place where the blue part of the Image is distinct, than when the Paper is colour'd with the Painter's Powders, and much more vivid.

The 7th Figure shews the Projection of the Paper ting'd with the Rays; and Fig.8. the Projection of it when painted: where a black Thread is wrapp'd round the red and the blue part, that the Distinctness of the Image of the Thread may shew when the red or when the blue part of the Image of the Paper is most distinct.

N.B. When the Candle enlightens the painted Paper, set an opaque Body as B between the Candle and Lens; left the Image of the Candle being also projected should disturb the Experiment.

EXPERIMENT IV.

Having made an Hole of \( \frac{1}{4} \) Inch Diameter in the Window-Shut of the darkned Room, I suffer'd a Sun-Beam to come
come into the Room, which I intercepted with a Prism at the Distance of 5 Inches from the Hole; and after its Refraction in passing thro' the Prism, I receiv'd it upon a Sheet of white Paper, where it was colour'd, making an oblong Image of the Sun or Spectrum of about 9 Inches in length and 2 in breadth, which Breadth was nearly equal to the Diameter of the round Image of the Sun received upon a Paper at the same Distance from the Hole, which here was 18 Foot. Or if the Sun be too high, a Looking-Glass being put in the room of the Prism will throw a white round Spectrum upon the Paper, which held at the said Distance of 18 Foot, will have its Diameter equal to the Breadth of the colour'd Spectrum.

The Colours of the Spectrum were these; Red, Orange, Yellow, Green, Blue, Purple and Violet, tho' the Violet was so faint in this as to be scarce perceivable. See Fig. 9.

N.B. The Axis of the Prism in this, and all the other Experiments hereafter mention'd must be perpendicular to the Ray that falls on it; and the Plane into which the Ray enters must be held in such a Position, that the Angle which such a Ray makes with that Plane when it enters, may be equal to the Angle made by the middle Line of those Rays which emerge after Refraction, on the other Side of the refracting Angle of the Prism, with the Plane out of which they emerge. That is \( \angle B D G = \angle A E H \)

If the Plane \( AC \), on which the Sun-Beam falls, be turned nearer to a perpendicular to the Sun Beam than before, the Spectrum will be much longer: if it be more inclin'd to the said Beam, the Spectrum will be shorter, and in both Cases less distinct. See the Spectrum \( DE \) and the Spectrum \( de \) in Fig. 10 and 11, where \( H, h \), represents the Hole in the Window Shut in each Case; \( AC, ac \) the Plane of the Prism on which the Rays enter; \( BC, bc \) that out of which they emerge; \( P, p \) the perpendicular, and \( C, c \) the refracting Angle.
If the Plane $AC$ be still more oblique to $HF$, all the Light will be reflected, and there will be no colour'd Image or Spectrum made by Refraction at all. Fig. 12.

But if it be held so as to be more nearly perpendicular to the Sun Beam than in Fig 10, the whole Beam will indeed enter the Prism; but meeting with $BC$ the lower Surface of the Prism, or rather the Surface of the Air contiguous to it, some of the Light will by the Plane $BC$ be reflected to $de$, passing almost perpendicularly thro' $AB$; and the rest will emerge thro' $BC$, and by Refraction make the imperfect Spectrum $DE$. See Fig. 10.

If the Sun-Beam enter $AC$ perpendicularly and in the middle of it, the Light will be all reflected as in Fig. 13. some of it by the Plane $BC$ to $R$, and the rest by the Plane $AB$ to $p$. But if the Beam fall nearer to $A$ (still perpendicularly) it will all be reflected by the Plane $AB$; if nearer to $B$, it will be all reflected by the Plane $BC$.

In order therefore to have the colour'd Spectrum as it ought to be, care must be taken that the emerging coloured Light may make the same Angle with the Plane $BC$, as the immersing Light does with the Plane $AC$; that is, the Angle $AEH$ must be equal to $BDG$, as was said before, Fig. 9. which may also be seen on the enlightened Dust in the Air. But the best way is to turn the Prism on its Axis, and at the same time look at the colour'd Spectrum, which will rise and fall and become longer or shorter as you turn the Prism; and between the Ascent and Descent of the Image, it will appear stationary: thereto stop the Prism, and the Reflection will be such as is required for all the Experiments hereafter mention'd.

In order to have the Prism move freely on its Axis, and stop any where, I fix'd each End of it into a triangular Collar of Tin, from the End of which came a Wire, which was the Axis of the Prism produc'd; and so I laid it on two wooden Pillars, with a Notch on the Top to receive the
the Wires, and fix'd it to a small Board just broad enough to stand fast. See Fig. 14.

EXPERIMENT V.

I took the Prism $CD$, and thro' it look'd at the coloured Spectrum $RP$, which appear'd then round and white as at $S$, just as if it had been the Sun's Light received on a Paper from the Hole $H$, and seen with the naked Eye. In this case the Prism $CD$ must be held in directum with $AB$, and the refracting Angles in the two Prisms must be equal. This Spectrum appearing white but just in one Point, is not so readily found; but the best way is to look thro' the same Prism $AB$ which makes the Spectrum, which may easily be done if it be pretty long, and then $RP$ will be seen white and round, and as at $S$, as if coming directly from $H$. See Fig. 15.

EXPERIMENT VI.

I held a broad Lens $LL$, ground to a Radius of $\frac{1}{2}$ Feet, in such manner that the whole colour'd Spectrum fell upon it; and after Refraction all the Colours appear'd to converge, if receiv'd on a Paper at $pp$; but when the Paper was held in the Focus at $P$ in the position $F \tau F \tau$, the Spectrum was round and perfectly white by the Union of all the colour'd Rays. If the Paper was held at $\Pi \Pi$, the Colours appear'd to diverge from each other, but then the Red was uppermost, which before us'd to be the lowest, and so on in an inverted Order.

I try'd the same Experiment with a Lens of one Foot Radius, with one of 9 Inches, and with another of 7, and the Success was the same. See the 16th Figure, where the $R, O, I, G, B, P, V$, express the Colours.

N. B.
N.B. Care must be taken that the very end of the Red, and the Extremity of the Violet be taken in by the Lens; otherwise the Spectrum will not be perfectly white at the Glass's Focus.

There is no fix'd Distance of the Prism from the Lens, but it ought to be brought so near the Prism that the two Ends of the Spectrum may fall nearer the Axis of the Lens than the Edges of the Lens; because there the Refraction is not so regular.

Behind the Lens $L$, which made the Colours converge into White at the distinct Base or Focus $F$, I plac'd the Lens $l$, which made the White be at $f$ the distinct Base of the two Glasses combin'd; and the Experiment succeed-ed as before. Fig. 17.

When the Paper was held in the Focus of the Lens, so as to receive the white Image of the colour'd Spectrum projected by the Lens; if with a Card I intercepted the red Ray, the White appear'd ting'd with Purple, and if I intercepted the Violet or purple Ray, or both, the White appear'd ting'd with Red; and if the Red was intercepted at the same time, the Spectrum appear'd to be a Mixture of Yellow, Green and Blue. If any single Colour was suffer'd to fall upon the Lens, the rest being intercepted, that Colour would continue the same; only it would be more intense in the Focus of the Lens.

**Experiment VII.**

I took a Board (Fig. 18.) $q b s$ which stood reclining on a Prop $t$, having an Hole of a Quarter of an Inch Diameter at $b$, and behind it a Prism $B$ supported on two Props, as above-mention'd, so as to turn easily about its Axis; and having set this Board on the Ground with the Prism behind it at $B$; by turning the Prism $A C$ about its Axis, I first made the red Ray of the colour'd Spectrum
pas thro' the Hole $h$, and fall obliquely upon the second Prism $B$. This Ray after its Refraction in passing thro' the second Prism, was carried up to the Ceiling of the Room at the place mark'd $R$; then I made the purple Ray fall upon the Board, and pas thro' the Hole $h$, as the Red had done before; and after Refraction thro' the Prism $B$ it was carried up to the Ceiling at $P$. And the green Ray being afterwards made to pas the second Prism in the same manner, went up to $G$; and so of all the intermediate Rays, which were by this second Refraction thrown to the intermediate places on the Ceiling between $R$ and $P$.

Care is to be taken that the second Prism be plac'd oblique to the Rays which come thro' the Hole $h$, least they be reflected, as they wou'd be, if the Board being in the Position $Q S$, and the second Prism in the Position $L N M$, the Ray from the first Prism be $p h$; for then it will be reflected upwards to $s$ instead of being refracted (Fig. 19.). Neither must the Plane of Immersion be too oblique, least the Incident Ray be reflected downwards by it, as the Ray $R h$ is by the Prism $B$ thrown to $E$, in Fig. 20. Several have confess'd to me that they at first us'd to fail in this Experiment, for want of sett'ing the second Prism in a due Inclination.

Tho' the Colours by the second Refraction on the Ceiling appear'd unchang'd, when seen by the naked Eye, yet if view'd thro' a Prism, they afforded new Colours (except some part of the Red, and some part of the Violet) which was owing to their not being fully separated; for which reason I made the following Experiment, to prove that if the Colours be well separated, they are truly homogeneous and unchangeable.

$N.B.$ When the Prisms are good, and no Clouds are near the Sun, the Extremity of the Red or Violet will afford unmix'd Colours in this Experiment; otherwise not.
EXPERIMENT VIII.

Having made a Hole in the Window-Shut 2 inches wide (Fig. 21.) I applied to it a Tin Plate, which sliding up and down hid all this Hole in the Wood, and only transmitted a small Beam thro’ it’s own Hole $H$, whose Diameter was $= \frac{1}{2}$ Inch. This Beam, by means of the Looking Glass $L$, plac’d on the Board of the Window $XW$, I reflected horizontally to the other end of the Room. But to correct the Irregularity of the Reflection of the Looking-Glass, I made use of the Frame of Past-Board $PP$, which had an Hole in it $b$ of $\frac{1}{4}$ Inch likewise: and placing it at $PP$ I suffer’d some of the reflected Beams to pass thro’ it, so as to fall upon the Lens $FE$ (convex on both Sides, and ground to a Radius of $4\frac{1}{2}$ Feet) at the Distance of 9 Feet, so that the Image of the Hole $b$ was projected to $f$ on the other Side of the Glass, at the Distance of 9 Feet more. Just behind the Lens, which by a Screw in the Stand $S$ might be rais’d or let down, so as always to receive the Beam along its Axis, I plac’d a Prism $A$ (upright on one of its Ends and easily moveable about its Axis, by reason of its Wire turning freely in an Hole in the solid piece of Wood $T$, which stood on another Stand behind the Lens) as near as I cou’d to the Lens $FE$, so that the Image of $b$ instead of being round, white, and projected to $f$, was cast sidewise on a white Paper stretch’d on a Frame, and appear’d colour’d, and 30 or 40 times its Breadth, as at $MN$. The Colours in this Case were very vivid and well separated, only the Violet had some pale Light darting from its End, upon account of some Veins in the Prism $A$, and the Light not coming directly from the Sun, but reflected; which ought not to have been, if the Sun had been low enough to have thrown the Rays a good way into the Room without the Help of a Looking Glass.
To shew that the Colours in this Spectrum were simple and homogeneal Lights, I made the following Experiments.

**EXPERIMENT IX.**

Having made an Hole $h$ in the Paper which receiv'd the colour'd Spectrum, I suffer'd the red Light to pass; which being refracted by a second Prism, fell upon another Paper at $T$, where it appear'd still Red whether seen with the naked Eye or Prisms of different refracting Angles. To the Eye which saw it thro' the Prism $V$, it appear'd indeed lower as at $t$, but red, round and unchang'd. I made the Experiment upon all the Colours, which by this means appear'd to be simple and homogeneal. See Fig. 22. Where the same Letters denote the Lens, Prism and first Paper.

Thro' the same Lens and Prism the Spectrum was made to fall on a Book; then thro' the Prism $F$ it appear'd unchang'd; and the Letters in the Book which cross'd the Spectrum, were as distinct as when seen with the naked Eye. See Fig. 23.

*NB.* The Axis of the Prism $F$ ought to be perpendicular to the long Axis of the Spectrum $s$ m thrown on the Book, which will appear as at $s \mu$; and the Prism in the Position represented at $F$, with its flat Side towards the Nose, for that is the most convenient Position for looking at the Spectrum in these Experiments.

I suffer'd the purple Ray only to pass thro' the Hole $h$ and fall upon a Book at $P$, the Letters of which appear'd at $\pi$, and were as distinct thro' the Prism $Q$ as when seen with the naked Eye; and I had the same Success with all the other Rays. See Fig 24.

But if a Sun-Beam as $r$ comes thro' the Hole $H$ directly upon the Book at $W$, an Eye looking at it thro' a Prism at
at $X$ will see this Beam at Toblong and colour'd, and the Letters on which it falls, confus'd. See Fig. 24.

**N.B.** The Lens ought to be very good, without Veins or Blebs, and ground to no less a Radius than I mention'd in the Experiment; tho' a Radius of a Foot or two longer is not amiss. The Prism ought to be of the same Glass as the Object-Glasses of Telescopes, the white Glass, of which Prisms are usually made, being commonly full of Veins. And the Room in these last Experiments ought to be very dark.

A few Days after, having got very good Prisms made for the purpose of the above mention'd Glass, I made all the Experiments over again before several Members of the Royal Society with better Success; and had the Spectrum very regularly terminated, without any pale Light darting from the Ends of it.

For a further Account of Experiments to this purpose, see Sir Isaac Newton's Opticks B. 1. Part. 1. to which I might have referr'd the Reader altogether; but that I was willing to be particular in mentioning such things as ought to be avoided in making the Experiments above-mention'd; some Gentlemen abroad having complained that they had not found the Experiments answer, for want of sufficient Directions in Sir Isaac Newton's Opticks; tho' I had no other Directions than what I found there.
III. A plain and easy Experiment to confirm Sir Isaac Newton's Doctrine of the different Refrangibility of the Rays of Light. By the same.

After the Experimentum Crucis made by two Prisms, I shou'd not give the following Experiment, but that it is so easy to be made, that by it those who want the Apparatus (or are unwilling to be at the pains) to make the Experimentum Crucis, may at any time satisfy themselves of the Truth of the fore-mention'd Doctrine.

Let the Candle $A$ be set before the Bar of a Chimney Looking-Glafs, such as is represented by $HH$ (Fig. 25) which is a Piece of Looking-Glafs Plate consisting of four Planes, seen in the Section of it $\alpha f d \beta$, viz. $d \beta$ which is quick-silver'd behind, $f \alpha$ a Plane parallel to it, $f d$ one of the Side-planes bezell'd towards $d \beta$, or inclin'd to it in an Angle of about 40 Degrees (tho' from 30 to 40 will do, but the greater the Angle the better, if it does not exceed 45°.) $\alpha \beta$ the other Side-Plane inclin'd in the same Angle to $\beta d$.

The Rays of the Candle which come from $A$ to $\gamma$ fall obliquely on the Plane $\alpha \beta$, so that instead of going on to $a$, they are by Refraction made to incline more towards the Perpendicular $p p$, namely to go on in the Line $\gamma c$, and then are reflected from the Point $c$ on the quick-silver'd Surface, in the Direction $c x$, so as to make the Angle $x c d = \gamma c \beta$. Now as the Rays which wou'd go to $x$, if not refracted, emerge obliquely from the Plane $\alpha \beta$, they leave the Direction $c x$, and decline from the Perpendicular $\pi \pi$, and, being differently refracted, open into four differently colour'd Rays; viz. $b R$ a red Ray, $b T O$ a Ray made
made up of Orange and Yellow; $b \, G \, B$ a Ray made up of Green and Blue or a Sea-Green, and $b \, P$ a purple Ray.

If from the place $E \, e$ you look full upon the Point $b$, the Spectrum or Image of the Candle at $b$ will appear double, but not mix'd; that is, there will appear a Sea-green Spot and a red Spot, as it were, one upon another; but not so as to produce a mix'd or intermediate Colour. Then if the right Eye or Eye at $E$ be shut, there will appear only a green Spot to the Eye at $e$; if the Eye at $e$ be shut, the Eye at $E$ will see only a red Spot.

If you come nearer to $b$, so that the Eyes at $e_1$, $e_2$ receive the most and the least refrangible Rays, there will be a double Spectrum, viz. a red and a purple one just touching, or upon one another: and the Phænomenon will answer as before. (*Fig. 25.*)

If keeping both Eyes open, you direct their Axes towards $O$ a Point nearer than the usual place of the compound Spectrum $S$, (*Fig. 26.*) which Point is in a Line from the Nose $N$ to the Point $S$; or in other Words, if you look full at $O$, or at the End of your Finger held in $O$, the red and the blue (or purple Spot) will appear to be divided from each other after the manner represented at $p \, r$ (*in Fig. 27.*) where the Red will appear to be on the Right-hand, and the Blue on the Left.

To make plain what is meant by *seeing* the Spectra $p$ and $r$ whilst we look full at $O$, I beg leave to explain the Distinction between *looking* and *seeing*; that I may the better shew how this Phænomenon proves that the Sensation of different Colours is caus'd by Rays differently refracted.

I. Definition.

The **Optic Axis** is a Line which going thro' the Center of the Convexity of all the Coats and Humours of the Eye.
Eye, falls upon the Middle of the Retina, as \(a a\) or \(A a\). Fig. 28.

II. Definition.

To look at any Point, is to turn both Eyes towards it in such manner, that the Optic Axes making an Angle at the said Point as \(a\), the Rays from \(a\) may have the Optic Axis for their Axis, and (by their Convergence upon the Retina after Refraction in the Eye) may paint the Image of the said Point upon the Middle of the Retina of each Eye, where the Optic Axis in each Eye falls.

III. Definition.

To see without looking, is to direct the Optic Axes to some other place than to the Point which is then seen; and in such a case, the Image of the Point seen will be projected upon a part of the Retina of each Eye, where the Optic Axis does not fall, namely either nearer to the Nose \(N\) as in (Fig. 26.) at the Points of the Retina mark'd \(nn\); or farther from the Nose than the Middle of the Retina, as at \(oo\) in Fig. 29.

Whatever is seen, by being look'd at with both Eyes, always appears single, by reason of the Communication between the Middle of the Retina in one Eye, and the Middle of the Retina of the other: there being no such Communication between any other part of the Retina in one Eye, and the Correspondent part of the Retina in the other, when these correspondent parts are equally distant from the Nose.

There is indeed a Communication between the Nervous Fibres on the Right-side of the Retina of one Eye, and the Nervous Fibres on the Right-side of the Retina of the other Eye, and so of those on the Left; but no single Object can be so painted in each Eye, as to have its Image on the Right or Left Part of one
one Retina that communicates with the Right or Left part of the other, of the same bigness and at the same time as in the other; because in whatever Position the Object is, it must be nearer to one Eye than to the other, except it be just in a Line from the Nose betwixt the two Eyes straight forward.

Hence it is that if there be two Candles set before any one, the First at the Distance of one Foot, and the Second at the Distance of two Feet, from the Eyes; he that looks at the second Candle at \( B \) will see it single, but see the first Candle or the Candle \( A \) double; one Appearance being in the Line \( AD \gamma \), the other in \( oAE \), because it paints it self upon \( oo \) in the Retina of each Eye, which Points are not the middle Points, but farther from the Nose than the middles \( mm \).

So if \( B \) be the first Candle, and \( C \) the second, he that looks at \( B \) will see \( C \) double, because it is painted in the Retina at the Points \( nn \) nearer the Nose than \( mm \); and so will appear to be in the same Position as \( pr \) in Fig. 27.

If \( \gamma p \) be two Candles so disposed, Fig. 30. that by the Interposition of a perforated Board \( FF \), \( \gamma \) can paint it self only in the Eye \( R \), and \( p \) in the Eye \( L \). Upon making the Optic Axes meet at \( B \) and to tend towards \( p \) and \( \gamma \), \( p \) and \( \gamma \) will each paint an Image on the Middle of the Retina of each Eye, by crossing their Rays at \( B \): and thus the two Candles will appear to be but One, or rather to be in one Place, upon the account of the Communication of the Middle of each Retina. But if instead of the Candles, \( p \) be a piece of red Silk, and \( \gamma \) a piece of green Silk, the same Position of the Eyes will make an Image at \( B \), appearing like a red and green Spot together without a Mixture of the Colours. If \( p \) be a red hot Iron, and \( \gamma \) a Candle of Sulphur, the Phænomenon will be more distinct.

If the Optic Axes be turn'd directly towards \( \gamma \) and \( p \), as if there was no Board \( FF \) in the way, there will appear two 

\[ A a a a 2 \]
Holes in the Board, the one having the red hot Iron in it, the other the Candle.

Now if, of the refracted Rays of the Candle in the first Case (Fig. 25.) those which diverge from each other, so as to fall into each Eye, cause the same Sensations respectively, as the Rays which come from a red hot Iron and those which come from a blue Candle; it is evident that the Candle in the first case affords red-making and blue-making Rays after Refraction, and that those Rays are differently refrangible; the red $bR$ (Fig. 25.) the least refrangible, as declining less from the Perpendicular $\omega \omega$; and the Purple as $bP$ declining most from the said Perpendicular.

The same will (cateris paribus) be found true in the intermediate Rays; and to be certain that the Experiment is as I have related it, the Planes $\alpha f$ and $fd$ of the Barr may be covered with Paper.

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IV. An Account of what appear’d on opening the big-belly’d Woman near Haman in Shropshire, who was suppos’d to have continued many Years with Child. Communicated by Dr. Hollings M.D. from Shrewsbury.

A Marry’d Woman, near Haman about Three Miles from Shrewsbury, about the 40th Year of her Age, had then first the common Reasons to believe she was with Child: at the Time of her Account she had the usual Signs of Labour, and a good Midwife, tho’ mistaken, assur’d her it was so, but that the Child was so big she could not be delivered without bringing it away in pieces. She not submitting to that, her Pains went soon off, and she continued without any other Disorders Nine Months longer, when
when she had again the Signs of Labour; and the same Midwife assist’d her as before, and she persisting in her former Resolution, her Pains, after a Day or two went off. Soon after her Belly swell’d to a surprizing Size, by which she got Subsistance for her Family by being seen as a Shew. I saw her first above twenty Years since, when her Belly was almost even with her Chin, the Weight of it so great, that she was oblig’d to support it with a Stool. She could not stand without the help of a Rope from the Ceiling, which assist’d her in changing her Posture of sitting. She slept commonly with her Arms folded on her Belly, and her Head rested between them. She had no swelling in her Legs: every other Part emaciated as usual in the like Cases. Thus this poor Creature liv’d without any other considerable Complaint above Thirty Years, the most remarkable Circumstance, I think, in her Case. She died in May 1715, when this appear’d to be an Ascites.

I need not mention the State the common Teguments must necessarily be in from so great a Distention, which had distorted many of her Ribs, and forc’d the Diaphragm so high, that it was surprizing to find her breathing could be so long continu’d. The Water was all contain’d in the Duplication of the Peritoneum, 13 Gallons besides a Quart that was spilt: it was saltish, with some little fat upon it, and towards the latter Running ting’d with Blood as usual. There was not any Water in the Cavity of the Abdomen, except what was contain’d in a kind of Bladder of the Shape I have sent, Fig. 3r. which lay a-cross the Fundus Uteri. This was divided by a Cartilaginous Substance into two Cavities; in one there was a Pint and a half, in the other three Parts of a Pint of Water. I believe it was this (I know not how) impos’d on the Midwife. The Uterus was of the natural Size without any Alteration, except that the Os Tinee and Collum minus were fill’d with a gritty Substance, hard as Stone, which I take to be the Humour sepse.
separated there, and coagulated by Time. Mr. Cooper Tab. 15. Fig. 4. says he found the same Parts fill'd with a glutinous Matter, which he thinks is useful to prevent Abortion; which if vitiated, Impregnation is hinder'd.

The Liver and other parts contain'd in the Abdomen, were forc'd into an incredible small Compass (and by that Pressure a little chang'd in Shape) to perform their Office so long; to which the Muscles of the Abdomen, distended so as to be scarce discernible, could give but little, if any, Assistance.

The Awe that People have here for dead Bodies, tho' never so prejudicial to the Living, would not suffer her Friends to let me make any farther Enquiry; so that I can send no Account of any other Part. The same Error hinder'd me examining another Woman, who died here about a Week after, of an Ascites which she had had Forty Years, any farther than to be satisfied she had Seven Gallons of Water contain'd between the Duplicatures of the Peritoneum, and none in the Cavity of the Abdomen.


Plurima sunt maxime quidem paradoxo, omnemque fidem apud vulgus superantia, quæ tamæ adhibitis Mathematicarum Scientiarum principiis levi negotio enodantur. Ac fane nullum problema magis arduum ac difficile videbitur, quam est Solis à Terra distantiam vero proximam determinare; quod tamæ obtentis accuratis qui-
quibusdam observationibus, ad electa & prævilia tempora peractis, non multo opere efficietur. Id quod inclytæ huic Societati, quam immortalem foœ auguror, in hac dissertazione ob oculos ponere libet, ut junioribus nostris Astronomis, quibus forsan hæc observare ob minorem ætatem obtringere potest, viam præmonitrem, quà immensam Solis distantiam intra quingentesimam sui partem rite dimiri poterint.

Norum autem vobis est hanc distantiam à diversis Astronomis autorebus diversam fingi, prout cuique ex conjecturâ probabile visum est: à Ptolemeo quidem ejusque asseclis, uti & Copernico & Tychoni Bratable, Terra semidiametris mille & ducentis, Keplero ter milli quingentis fere. Ricciolus distantiam Keplerinam duplicat, quam ræmen Hevelius dimidio tantum auget. At vero visis in Solis difco ope Telecopii Planetis Venere & Mercurio mutato fulgore nudatis, tandem compertum est Planetarum diametros visibiles multo minores esse quam eatenus haberentur; Venerisque Semidiametrum è Sole visam, non nisi quartam minuti primi partem vel quindecim secunda subtendere; Mercuriique semidiametrum, ad mediam ipsius à Sole distantiam, sub angulo decem tantum secundorum conspici; atque sub codem etiam Saturni semidiametrum è Sole videri. Jovis autem Planetarum maximis semidiametris non nisi tertiam minuti primi partem apud Solem subtendere. Unde, servant analogia, nonnullis è modernis Astronomis visum est, Terra quoque semidiametrum è Sole confpectam, medio loco inter Jovis majorem & Saturni & Mercurii minorem angulum subtendere, Venerisque aqualem, nempe quindecim secundorum: adeoque Solem à Terra quatuordecim fere millibus semidiametrorum Terra distant. Item autem Authoribus, aliud argumentum paulo ampliavit hanc distantiam: quoniam enim Luna diameter paulo major est quartà parte diametri Terra, si Parallaxis Solis ponatur quindecim minutorum secundorum, fieret Luna.
Lunæ corpus corpore Mercurii majus, Planeta scil. secundarius primario major; quod concinnitati Systematis mundani contrariari videtur. E contra vero Venerem inferiorum & Satellitio deslitutam, majorem esse Terræ nostræ superiori & tam insignem comitem nàdà, vix concedere videtur eadem concinnitas. Ut itaque medio loco incedamus, ponatur Terræ semidiameter è Sole visa, scu quod idem est, Solis Parallaxis horizontalis, duodecim secundorum cum semidis: unde Luna minor erit Mercurio & Terræ Venerum major; ac proveniet Solis a Terræ distantia sedecies mille cum quingentis Terræ semidiametris proxime. Huic autem distantiae in praesentiæ adensum præbeo, usq; dum Experimento quod proponimus quantus sit certius constet. Nec moror authoritatem quantumvis gravemeorum qui Solem ultra hos terminos in immensum evehunt, freti observationibus vibrantis Penduli, determinandis his angu- lorum minutiis, uti videtur, haud fatis fidis: saltam hác methodo tentanti Parallaxis aliquando nulla, aliquando etiam negativa occurret; hoc est distantia vel infinita fiet, vel infinito major: quod absursum. Et, ut verum fatare, minuta secunda vel etiam dena secunda instrumentis quantumvis affabre factis certo distinguere vix homini datum est; atque adeo minime mirandum, si tantorum Artifitium multos & ingeniosos conatus haecènus elusérit rei ipsius maxima subtilitas.

Dum autem, ante 40 fere annos, in Insula Sanctæ Helena, syderum polum Australen ambientium observationibus operam darem; contigit mihi Mercurium sub Solis disco transirentem omni adhíbita diligentiá observare: quadque mihi præter spem feliciéter succéssit, momentum quo Mercurius ingrediens Solis limbum interius contingere visus est, pariterque momentum quo egrediens limbum Solis inflammavit, facto angulo contactûs interioris, Tubo optimo viginti quatuor pedum accuratissime obtinui. Unde pro comperto habui intervallum quo Mercurius totus intra Solis discessi.
difcum tum temporis apparet, etiam absque errore uni-
us minuti secundis temporis. Nam filum luminis Solaris,
inter limbum planetae obscurum & Solis lucidum intercep-
tum, quantumvis tenue in oculos incurrere visum est; &
in isto oculi, denticulius in limbo Solis a Mercurio ingre-
diente factus evanescere, uti ab egredientes factus quasi
momento incipere. Hoc autem perspecto statim intellexi
Solis Parallaxin ex hujusmodi observationibus rite conclu-
di posse, si modo Mercurius Terris vicinior majorem hae-
ret parallaxon à Sole; etenim hæc parallaxium differentia
tantilla est, ut semper minor sit ipsà Solari quam quaer-
imus; proinde Mercurius, licet frequenter intra Solem vi-
dendus, huic nostro negotio vix satiis aptus habebitur.

Reflat itaque Veneris transites per Solis discum, cujus pa-
 rallaxis quadruplo fere major Solari, maxime sensibles
efficient differentias, inter spatia temporis quibus Venus Solem
perambulare videbitur, in diversis Terræ nostræ regioni-
bus. Ex his autem differentiis debito modo observatis,
dico determinari posse Solis parallaxon etiam intra speru-
li secundis exiguam partem. Neque alia instrumenta po-
stulamus præter Telescopia & Horologia vulgaria sed bona:
& in Observatoribus non nisi fides & diligentia, cum mo-
dica rerum Astronomicarum peritia desiderantur. Non
enim opus est ut Latitudo Loci sperulosè inquiratur, nec
ut Horæ ipsæ respectu meridiani accurate determinentur:
sufficit, Horologis ad Cæli revolutiones probe correctis,
fi numerentur tempora à totali Ingressu Veneris infra di-
cum Solis, ad principium Egressus ex eodem; cum scili-
cet primum incipient Globus Veneris opacus limbum Solis
lucidum attingere; quæ quidem momenta, propria expe-
rientia novi, ad ipsum secundum temporis minutum ob-
servavi possé.

Ob leges autem motuum admodum arctas, ra-
ristime intra Solis orbem conspicitur Venus, ac per plus
quam centum & viginti annorum decidit, ne semel

B b b b quidem
quidem ibidem videbitur; nempe ab anno 1639. (cum præclaro Juveni Horroxio nostro, cique primo & soli à rerum conditu, jucundissimum hoc spectaculum obrigid,) usque in annum 1761, quo juxta Theorias quas hactenus coelo conformes experimur, Stella Veneris iterum subtercurret Solem, Maii 26. mane; * ita ut Londini, horâ fere sextâ matutînâ in medio disci Solaris expectanda sit, nec nisi quatuor minutis centro Solis Australior. Duratio autem hujus transitus erit oto fere Horarum, nempe a secunda usque in decimam fere matutinam. Atque adeo ingressus minime Anglis conspicuus erit: cum autem Sol tum temporis occupaturus sit 16. Geminorum gradum, vigil-ti tres ferme gradus in Boream declinans; per totam quasi Zonam frigidam Septenttrionalem inocciduus conspicietur: ac proinde qui lîtūs Norwêgiae incolunt ultra Urbem Ni-drosiam, quam Drontem vocant, usque ad Promontorium ejus Boreâle, Venerem Solis dicicum subingredientem observare poterunt; ac fortasse Scottis Borealioribus & Infus- lœ Hetlandie, olim Thylen dixte, incolis, in oriente Sole ingressus ille conspicui poterit. Quo tempore vero Venus Solis centro proxima erit, Sol verticalis erit supra littora Borealia sinus Gangetici, vel potius regni Peguani; ac proinde in Regionibus circumvicinis, cum Sol in ingressu Veneris quatuor fere horis distabit ad ortum, & in egredi-fu to idem fere ad occasum, accelerabit motus apparentes Veneris intra Solem duplo fere parallaxeos horizontalis Vene-ris à Sole; quia Venus tunc ab ortu in occasum fertur retrogradâ, interea dum oculus ad Terra superficiem positus in contrarias partes ab occasu in ortum gyratur.

Postâ autem parallaxi Solis, uti diximus, duodecim secundorum cum femisse, erit parallaxis Veneris 43um secundorum; & sublatâ parallaxi Solis, restabit saltum femintum pro parallaxi Horizontali Veneris à Sole, ac proinde dôdrante saltum minuti promovebitur Veneris motus à parallaxi illa, interea dum Solis dicicum percurre, in iis

* Vide Phil. Transact. No. 193.
feelicet Poli altitudinibus que Tropico vicine sunt; atque adhuc amplius in vicinia Equatoris. Venus autem tum temporis fatis accurate quatuor minuta prima singulis horis intra Solem coniciet; ac propterea dodranti minuti undecim saltem temporis minuta prima competunt, quibus duratio Eclipseos hujus Venereae ob parallaxin contrahetur. Atque ex hac contractione solà liceret de parallaxi quam quærimus tutò pronunciare, si modo darentur Solis diameter Venerisque Latitudine in minimis accuratae; quas tamen ad computum postulare, in re tam subtili, haud integrum est.

Procuranda est igitur alia observatio, si fieri posset, in locis illis ubi medium Solis occupat Venus in ipso Mediodio; nempe sub Meridiano priori opposto, i.e. sex quasi horis vel 90 gradibus Londino occidentaliore, & ubi Venus paulo ante occasum Solem subintrat, paulo post ortum, exit; id quod fieri in dìcto Meridiano, sub altitudine Poli Borei-quinquaginta sex circiter gradum: hoc est, in eo Sinu qui Hudsoni dicitur, ad Portum ejus cui nomen Nelsoni inditum. In locis enim huc circumvicinis parallaxis Veneris durationem transitus prostrahet, & sex saltem temporis minutis longiorem efficiet; quia dum Sol ab occasu in ortum sub Polo tendere videtur, ea loca in disco Tere, motu contrario in occasum ferri videbuntur, hoc est motu cum motu proprio Veneris conspirante; proinde tardius moveri videbitur Venus intra Solem, ac cum diurnumora mora discum ejus pertransire.

Si itaque in utroque loco hic transitus ab Artificibus idoneis contigerit debite observari, manifestum est totis septendecim minutis longiorem futuram esse moram in portu Nelsoni observabilem, quam quæ apud Indos orientales expectanda est: nec multum refert an ad Fortalitium Sandi Georgii vulgo Maderas dictum, vel ad Benouem in litore occiduo Infulæ Sumatrae prope æquatorem capitatur observatio, si Anglis tum temporis hac studia curæ futuris erint.
erint. Si vero Gallis his rebus invigilare placuerit, non incommode apud Pondichery se fisset Observator in litore Sinus Gangetici occidentali, sub altitudine Poli duodecim fere graduum. Batavis autem celeberrimum Bataviae suæ Emporium Observatorium huic negotio latis aptum ministrat, si modo illis eriam animus fuerit hac in parte caelorum scientiam promovere. Ac fane vellem diversis in locis ejusdem Phænomeni observationes à pluribus institui, tum ad majorem adstruendam ex consensus fidem, tum ne Nubium interventu frustraretur singularis Spectator, eo spectaculo quod nescio an denuo visuri sunt hujus & subsequentis seculi Mortales; & à quo pendet Problematis nobilissimi & aliunde inacessi solutio certa & adæquara. Curiosis igitur syderum scrutoribus, quibus, nobis vita functis, hæc observanda reservantur, iterum iterumque commendamus ut, moniti hujus nostri memores, observationi peragendae strenue totisque viribus incumbant; iisque fausta omnia exoptamus & vomemus, præprimis ne nubili caeli importuna obscuritate exoptatissimo spectaculo priventur; utque tandem Orbium celestium magnitudines intra arctiores limites coercitæ in eorum gloriam famamque sempiternam cedant.

Diximus autem hac ratione Solis Parallax in intra quingentesimam sui partem investigari posse, id quod nonnullis mirum sine dubio videbitur. Veruntamen si in utroque è locis nuper designatis accurata habeatur observatio; jam monstravimus, totis septendecim minutis differre inter se durationes Eclipson harum Venercarum, ex Hypothesi sicilicet quod Solis parallaxis fuerit duodecim cum dimidio minutorum secundorum. Quod si major vel minor reperiatur ex observatione hæc differentia, in eadem fere ratione major vel minor erit Solis parallaxis. Cumque 17 minuta prima temporis competant duodecim secundis cum dimidio parallaxeos Solaris; pro unoquoque parallaxeos minuto secundo, orietur differentia plusquam 80 secundorum.
dorum minutorum temporis; adeoque si habeatur differentia hæc intra bina secunda vera & comprobata, intra quadragesimam partem unius secundi minuti constat quanta sit Solis Parallaxis; ac proinde distantia ejus determinabitur intra quingentesimam sui partem, saltem si parallaxis non minor reperiatur ea quam suppositimus: quadrages enim duodecim cum dimidio sunt quingenti.

Hactenus Astronomice docitis latis superque rem indicasse mihi videor, quos etiam monitos velim, me in hoc argumento, Latitudinis Planete rationem non habuisse, tum ad vitandas calculi intricatiois molestias, conclusionem etiam minus evidentem redditas; tum ob motum Nodorum Veneris nondum compertum, nec nisi ex hujusmodi corporalibus Planetæ cum Sole Conjunctionibus rite determinandum. Non enim conclusum est Venerem quatuor minuta infra Solis centrum transitorum; nisi ex Hypothesi quod Planum Orbitæ Veneris, in Sphæra stellârum fixarum immobile, Nodos suos iidem in locis habiturum sit, ubi anno 1639 inventi sunt. Quod si tramite Australiori transeat anno 1761, liquido patebit regredi; si vero Borealiori, progresi inter Fixas; idque in ratione $5\frac{1}{4}$ min. in centum annis Julianis, pro unoquoque minuto, quo via Veneris tum temporis plus vel minus distabit à Solis centro quam dictis quatuor minuitis. Differentia autem inter durationes harum Eclipium paulo minor sit septendecim minutis, ob Latitudinem Veneris Australium; major vero futura, si, procedentibus Nodis, ad Boream centri Solem transierit.

In eorum autem gratiam, qui cum observandis sydribus oblectentur, nondum tamen integram Parallaxium doctrinan hauserint, libet Schematico simulque Calculo paulo accuratior, rem plenius exponere.

Ponamus igitur, anno 1761, Maii 25°. 17h. 55'. Londini, Solem occupaturum \(= 15^\circ. 37'\), ac proinde ad centrum ejus Eclipicam tendere in Boream angulo 6°. 10'.

Veneris
Veneris autem visibilem intra Solis discum Viam tum temporis descendere in Austrum, facto angulo cum Ecliptica 8°. 28': proinde via Veneris tendet parum in Austrum respectu æquatoris, intersecans declinationis parallelos angulo 2°. 18'. Ponamus etiam Venerem ad dictum tempus Solis centro proximam fore, ac ab eodem quatuor minutis distare ad Austrum; singulisque horis etiam quatuor minuta prima intra Solem motu retrograde describere. Erit autem Solis Semidiameter 15°. 51". proxime, \( V \) erœ vero o°. 37" ½. Ac supponamus, experimenti gratia, differentiam parallaxium Horizontalium \( V \) erenis & Solis, quam quaerimus, c°. 31" esse, quali is ex supposita Solis Paralaxi o°. 12" ½ elicitar. Describatur itaque (Fig. II.) centro \( C \) circellus \( AEBD \), cujus semidiameter fit o°. 31''. discum Terræ representans, & in eo Ellipses parallelorum 22 & 56 grad. Latitudinis borealis, modo jam ad construendas Eclipes Solares ab Astronomis usitato, ut \( DabE, cde \): fit autem \( BCA \) Meridianus in quo Sol; ad quem inclinetur recta \( FHG \) Viam Veneris designans angulo 2°. 18', quæque distet à centro \( C \) 240 partibus qualium \( BC \) est 31; & de C cadat recta \( C\ell \) ipsi \( FG \) perpendicularis. Ac posito planeta in \( H \) ad 17 h. 55', vel 5 h. 55' mane, dividatur recta \( FHG \) in spatia Horaria III., IV., IV., V., V., VI., &c. ipsi \( CH \), hoc est quatuor minutis æqualia. Fiat etiam recta \( KL \), æqualis differentiae apparentium Semidiametrorum \( Solis \) & \( V \) erenis five 15°. 13" ½. Et Circulus radio \( KL \), centro vero quolibet puncto intra circellum Dici Terra descripsit, occurret rectæ \( FG \) in puncto denotante aliqua hora Londini numerabitur, cum in eo Terra superficie loco, qui sumptu in disco puncto subjacet, \( V \) ernen angulo contactus interioris \( Solis \) limbum continget. Ac si centro \( C \) radio \( KL \) descriptus circulus occurrat ipsi \( FG \) in punctis \( F & G \) erunt rectæ \( FH, HG = 14'. 41'' \), id quod percurrere videbitur \( V \) eneris tribus horis cum 40 min. Cadet igitur \( F \) in
Fin ii. 15', Londini; G. vero in IX h 35' mane. Unde manifestum est quod, si Terræ magnitudo, ob immensam distantiam, quasi in punctum evanesceret; vel si motu diurno deflectita Solem haberet eisdem puncto C semper verticalem, Eclipso hujus Mora integra per septem horas cum triente duraret. Verum Terrâ interea motu motui Veneris contrario gyrâ per 110 grad. Longitudinis suæ, ac proinde contradistà dictæ moræ duratione, puta 12 min. proveniæt ea 7 h. 8'. proximè, sive 107 grad.

Jam in ipso Meridiano Venus Solis centro proxima erit ad Ostium orientale fiuminis Gangis, ubi poli altitudo est 22 grad. circiter. Locus igitur ille utrique æqualiter disabit a Sole, in momentis introitus â exitus planæ, nempe 53° ½ grad. ut sunt puncta a, b, in parallelo majore D a b E. Erit autem Diameter A B ad distantiam a b ut quadratum Radii ad contentum sub Sinibus 53° ½ & 68° grad. hoc est, ut 1° 02" ad o° 46" 13' 11"; ac calculo rite instituto (quem ne Leātori tædio sit, omittæ praestat) invenio quod circulus centro a & radio K L descriptus occurret rectæ F H, in puncto M, ad 11 h. 20' 40"; centro vero b descriptus occurret ipsi H G in N, ad 1X h. 29' 22"; horis scilicet Londini numeratis; proinde tota Venus intra Solem conspicietur ad Gangis ripas, per 7 h. 8'. 42". Rectè igitur posuimus durationem fore 7 h. 8'; cum pars minuti hic nullius sit momenti.

Aptato autem calculo ad Portum Nelsoni, invienio, quod Sole jamjam occasuro, disçum ejus subitura sit Venus; flatim vero ab ortu ejus exitura ab eodem; Loco illo interea per Hemisphærium à Sole aversem de c ad d translato, motu motui Veneris conspirante. Mora igitur Veneris intra Solem diuturnior sit ob Parallaxin, puta quatuor minutis; ut sit omnino 7 h. 24'; sive 111 grad. æquatoris. Cumque Latitudo Locù sit 56 gr, erit ut Quadratum Radii ad contentum sub Sinibus 55° ½ & 34 grad. ita A B = 1°, 02" ad c d = 28° 33". Ac calculo rite praesto
rado confhabit, circulum centro c radio K L descriptum recte F H occurfurum in O, ad I h. 12' 45", centro vero d descriptum ipsi H G in P, ad IX h. 36". 37". Quocirca duratio Moræ ad Nelfoni portum erit 7 h. 23'. 52"; major scilicet quam ad oftia Gangis totis 5'. 10" temporis. Quod si Venus absque Latitudine transferit, fier diia differentia 18'. 40"; Si vero quatuor minutis Solis centro fuerit Borealior, ad 21'. 40" augebitur eadem differentia, multo major futura auta Planetae Latitudine Boreæ.

Londini autem, ex predictis Hypothesibus, consequitur Venerem jam tum infra Solem ingressam orituram; & ad 9 h. 37' mane, in Egressu Solis limbum interius concaeturam; ae denique non nisi horæ 9 h. 56', orbem ejus integrum relieturam esse.

Iisdem stiam Hypothesibus constat Venerem extremum Solis limbum Boreum quasi centro suo stringere debere, Anno 1769, Maii. 23°. 11 h. 00', ita ut, ob Parallaxin, in Borealibus Norwegia partibus, tota infra Solem inocciduum apparere poterit: dum in litoribus Peruviæ & Chili, vix exiguus sui segmento cadentis Solis disco quasi inequetare videbitur; uti in Infulis Moluccis earumque vicinitis, oriente Sole. Quod si Nodi Veneris retrocedere reperiantur (ut ob nuperas quatuor observationes suspicio est) tum toto corpore intra orbem Solis ubique conspicua, maximâ harum Eclipsem differentiâ argumentum Parallaxeos Solaris praebebit adhuc multo luculentius.

Quomodo autem ex observatis alicubi apud Indos Orientales; anno 1761, Ingressu & Egressu Veneris, & cum Exitu ejus apud Nos observabili collatis, eadem Parallaxis derivari poterit; aptando scilicet angulos Trianguli specie dati in trium Circulorum æqualium circumferentias, alia occasione docebitur.

FINIS.
PHILOSOPHICAL TRANSACTIONS.

For the Months of July, August and September, 1716.

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I. An Account of the Cause of the late remarkable Appearance of the Planet Venus, seen this Summer, for many Days together, in the Day time. By Edm. Halley, R. S. Secr.

It may justly be reckoned one of the principal Uses of the Mathematical Sciences, that they are in many Cases able to prevent the Superstition of the unskilful Vulgar; and by shewing the genuine Causes of rare Appearances, to deliver them from the vain apprehensions they are apt to entertain of what they call Prodigies; which sometimes, by the Artifices of designing Men, have been made use of to very evil purposes.

Of this kind was the late Appearance of Venus in the Day time, generally taken notice of about London and elsewhere; and by some reckoned to be Prodigious. This put me upon the enquiry, how it came to pass that at that time the Planet should be so plainly seen by Day, whereas she rarely shews her self so, unless to those who know exactly where to look for her. To resolve this, the following Problem arose, viz. To find the Situation of the Planet in respect of the Earth, when the Area of the illuminated part of her Disk is a Maximum.

To investigate this Maximum, I found it requisite to assume the following Lemmata. I. That the visible Areas of the Disk of the same Planet, at differing Distances, are always reciprocally as the Squares of those Distances; which is evident from the first Principles of Opticks. II. That the Area of the whole Disk of the Planet is to the Area of the illuminated Part there-of, as the Diameter of a Circle to the Versed-Sine of the exterior Angle at the Planet, in the Triangle at whose Angles are the Sun, Earth, and Planet. III. That in all plain Triangles, four times the Rectangle of the Sides containing any Angle, is to the excess of the Square of the Sum of the Sides above the Square of the Base, as the Diameter is to the Versed-Sine of the
the Complement of the contained Angle to a Semicircle, which I call the exterior Angle: This is a new Theorem of good use in Trigonometry, and easily proved from the 12th and 13th of the II. Elem. Euclid.

This premised, putting \( m \) for the Distance of the Sun and Earth, and \( n \) for that of the Sun and Venus, and \( x \) for the Distance of the Earth and Venus, or the third Side of the Triangle which we seek; by the third Lemma, \( 4n x \) will be to the excess of the Square of \( n + x \) above the Square of \( m \), as the Area of the whole Disk of Venus to the Area of the part illuminated; and by the first Lemma, the Area's of her whole Disk are at all times as the Squares of \( x \) reciprocally; whence the Quantity \( \frac{mn + 2nx + xx - mm}{4nx^3} \) will in all Cases be proportional to the Area of the illuminated part.

Now that this should be a Maximum, it is required that the Fluxion thereof be equal to zero, or that the Negative parts thereof be equal to the Affirmative, that is, that \( 2nx + 2xx \times 4nx^3 = 12nx^2 \times mn + 2nx + xx - mm \); and dividing all by \( 4nx^2 \), the Equation becomes \( 2nx + 2xx = 3mn + 6nx + 3xx - 3mm \). Consequently \( 3mn + 4nx + xx = 3mm \), and therefore \( x = \sqrt{\frac{3}{2}} \frac{mm}{mn - 2n} \).

From hence a ready and not inelegant Geometrical Construction (if I may be allowed to say so) becomes obvious; for with the Center \( S \) and Radius \( ST = m \), describe the Semicircle \( TD \cdot A \); and with the same Center and Radius \( SE = n \), the Semicircle \( EVB \); which two Semicircles shall represent the Orbs of the Earth and Venus. Make the chord \( AD \) equal to the Radius \( ST \), and from \( D \) towards \( A \), lay off \( DF = SE \); draw \( TF \), and thereon place \( FG = BE = 2n \), and with the Center \( T \) and Radius

\( F \) to \( G \).
TG describe the arch GV, cutting the Semicircle B V E in V; and draw the lines SV, TV: I say the Triangle STV is Similar to that at whose Angles are the Sun, Earth and Venus, at the time when the Area of the enlightened part of that Planet's Disk, as seen from the Earth, is greatest. How this Geometrical Effect follows from the Equation is too evident to need repetition.

In consequence of this Solution, I find this Maximum always to happen, when the Planet is about forty Degrees distant from the Sun; and the times thereof, about the middle between her greatest Elongations on both sides from him, and her retrograde Conjunctions with him; when little more than a quarter of her visible Disk is luminous, and resembling the Moon of about five Days old; and notwithstanding that her Diameter is at that time but 50 Seconds, yet she shines with so strong a Beam, as to surpass the united light of all the fixed Stars that appear with her, and casts a very strong Shade on the Horizontal plain whereon they all shine: an irrefragable Argument to prove that the Disks of the fixed Stars are unconceivably small, and next to nothing; since shining with a native Light, so many of them do not equal the reflex Light of one quarter of a Disk of less than a Minute Diameter.

In this situation Venus was found in July last, on the tenth Day, about which time, when the Sun grew low, she was very plainly seen in the Day time, for many Days together: as she might have been in the Mornings, about the latter end of September. But this, arising from the Causes we have now shewn, is nothing uncommon; for every eighth Year it returns again, so that the Planet may be seen on the same Day of the Month and Hour, very nearly in the same place; as all acquainted with the Heavenly Motions must know.

Lastly, it may not be amiss to note that the Equation \( x = \sqrt{\frac{3}{2} m m + n n} - 2 n \) has a Limit; for if \( n \) be equal to \( \frac{1}{4} m \), the point \( V \) will fall on \( B \); and the whole Disk of a Planet at that distance from the Sun would be the Maximum, viz. when in its superior Conjunction with the Sun. And the like if \( n \) were less than \( \frac{1}{4} m \); the Arch GV in such Case not intersecting the Semicircle B E.

SIR,

I am about to give you the best Account I can of what is remarkable, and known almost to all hereabouts, concerning the pressing forward of the Cliffs, and sinking of the Hills in the Neighbourhood of our Town of Folkestone. I begin with giving you a Sketch of the Situation of the Country. This I shall do by describing a strait Road from what we call the Mooring-Rock, to Tarlingham-House; the manner of the Country, as to the Rising and Falling, being much the same, for about a Mile on either Hand of the Road described.

A. The Mooring-Rock, about half-way between high and low Water-Mark.

D d d d

B
B. The Foot of the Cliff, 50 Yards from the Rock.
C. The Top of the Cliff, about 6 Yards high.
C D. A Plain of 50 Yards.
D E. A cragged Cliff, of 60 Yards high.
E F. A Plain above a Mile long.
F G. An Hill of steep Ascent, near half a Mile.
G H. The Land from the Top of the Hill to the House, near a Mile.
I. Tarlingham House, lying near 2 Miles and a half 
N. N. W. from the Rock.
E G H. A Line of Sight.
K B L. The Shore at High-Water Mark.

I hope Sir, you will understand the Situation of the Place pretty well, tho' I have not observed exact Proportion in the Sketch; which the Paper would not allow after I had taken the Rise of the Cliffs so high, which I thought proper for the more particular Describing of them.

The Mooring-Rock (tho' it lies surrounded with great numbers of other Rocks) is it self a most noted one, known by this Name, time out of mind. At this Vessels use to be moored, while they are loading other Rocks; which they take from hence, not only for our own Pier Heads, but for those of Dover Pier; and a very great Quantity of them were Shipt, in the time of Oliver's Usurpation, and carried to Dunkirk, for the Service of that Harbour.

This Rock has remain'd fixt thus, for the memory of Man; and old Men have observed, that, for forty Years and upwards, the distance between it and the Foot of the lesser Cliff AB has been much the same; neither can they be much out in their Guesss, the Distance being so small. Tho' there seems nothing extraordinary in this, yet its what they take special Notice of, to their great Surprize; for they say, and prove by good Marks and To-
kens, that the lesser Cliff \( B C \) has been constantly falling in, insomuch, that from time to time, in their Memory, near \( 10 \) Rods forward to the Land has been carried away by the Sea. From whence, as it appears that the Plain between the Top of the lesser Cliff and the Foot of the higher \( CD \) has been formerly double the Breadth that it is at present, so the distance been the Rock and the Foot of the lesser or lower Cliff \( AB \) should have increased in Proportion, and would have been double at present, to what it has been formerly: But this Distance remaining the same (as is above noted) or rather less (in the Opinion of many) is what is greatly wonder'd at: nor can it be accounted for otherwise, than by supposing that the Land press'ing forward into the Sea is wash'd away by the high Tides; and, as often as this happens, presses forward again. This press'ing forward of the Land into the Sea, would be incredible, were it not shewn to be matter of Fact; and that not only at this one Place of Observation, but by like Observations all along this Coast, as far as the Situation continues the same.

Now, Sir; let us climb both these cragged Cliffs, and place our selves at the Top of the higher One, at the Point \( E \). And here we are to observe, that (as old Men inform us) upward of forty Years ago, not so much as the Top of Tarlingham-House could be discern'd, neither from hence, nor yet a good Distance off at Sea; but it discover'd it self by degrees, till at this Day, not only the whole House, but a great Tract of Land below it, is plainly to be seen, as in the Line of Sight \( E G H \). The Tract of Land is more in Proportion than describ'd in the Sketch, between the Point at \( H \) and the House. In this there can be no Fallacy; and we can ascribe it to nothing less than the sinking of the Hills (for their Tops could never wear D d d d 2 away
away considerably, being always cover'd with Grass, and never broken up by the Plough or otherwise. These Hills are all of Chalk, and have probably very large Caverns within, Springs of Water always flowing plentifully from the foot of them; and I have had it observ'd to me, that upon their Tops frequent Cracks have been taken notice of. Whatever be the Cause of it, 'tis not to be doubted but that these Hills are greatly sunk. And this sinking of the Hills, the People at this Place believe, forces the Cliffs and all the Land forward into the Sea. The Cliffs consist of great ragged Sand-Stones till we come near a Yard (at some places more) of the Bottom; then we meet with what they call a Slipe, i.e. a slippery sort of Clay always wet. Upon this Slipe at the bottom, they presume that the hard Stony Land above slides forwards toward the Sea, as a Ship is launch'd upon tallow'd Planks. I thought it proper to give you this account of the Nature of the Earth; and withal to mingle with it the Opinion of the People, that you might perceive they are so far from doubting of the Truth of what is above written, that they endeavour to find some Solution of it, as being a thing not more strange than true. If I should take all the Hands that can be got to testify the Truth of this, it would make too large a Roll, so I shall choose only a few of the most antient and of best Credit.

I assure my self that I have Credit enough with you to be believed upon my own single Subscription, that I am,

S. I. R., Tours,

Folkestone in Kent,
February 24, 1715-16.

John Sackette.
We whose Names are underwritten do hereby testify the Truth of the Matters of Fact in the within written Letter related,

Benjamin Master, a Jurat of the Town, aged 74.
Robert Hammond, Senior, a Jurat of the Town, aged 77.
William Godden, a Fisherman, aged 74.
Thomas Marsh, a Fisherman, aged 72.
William Hall, a Fisherman, aged 73.
James Godden, a Fisherman, upward of 60.

III. Miscellaneous Observations made about Rome, Naples and some other Countries, in the Year 1683 and 1684; and communicated to the Publisher by Tancred Robinson M. D. R. S. S.

SIR,

You having been pleas'd to think some of my Observations might be agreeable to the Publick, I shall here freely give you them (such as they are) omitting those that were formerly extracted out of the MSS. Diaries of my Travels, some of which are Printed in several Philosophical Transactions, and others in some of Mr. Ray's English Tracts.

In my Journey from Rome to Naples I observ'd on the Rubbish of the Tre Taberne an unusual Vegetable for that place, remote from Town or House, which was the Ficus Indica Spinosa commonly call'd the Opuntia or Tuna, and by our Writers of America the Prickly-Pear, whose Juice gives the Urine a red Colour; when I came to Naples, I found it there near the Rocks, and in some wild solitary Places like a Native. If the Spaniards planted it, they chose desert Situations. On this Plant the Cochineel
Vermiculus is said to feed in great Numbers, before it changes into the Chrysalis or Aurelia of a Lady-Cow: but the Colour lies in the Nymph-Worm before it turns a Beetle. This gives me occasion to reflect upon the many Species of our European Vermiculi, some of which might be found to yield rich Colours (if try’d): We are certain the Maggot of our Ilex gives the Kermes, and a noble Scarlet Dye before it turns into a Fly. Many Shell-Fish (which are a sort of Insect) contain Purple Juices.

This brings on another Remark I made in passing the Apennines and Alps, where I noted in some Beds or Strata, and even in the midst of the hardest Rocks, great varieties of perfect Shells, that never occur’d to me on the Italian Shores, nor in any of the numerous Museums of that Country: So I guess they might be Exotic.

Going further on the Via Appia, I observ’d abundance of the Siliqua Arbor or Carob Tree, commonly call’d Panis S. Ioannis Baptistæ; on the Pulp whereof many poor People were feeding. The Husks tasted like Manna to me. Near them grew plenty of the Arbor Juda.

The Arbutus, or Strawberry Tree, was common in the woody places; if this grows wild in the South West parts of Ireland, as some affirm, I shall think them much warmer than any Counties of England.

Before I enter’d the beautiful Campania of Naples large Woods of Cork Trees grew on each side the Road, where the Inhabitants were decorticating them. I ask’d if the Trees did not perish: they answer’d, some did, but the Acorns return’d annual Supplies. The Women and Children wore Shoes made of the Bark.

Coming near Capua I observ’d a Species of Ash, or Ormus, on the Trunk whereof many Saccharin Concretions were visible. This prov’d the true Manna, that issues out thro’ the Incisions made in this Tree by the Inhabitants of Calabria. Swarms of Cicada’s were sucking the Body
Body and Boughs, and perhaps by wounding then made way for fresh Manna. Here I may note, that many Insects have not only a Proboscis to bore and draw out the Juices of Plants for Aliment, but other proper Instruments to convey their Eggs into Vegetables and Animals, where they may find Covert and Food when they come to hatch, in the Gall-Tumours, and other Excrences occasion'd by the Wounds of the Parent Insects, that make such variety of Cuniculi in all parts of Plants, and even in the cutaneous parts of living Creatures and in dead Flesh.

This confirms me, that many Gums and Exudations find their way out of Vegetables thro' the Wounds of Insects and other Apertures. Most Voyagers thro' the East Indies affirm, that Gum Lack is work'd and made by large Ants that cover the Trees. I rather think the Insects suck and terebrate the Tree, and so give vent to that peculiar Sap that hardens in the Sun. This may extend to most Balsamiferous, Gummiferous, and Saccharine Plants, especially in hot Climates where Insects abound, and are more active. In cold Climates the Saps of many Vegetables will boyl into Sugars, as that of Maple, Birch, Reeds, &c. Not but that the Fluids of Plants (like those of Animals) will spontaneously break thro' their Vessels in a Plethory, and make on the superficial Parts various Eruptions and Congestions.

Discoursing of Manna I may here take Notice there are many adulterations of this Drug: all pass for the Calabrian, whereas that of Brianston is from the Larix, that of Persia from the Myrica, and these frequently mixt with the Juices of Spurges, and other Purgative Ingredients. I must not here deny that Dew will sometimes in cool Mornings shoot, and congeal into a solid, sweet, white Substance, which I once observ'd in very hot Weather before Sun-rise.

Upon
Upon viewing the Vulcanos about Naples, Vesuvius on the East side, the Solfatara and Monte di Cinere on the West near Puzzuolo and Bajæ; I observ’d the same Face of Nature, which I believe runs thro’ all the other Vulcano’s of our Globe, viz. heaps of Pumice Stones and Cinders of Marchasites on the sides, with Beds of Flower of Brimstone on the tops. The Holes and Cavities in those calcin’d Minerals seem to be the Nidus of the Sulphur, which hath been sublim’d by the Heat and Fire of that vast Mass of Pyrites, that compose the Bowels of those Vulcano’s, and lye scatter’d thro’ many parts of the Earth, even under the Sea, where they sometimes germinate, ferment, and take Fire, throwing up little Islands. Earthquakes and other Chocs of the Globe may spring from the Mines of these combustible and explosive Minerals, loaden with Brimstone and Elatic Salts. Hence some Account may be given of Therma or hot Baths, whose Waters gliding thro’ these hot Beds take their Gas. Of such Medicinal boiling Waters and Stoves, there are more about Naples than in any place I ever saw or heard of, the whole Country being continually pervaded by hot Steams.

Walking round this City I found Palm Trees, some with unripe Dates hanging down, others without any Fruit: and there was another Species of Palm that sweats out the Gum Dragon: I suppose the Monks had transplant’d them out of Africa. I saw growing here many Sugar Canes, Rice, Maiz, abundance of the purging Senna, and Cummin Seed. Thro’ the whole Campania of Naples I observ’d the same Vegetables to be larger and more proud than in other parts of Italy, as the Platanus, the Lentiscus, the Terebinthus, the Pistaches, the Oleanders, Agnus Castus, Barba Jovis, the Tragacanth, the Styrax, the Capers, &c. The Melons, Jujubes, the Azaroles, and other Fruits were of a better Taste. The Gossypium, with the Cor-
ton breaking out of the husks, adorn'd some of the Fields; the Hedges full of Pomegranats, Almonds, Tamarisk, Sumach, Cedrus Lycia (a sort of Juniper or Savin) abundance of Phillyrea, Alaternus, Cistii, Cytiis, Myrtilles, Spanish Broom, Bays, Laurustines, &c. all wild, Indigenous of that warm Soil and kind Climate. The Water-Melons, the Olives, the Oranges, Lemons and Citrons were better than about Genoa or in Provence.

The Lotus Arbor or Nettle Tree, the Paliurus or Christ Thorn, the Ricinus or Palma Christi, common in the Hedges, with several Thymelaeas.

I saw them fishing for Coral, and Hippocampi: the first did not come soft out of the Sea; the hard Incrustation covers the Vegetable part that bears Seed, as the Alga's and Fuci do. They take the Sword-Fish by darting a Spear into him, as they do the Whales in the Greenland Fishery.

When dark Night came on, I could see Multitudes of Luminous Flies thro' the Campania of Naples: perhaps our Male Gloworm, or flying Cicindela, may abound there; not but that many other Insects may carry such Lanthorns about them. The Scorpions creep out about that time; and I have found them often in Bed, with the Punaises.

The Hedges are full of Lizards of various Colours; and the Cicada's chirp and sing towards Evening. I observ'd several Species of stinging Spiders in the Corn Fields, some of which, in hot Harvests, may prove Tarantula's; the Poysons of Animals and Plants increasing with the approach of the Sun, and the Heats of Climates. Abundance of Silk-Worms were spinning on the Trees and Shrubs; the Birds prey'd upon them, before they could change into Papilio's, as they do upon swarms of Locusts.
I eat often their young Frogs, Tortoises and Snails, served up with Oyl and Pepper, which agreed well with me: so did their Sea Urchins, and the Urtica Marina, (called Sea Gelly or Blubber, tho' it be an Animal, having a true Heart, and Vessels for the Circulation of Fluids) Some of their Thistles are no ungrateful Sallet.

I saw some Vitriol Works about Siena, Rome and Puzzuolo; those of Alum only about Civita Vecchia. Amongst the Sands of the Adriatic Sea I observ'd many white, clear, shining Flints; which they told me were carried to Venice, to make the fine Chrystal Glass at Muran.

Upon reading our ingenious Dr. Musgrave, de Geta Britan. & Synop. Chronol. Dom. Sever. I consulted my Diary taken at Rome. The Magnificent Septizonium figur'd by him stood near the Foot of the Palatine Hill, on the E.S.E. side, overlooking the Via Appia and the Circus Maximus, the Amphitheatre of Titus being near on the other Side. By the number of Portico's (which were Seven) it might contain Multitudes of People, as Spectators of the Triumphant Entries and the publick Games. But I would not be thought to differ from our Learned Countryman, who with good Authority, thinks it the Sepulchretum of that Imperial Family; tho' most of the Ancient Mausoleum's, (at least those I saw) were Rotondas, or Columbaria's, for the more convenient placing the Urns of the Kindred; as that of Augustus near the Campus Martius; that of Adrian on the other Bank of the Tyber; those said to be of Scipio, of Cicer, and Munatius Plancus, near Gaeta and the Via Appia; that of Virgil on the side of Mount Paullippus; that of C. Metella and some others on the Via Flaminia. Some were Pyramidal as that of Cestius in the Wall of Rome, and a few others on the public Roads. This Septizonium Severi seems to differ from the rest of those Ancient Sepulchretums, which
might be varied according to the Fancy and Humour of great Families.

This Urn Burial was only in Fashion amongst the Gentes Majores: as for the dead Bodies of the Plebeians and Slaves, they were generally laid in places where they had dug Stone; and those Quarries became Catacombes. The Laws prohibited them to bury within a City, unless the Bodies were first reduc’d to Ashes.

I observ’d in many of the Ruins about Rome and Naples, great Stones laid close, and wedged very fast with little or no Cement; the Bricks towards the middle of a Building, were generally of a Rhomboïdal Figure, very Smooth, Shining and Hard, laid in Plaister as firm as Marble. Their Mortar was much more durable then ours, as appears at this Day by their Aqueducts and Piscina’s, the Cento Camare, and Caligula’s Bridge under Water at Bajæ. Pliny says, they made use of the Terra Puteolana, but the present Inhabitants have lost the way of tempering it.

During my abode at Genoa, Leghorn, Oßia and Civita Vecchia, I observ’d many Torpedo’s or Cramp Fishes, most accurately Anatomized by S. Lorenzini; plenty of Sphyraena’s, (a Species of Sea Pike, a-kin to the Needle-Fishes) The Uranoscopus, call’d Bocca in Capo and Prete. The Mola or Sun Fish. The Dentex or Pentalis, Altavela’s a sort of Pastinaca. The Pesce Balestra or Capriceus. The Pesce Pettine or Novacula. The Zygrena or Ballance Fish, as large as the Saw-Fish or most Sharks. The Scolopax or Trombetta, call’d by our Seamen the Bellows or Trumpet-Fish. The Draco Marinus: The Tunny-Fish. The Centrina or Pesce Porco. The Aquila. The Scorpius Major, with Varietyes of Turdi in the Markets. But what pleas’d me most, were some odd Sea Animals, as the Lepus Marinus, (a Species of naked Snail) the Hystric Marinus, or Eruca, call’d by the Seamen Pincio, with a Brush hanging out of the Tail, like the Byssus or Silk.
Silk of the Pinna. Many Tamburo's or Drum-Fishes; Plenty of Murana's. I observ'd a strange Sea Animal, call'd the Microcosmo marino, with many Shells, Tubuli and Vegetables growing or sticking to the Back of it, this appear'd to me a kin to the Echini Marini, or rather to the Stella Marina, being Triangular, and sometimes Pentadactylous.

I embark'd once with the Fishermen, who shew'd me several Loligo's, Polypi, and Sepia's, or Cuttle-Fishes, (all Crustaceous) some of them were casting out their Ink in the Water: I suppos'd some Sharks, Dog-Fishes, or other Enemies, were near them; this black Liquor may be the Gall of those Animals. In the Nets, I often found Sea Insects, and Vegetables; and indeed a new World, undescrib'd by natural Writers, at least unknown to me: but for want of the Art of Designing or Drawing, abundance of things escap'd me, and were utterly lost; therefore I would advise all Travellers to be conversant in that most useful Science.

I observ'd the Italians near the Alps and Appennines, call'd several Birds Francolino's, as our Red, Grey and Black Game; and even their red and white Patridges; the different Colours of the Hens from the Cocks, the many Variegations in Feathers, the different Ages and Places, have all given occasion to multiply Names and Species; the same may happen in Fishes, Quadrupeds, Insects, and all the Divisions of Zoology; and even in Botany and Mineralogy.

The Italians call many of their little fat Birds Beccafico's, that feed upon Figs, Grapes, and other sweet Fruits. So the French multiply their Ortulans, taken in the Vineyards and Gardens. Some of the Antient Writers take Notice that the Romans used to feed their Geese and other Birds with Figs, when they intended to swell their Livers to a monstrous Bigness.
The Merops or Apiaster is common on their Brooks: it flies like our Kings-Fishher, and preys not only upon Insects but Fish. There is a very beautiful Bird in Italy, that suspends its Nest down from the Boughs of Trees. When I saw it fly by me, I took it for an Indian, from the brightness of its Colours; it is as large as our Mistel-Bird and Thrush an Iterus Flinii.

The great Cock of the Wood (said to be found in Ireland) is common on the sides of the Italian Hills, and brought frequently to the Markets. I saw twice or thrice the Himantopus, and the Phenicopterus or Flamingo, (whose Tongue was a Dainty amongst the Romans, when they grew Luxurious). I observ'd some Spoon-bills: these three last Birds were wading in the Rivers and Marshes, near the Sea. Once I spy'd some Pelecans on the Adriatic, near the mouth of the Po. The Avis Diomedaea was hung up dry'd in one of the Museums at Florence, but they told me it had been taken on some of the Isles of the Archipelago.

On the Laguna of Venice, I saw several Species of Mergi, Lari, Colymbi, and other Water Fowls, most of which Div'd. I was surpriz'd with the Variety of them, having not seen so many on other Coasts: perhaps the hard Winter had forc'd some unusual Birds thither. The Monks and Fryers told me, they eat some of those Sea Birds in Lent and on Fast Days, because they liv'd upon Fish, and had a piscose Taste, as the French pretend their Macreuse to have, which is a sort of Sea Duck, common on the Coast of Normandy, and brought to the Markets, even at Paris on Maigre Days; of which I gave a long History in the Philos. Transact. An. 1685. No. 172.

Buffalos are common in the Kingdom of Naples, and in some parts of Lombardy, where they plough and draw with
with them. A peculiar Cheese is made of their Milk (call'd Caño di Cavallo) roll'd up like stiff pieces of Ribbon. Out of their black shining Horns they make Snuff-boxes and Combs. The Creature is unruly, and therefore they lead them with Iron or Brass Rings drawn thro' their Noses. They make a Buff Leather of their Skins. I once saw some hairy Sheep feeding on a Common; perhaps they had been brought from Africa.

In passing the high Alps, I had a View of the Ibx or Steinbock, whose large Horns are recurvated almost as far back as the Tail; they are very ponderous for the bulk of the Animal, having many knotty Rings, that may help them in climbing. They are rarely taken.

The Rupicabra or Chamois is very common on the sides of the Cliffs, whose Skins afford the soft Leather. The Mus Alpinus, or Marmota, is as large as a Rabbet, will soon grow tame in Houses, tho' brought down from the Summits of the highest Mountains, where it will grow fat.

I have seen in several Towns of Italy fresh strong Porcupines, which the Inhabitants told me were taken in the Hedges and Ditches thereabouts, tho' much more rare than our Land Urchins. In the Grisons Country, and in some Cantons of Switzerland, I have often observ'd the Ranunculus Viridis or small Tree-Frog, perching on the Boughs and Leaves.

In the Northern Parts of Germany I saw several Elk-Skins, and those of the Rhin-Deer stuffed, and set up in Museum's, but never alive: tho' the Animals are said to be common in Muscovy and Lapland, and sometimes seen in the Forests of Prussia.

The Skins of Hippopotami (said to be the Behemoth) are in some Collections of Curiosities in Italy and Holland: so are those of the Musk-Deer, one of which is in the Museum of our Royal Society.
Give me leave here-to reflect a little upon the late Aurora Borealis, whose Phenomena you have so well describ'd and explain'd in your late Philosophical Transacti-
on, No. 347. I am of your Opinion, that those Phosphorous or Luminous Appearances in the Firmament, proceed from the various Effluvia perspir'd out of our Globe, or passing thro' it; for I have seen those Lights over Vesuvius, the Strombulo Islands, and towards ætna in dark Nights, when those Vulcano's were not flaming nor burning, their Sides and Tops being passable to Travellers at that time, and all their outward parts quiet. We are certain that Iceland and Greenland abound with Vulcano's; so may North East Lapland, North Russia and Tartary, where vast Chains of Mountains are said to run. The Jesuits, and other Travellers, relate many prodigious Eruptions of Fires, and Earthquakes towards the North of China; but nearer the Pole the Earth must be clos'd and pent up many Months, by the long severe freezings and continual Snow and Ice, which relaxing towards Spring, may give vent to that vast Mass or Magazine of perspirable Matter, that had been kept so long in hot Subterraneous Prisons. This may be one Reason why Animal Bodies themselves are often sensible of Changes at that Season in our Climate, when Perpiration is upon such an Increase; but I will not take up your time any longer, especially upon a Subject that you understand so well.

S I R,

Your most humble Servant,

Tancred Robinson.

IV. An
IV. An Account of the Mischiefs ensuig the swallow-
ing of the Stones of Bullace and Sloes: By the Re-
verend William Derham, Prebend of Windsor,
and F. R. S.

Among the Accounts which the Royal Society hath
had of the Mischiefs ensuig the swallowing of
divers sorts of Stones, I do not remember any Case
wherein the leffer Stones of Fruits (such as Sloes parti-
cularly and Bullace) have produced any dangerous
Symptoms, especially in the Stomach alone. The larger
Stones, I know, of Prunes, and other great Plumbs, have
produced very fatal Effects; but the leffer Stones of
Sloes, Cherries, &c. many swallow rather out of choice,
than with any apprehensions of Danger, thinking them
useful in preventing a Surfeit from the Fruit. But the
following Case will shew the Danger even of these
leffer Stones. And I have acquainted the Society with
it, on purpose to prevent Dangers, if it should be
thought fit to publish it in the Transactions, for a warning
to others.

The Case is this. About two Years ago the Man-
Servant of a Neighbouring Clergyman complained to
me of excessive Pains in and about his Stomach; that he
lay under a great Dejection of Appetite; and whenever
he eat, that he could not retain it, but in a little time
vomited it up. By which means he was, in a short time,
reduced to a very low and languishing Condition, in-
foruch as they began to despair of his Life.

Upon this he applied himself to some Practitioners in
Physick: One of which ply'd him with strong Vomits
eight Days together, with very little Signs of Success.
But some time after, having Occasion to ride somewhat
more than ordinary, he found himself very sore in his
Stomach, and Sick; which ending in violent Vomiting
and Straining, brought up the first Stones he ever per-
ceived to come from him, which were about Twenty
in number.

After this he had frequent returns of the Vomiting
up of Bullace and Sloe-Stones, especially upon strong
Exercises; particularly moving and stooping much in
Weeding in the Garden; in Riding also, although it
was only to water his Master's Horse. Upon these
Occasions he would be seized with acute Pains in his
Stomach, and soon after Vomit up more of those
Stones.

He hath counted above One hundred and twenty
Bullace and Sloe-Stones that have been discharged; and
many others he could not number, by reason they
came up when he was in Riding, or in his Business. He
is not yet free of them, but is in Pain oftentimes, and
Vomits them up, especially in Riding; but after he hath
discharged them, he is much easier for a while. He
commonly brings up a slimy Matter with them, mixed
with Blood or something very like Blood.

The Cause of all this Disaster the Man assures him-
sell was this, namely, being in his Youth a great lover of
Fruit, he used greedily to devour all sorts he could come
at, and Bullace and Sloes being the easiest to be gotten,
he used to ingurgitate great quantities of them, without
evacuating many of the Stones by Stool, as he well re-
members, and as he observed others did. These Stones
he thinks have lain in his Stomach (some of them at
least) above ten Years; but he felt no Pains till about
four Years ago. And these at first were not so violent,
nor attended with such severe Fits of Vomiting, and
F f f f l o s
Foods of Appetite, as they by degrees came to be afterwards.

Thus having related the Cafe' as the Man told it me, I shall leave the Ethiology of it to the learned Physicians, it being sufficient for me to relate the matters of fact, and thereby testify the Duty and Respects owing to the Society by

Their most obedient
Humble Servant,
W. Derham.

V. Observations and Experiments relating to the Motion of the Sap in Vegetables. By Mr. Richard Bradley, R. S. S.

Of Plants in general we may first observe, that they are either Terrestrial, Amphibious, or Aquatic; and so nearly do Vegetables agree with Animals in most points, except Local Motion and its Consequences, that from the Knowledge of the one we are reasonably led to the Discovery of the other.

Those Plants which I call Terrestrial are such as Trees, Shrubs and Herbs, which grow only on the Land. These like Land Animals have diversities of Food, a Method of Generating, and certain Periods of Life.

Of the Amphibious race, which live as well on Land as in the Waters, are the Willows, Rushes, Minths, &c. These are not unlike in many respects to the Otter, Tortoise, Frog, &c.

The Aquatics, whether of Lakes, Rivers, or the Sea, are very numerous; these may be compared with the Fish-kind, and like them will not live out of their proper
per Element. In Fresh Waters are the Water-Lilly's, Plantains, &c. and in the Sea, Corals, Fuci, &c.

Plants seem to possess only the next degree of Life below the most stupid Animal; or where Animal Life leaves off the Vegetable Life seems to begin.

The Seasons of Motion in Plants are the same with those of Animals, which sleep during the Winter. An Artificial Heat will give Motion to either of these in the Coldest time.

The common Opinions relating to the Sap's Motion are as follows. First, The Sap does not rise by the Pith; because some have observed the Trunks of large Trees to be without that part, and yet the same Trees have continued to put forth Fruit, and Branches on their Tops. I have observed, that the Pith is not found in those Branches of a Tree which exceed two or three Years growth; and it is certain, that the Pith which is in a Branch of this Year, will (the greatest part of it) be distributed into those Boughs which form themselves the next Season.

It is said by some, that the Tree does not receive its Nourishment by the Bark, for that Trees having lost that part, will still continue their Growth. Others tell us, that if the Bark be cut away round the Trunk of a Tree, it will presently die. These various opinions seem to have been set on foot without extraordinary Consideration, upon the belief that a Tree has but one Bark: Whereas, upon Examination with the Microscope, we find four distinct Coverings to each Branch, without the woody parts. The two outermost Barks may be taken from a Tree without great Damage, but the other two which lye nearer the Wood being strip'd off will kill the Tree.

Some affirm, that the Sap doth neither rise nor fall in the woody part of a Tree, because they have not been able
able to discern any Sap to issue out of that part, when a Branch has been cut. The Microscope plainly shews us the Vessels in the Wood, through which the Sap riseth from the Root; but as these Tubes are not large enough to admit into them any thing more gross than Vapour, so they have not been esteem'd to be of any great Use. But I hope the Explanation of the adjoynd Figure will in some measure discover the Office of these, and of such other parts of a Plant as are severally design'd for the Growth of Vegetables; but it will first be convenient to enquire a little into the Nature of the Root.

The Root of a Tree is chiefly composed of a Parenchyma; more gross than that in the Stem or Body of the Tree; it has likewise Vessels and a Covering, which I shall better explain in another Paper. The Root, that is, the principal part of it, receives into it such Juices of the Earth as are proper for it, and no other. Somehow like a Week of Cotton, which having been impregnated with Oil, will only admit Oil into it. This Provision being made in the Stomach of the Plant (as I call it) chiefly in the Autumn Months; the Tree is prepared for Germination so soon as the Earth is sufficiently warm'd, either by the Sun's Beams, or an artificial Heat, such as Horse dung, Bran and Water, or other such like Ferments. These Heats raise into Vapour the Juices contained in the Root, and by that means cause Vegetation.

Figure I. which I am about to explain, is part of the Branch of an Apple Tree made in May 1715, and cut in April 1716. It was cut in figure of a half Cylinder, the length somewhat more than the Diameter, which was about a quarter of an Inch. This being magnified with one of Campani's Microscopes, discovers the following parts, viz.
1, 2, 3, 4, 5, 6, 7. are Capillary Vessels, which run longitudinally through the Branch, in the Ligneous part, which was made in the Year 1715. Through these Tubes, the Steam riseth from the Root; the strength of which is well explain'd by the Engine for raising Water by Fire, invented by the late Captain Savory.

From A to B, we may view Vessels of the same sort, made at the same time.

8, 9. are Vessels of the same use with the former, now forming themselves for the use of the Year 1716.

By this means the Diameter of the Branch is increas'd, and additional Nourishment suffer'd to pass into those Buds which are to make new Branches. These are made out of the Fourth or innermost Bark, markt C, C.

The Mouths of the Capillary Tubes of the Years 1715, and 1716, are D, E. The Vapour which riseth from the Root, is continued in these Vessels, to the extremities of the Branches; where it meets with parts (not here describ'd) like to Glands; which Glands, if we may so call them, are likewise found at every Knot or Joyn't. At these places, the Vapour coming near the Air is condensed, and returns between the Barks, by means of its own weight, down F, G, H, leaving in each Bark mark'd I, K, L, such Juices as each of them naturally is inclin'd to separate from it; till at last, the more Oyly part passing to the Root, may lengthen the Fibres thereof, as Icicles are lengthned; and by its Oleous Particles, preserve them from Rotting by the Wet. The parts which compose the several Barks, are Parenchymous or Spongey.

The first mark'd M, is of a closer Texture than the second N, and the second closer than the third O, and so on till these Parenchymous Parts are interwoven with the longitudinal Wood-Vessels, where they are somewhat constrain'd, till they come to make the Pith mark'd P. Then they are much Larger than in any other
other part of the Tree; and by what I have observ'd, seem to contain a more finish'd Juice than the rest, and may well enough be stiled the Medulla.

We may note, that when the fourth or innermost Bark C, has once compleated its Sap-Vessels, and is firmly join'd to the Wooden Part, then the third Bark O takes its place for the succeeding Year; and so the rest, except that the first mark'd M, splits and divides its self, to supply the place of the second, as I shall demonstrate hereafter.

Before I conclude, I shall beg leave to recommend the following Enquiry to the Curious, viz.

If the several Barks, having different Texture of Parts, admit into each separate and different Juices from the rest: Whether those Juices may not be of very Different Vertues; the first more Astringent than the others, the second perhaps Emetick; and the third Cathartick. This seems to be worth Enquiry.

VI. Some Microscopical Observations, and Curious Remarks on the Vegetation, and exceeding quick Propagation of Moidiness, on the Substance of a Melon. Communicated by the same.

I had lately a large Melon-Fruit, which I split lengthways thro' the Middle, in order to observe the Vessels which composed the Membrane or Tunick of each Ovary; but my affairs at that time not permitting me to continue the Work I had began, I lay'd by the one half of the Melon, to be examin'd when I might have more Leisure.
At the end of four Days, I found several Spots of Moldiness began to appear on the fleshy Part of the Fruit, somewhat Green towards the Rind; and of a paler Colour towards the Middle of the Fruit. These Spots grew larger every Hour, for the space of five Days; at which time the whole Fruit was quite cover'd.

This surprising Vegetation made me Curious to examine, if there was any difference between those Parts which were Green and the others, besides their Colour. The first being seen with the Microscope, appear'd to be a Fungus, (See Fig. 2.) whose Cap was fill'd with little Seeds, to the number of about Five Hundred; which shed themselves in two Minutes after they had been in the Glasses.

The other Sort had many Grass-like Leaves, among which appear'd some Stalks with Fruit on their Top. Each Plant might well enough be compared to a sort of Bull-Rush. (Fig. 3.) They had their Seed in great Quantities, which I believe were not longer than three Hours before they began to Vegetate; and it was about six Hours more, before the Plants were wholly perfected: for, about seven of the Clock one Morning, I found three Plants at some Distance from any others; and about four the same Day, I could discern above Five Hundred more growing in a Cluster with them, which I suppos'd were Seedling-Plants of that day. The Seed of all these were then Ripe and Falling.

When the whole Fruit had been thus cover'd with Mold for six Days, this Vegetable Quality began to abate, and was entirely gone in two Days more. Then was the Fruit putrifid, and its fleshy Parts now yielded no more than a stinking Water, which began to have a gentle motion on its Surface, that continued for two Days without any other Appearance. I found then several small Maggots (Fig. 4.) to move in it, which grew
grew for the space of six Days; after which they laid themselves up in their Bags. Thus they remain'd for two Days more without Motion, and then came forth in the Shape of Flies. (Fig. 5.) The Water at that time was all gone, and there remain'd no more of the Fruit than the Seeds, the Vessels which composed the Tunicks of the Ovarys, the outward Rind, and the Excrement of the Maggots; all which together weigh'd about an Ounce. So that there was lost of the first weight of the Fruit when it was cut, above twenty Ounces.

We may Judge from this, and other Cases of the like nature, how much Vegetable Life is dependent on Fermentation, and Animal Life on Putrifaction.

VII. The Art of Living under Water: Or, a Discourse concerning the Means of furnishing Air at the Bottom of the Sea, in any ordinary Depths.

By Edm. Halley, LL. D. Secretary to the Royal Society.

There have been many Methods proposed, and Engines contrived, for enabling Men to abide a competent while under Water: And the Respiring fresh Air being found to be absolutely necessary to maintain Life in all that breath, several ways have been thought of, for carrying this Pabulum Vita down to the Diver, who must, without being somehow supplied therewith, return very soon, or perish.

We have heard of the Divers for Spunges in the Archipelago, helping themselves by carrying down Spunges dipt in Oyl in their Mouths: but considering how small a Quantity of Air can be supposed to be contained in

the
the Pores or Interfaces of a Spunge, and how much that little will be contracted by the Pressure of the incumbent Water, it cannot be believed that a Supply, by this means obtained, can long suffice a Diver. Since by Experiment it is found that a Gallon of Air, included in a Bladder, and by a Pipe reciprocally inspired and expired by the Lungs of a Man, will become unfit for any further Respiration, in little more than one Minute of Time; and though its Elasticity be but little altered, yet in passing the Lungs, it loses its vivifying Spirit, and is render'd effete, not unlike the Medium found in Damps, which is present Death to those that breath it; and which in an instant extinguishes the brightest Flame, or the shining of glowing Coals or red hot Iron, if put into it. I shall not go about to shew what it is the Air loses by being taken into the Lungs, or what it communicates to the Blood by the extream ramifications of the Aspera Arteria, so intimately interwoven with the Capillary Blood-Vessels; much less to explain how 'tis performed, since no discovery has yet been made, to prove that the ultimate Branches of the Veins and Arteries there, have any Anastomoses with those of the Trachea; as by the Microscope they are found to have with one another. But I rather choose to leave this Enquiry to the Curious Anatomist, to whom the Structure of the Lungs is better understood; and shall only conclude from the aforesaid Experiment, that a naked Diver, without a Spunge, may not be above a couple of Minutes enclosed in Water, (as I once saw a Florida-Indian at Bermudas) nor much longer with a Spunge, without Suffocating; and not near so long without great Use and Practice: ordinary Persons generally beginning to stifle in about half a Minute of Time. Besides if the Depth be considerable, the pressure of the Water on the Vessels is found by Experience to make
the Eyes Blood-shot, and frequently to occasion spitting of Blood.

When therefore there has been occasion to continue long at the Bottom, some have contrived double flexible Pipes, to circulate Air down into a Cavity enclosing the Diver as with Armour, to bear off this pressure of the Water, and to give leave to his Breast to dilate upon Inspiration: the fresh Air being forced down by one of the Pipes with Bellows or otherwise, and returning by the other of them; not unlike to an Artery and Vein. This has indeed been found sufficient for small Depths, not exceeding twelve or fifteen Foot: but when the Depth surpasses three Fathoms, Experience teaches us that this Method becomes impracticable: for though the Pipes and the rest of the Apparatus may be contrived to perform their Office duly, yet the Water, its weight being now become considerable, does so closely embrace and clasp the Limbs that are bare, or covered with a flexible Covering, that it obstructs the Circulation of the Blood in them; and presses with so much force on all the Junc-
tures, where the Armour is made tight with Leather, Skins or such like, that if there be the least defect in any of them, the whole Engine will instantly fill with Water, which will rush in with so much violence, as to endanger the Life of the Man below, who may be drown'd before he can be drawn up. Upon both which accounts, the danger encreases with the Depth. Besides a Man thus shut up in a weighty Cage, as this must needs be, cannot but be very unwieldy and inactive, and therefore unfit to execute what he is designed to do at the Bottom.

To remedy these Inconveniences, the Diving-Bell was next thought of; wherein the Diver is safely conveyed into any reasonable Depth, and may stay more or less time under Water, according as the Bell is of greater or lesser Capacity. This is most conveniently made in form of
of a Truncate Cone, the smaller Basis being closed, and
the larger open; and ought to be so prorized with Lead,
and so suspended, that the Vessel may sink full of Air,
with its greater or open Basis downwards, and as near
as may be in a situation parallel to the Horizon, so as to
close with the Surface of the Water all at once. Under
this Coverecle the Diver setting, sinks down together
with the included Air into the Depth desird; and if the
Cavity of the Vessel may contain a Tun of Water, a
singe Man may remain therein at least an Hour, without
much inconvenience, at five or six Fathom Deep. But
this included Air, as it descends lower, does contract it
self according to the weight of the Water that compriss
es it; so as at thirty three Foot deep or thereabouts, the
Bell will be half full of Water, the Pressure of it being
then equal to that of the whole Atmosphere; and at all
other Depths, the space occupied by the compriss Air in
the upper part of the Bell, will be to the under part of its
Capacity full'd with Water, as thirty three Feet to the
depth of the Surface of the Water in the Bell below the
common Surface thereof. And this condensed Air, being
taken in with the Breath, soon insinuates itself into all
the Cavities of the Body, and has no sensible effect, if
the Bell be permitted to descend so slowly as to allow
time for that purpose. The only inconvenience that at-
tends it, is found in the Ears, within which there are Cavi-
ties opening only outwards, and that by Pores so small
as not to give admission even to the Air itself, unless they
be dilated and distended by a considerable Force. Hence
on the first descent of the Bell, a Pressure begins to be felt
on each Ear, which by degrees grows painful, like as if
a Quill were forcibly thrust into the Hole of the Ear;
till at length, the force overcoming the Obstacle, that
which contringes these Pores yields to the Pressure, and
letting some condensed Air slip in, present Eafe ensues.

But
But the Bell descending still lower, the Pain is renewed; and again eas'd after the same manner. On the contrary, when the Engine is drawn up again, the condensed Air finds a much easier Passage out of those Cavities, and even without Pain. This Force on the auditory Passages might possibly be suspected to be prejudicial to the Organs of Hearing, but that Experience teaches otherwise. But what is more inconvenient in this Engine, is the Water entring into it, so as to contract the bulk of Air (according to the aforesaid Rule) into so small a space, as that it soon heats and becomes unfit for Respiration, for which reason it must be often drawn up to recruit it: and besides the Diver being almost covered with the Water thus entring into his Receptacle, will not be long able to endure the Cold thereof.

Being engaged in an Affair that required the Skill of continuing under Water, I found it necessary to obviate these Difficulties which attend the use of the common Diving-Bell, by inventing some means to convey Air down to it, whilst below; whereby not only the Air included therein, would be refresh'd and recruited, but also the Water wholly driven out, in whatever Depth it was. This I effected by a Contrivance so easy, that it may be wondred it should not have been thought of sooner, and capable of furnishing Air at the bottom of the Sea in any quantity desired. The description of my Apparatus, take as follows.

The Bell I made use of was of Wood, containing about 60 Cubick Foot in its Concavity, and was of the form of a Truncate-Cone, whose Diameter at Top was three Foot, and at Bottom five. This I coated with Lead so heavy that it would sink empty, and I distributed the weight so about its bottom, that it would go down in a perpendicular Situation and no other. In the Top, I fixed a strong but clear Glass, as a Window to let in the Light.
Light from above; and likewise a Cock to let out the hot Air that had been Breathed; and below, about a Yard under the Bell, I placed a Stage which hung by three Ropes, each of which was charged with about one Hundred Weight, to keep it steady. This Machine I suspended from the Masts of a Ship, by a Spritt which was sufficiently secured by Stays to the Masts-head, and was directed by Braces to carry it over-board clear of the Ship side, and to bring it again within-board as occasion required.

To supply Air to this Bell when under Water, I caused a couple of Barrels, of about 36 Gallons each, to be cased with Lead, so as to sink empty; each having a Bung-hole in its lowest Part to let in the Water, as the Air in them condensed on their descent; and to let it out again, when they were drawn up full from below. And to a Hole in the uppermost Part of these Barrels I fixed a Leathern Trunk or Hose, well liquored with Bees-Wax and Oyl, and long enough to fall below the Bung-hole, being kept down by a Weight appended; so that the Air in the upper Part of the Barrels could not escape, unless the lower ends of these Hose were first lifted up.

The Air-Barrels being thus prepared, I fitted them with Tackle proper to make them rise and fall alternately, after the manner of two Buckets in a Well; which was done with so much ease, that two Men, with less than half their Strength, could perform all the Labour required: and in their descent they were directed by Lines fastned to the under edge of the Bell; the which past through Rings placed on both sides the Leathern Hose in each Barrel; so that sliding down by those Lines, they came readily to the Hand of a Man, who stood on the Stage on purpose to receive them, and to take up the ends of the Hose into the Bell. Through these Hose, as soon as their ends came above the Surface of the Water in
the Barrels, all the Air that was included in the upper Parts of them was blown with great force into the Bell, whilst the Water entred at the Bung-holes below, and fill'd them: and so soon as the Air of the one Barrel had been thus received; upon a signal given, That was drawn up, and at the same time the Other descended: and by an alternate Succession furnished Air so quick, and in so great Plenty, that I my self have been One of five who have been together at the Bottom, in nine or ten Fathoms Water, for above an Hour and half at a time, without any sort of ill consequence: and I might have continued there as long as I pleased, for anything that appeared to the contrary. Besides the whole Cavity of the Bell was kept entirely free from Water, so that I sat on a Bench, which was diametrically placed near the Bottom, wholly drest with all my Cloaths on. I only observed, that it was necessary to be let down gradually at first, as about 12 Foot at a time; and then to stop and drive out the Water that entred, by receiving three or four Barrels of fresh Air, before I descended further. But being arrived at the Depth designed, I then let out as much of the hot Air that had been Breathed, as each Barrel would replenish with Cool, by means of the Cock at the Top of the Bell; through whose Aperture, though very small, the Air would rush with so much violence, as to make the Surface of the Sea boyle, and to cover it with a white Foam, notwithstanding the great weight of Water over us.

Thus I found I could do any thing that was required to be done just under us; and that, by taking off the Stage, I could, for a space as wide as the Circuit of the Bell, lay the Bottom of the Sea so far Dry, as not to be over-shoes thereon. And by the Glass Window, so much Light was transmitted, that, when the Sea was clear, and especially when the Sun shone, I could see perfectly well to Write or Read, much more to fasten or lay hold on any thing
thing under us, that was to be taken up. And by the
return of the Air-Barrels, I often sent up Orders, written
with an Iron Pen on small Plates of Lead, directing how
to move us from Place to Place as occasion required. At
other times when the Water was troubled and thick, it
would be dark as Night below; but in such Case, I have
been able to keep a Candle burning in the Bell as long as
I pleas'd, notwithstanding the great expence of Air requi-
site to maintain Flame.

This I take to be an Invention applicable to various
Uses; such as Fishing for Pearl, Diving for Coral,
Spunges and the like, in far greater Depths than has hi-
therto been thought possible. Also for the fitting and
plaining of the Foundations of Moles, Bridges, &c.
upon Rocky Bottoms; and for the cleaning and scrub-
bining of Ships Bottoms when foul, in calm Weather at
Sea. But as I have no experience of these matters, I
leave them to those that please to try. I shall only inti-
mate, that by an additional Contrivance, I have found it
not impracticable for a Diver to go out of our Engine,
to a good distance from it, the Air being conveyed to
him with a continued Stream by small flexible Pipes;
which Pipes may serve as a Clew to direct him back again,
when he would return to the Bell. But of this perhaps
more hereafter.

VIII. Observations on the Glands in the Human
Spleen; and on a Fracture in the upper part of the
Thigh-bone. By J. Douglass, M.D. and R. S. S.

That Anatomy, as well as Physick and Surgery, has
received much improvement from a careful and true observation of what was found in the Dissection of morbid
morbid Bodies, will appear from the two following In-
stances, among many more that might be adduced for
that purpose. For it is certain, that nothing has con-
tributed so much towards forming a right Notion of the
Nature of the several Diseases, and a true knowledge of
the Structure of many Parts of the Human Body, as
their appearance in a preternatural State.

My first Observation is of the Glands visible to the na-
ked Eye, that appear dispersed thro' the Fibrous Substan-
ce of the Human Spleen. The SubjeaT I found them in,
was a Boy of about 4 or 5 Years Old, that died of a
general Atrophy, or Consumption of all the Muscular
Fleshy Parts of the Body, occasioned without all doubt
from the numerous Glandulous Swellings scattered up and
down the whole Mesentery; which by compressing the
Lymphatick Vessels, called in this place Vasa lactea, pre-
vented the access and supply of the Chyle, so necessary
for the continued nourishment and increase of the Parts.
For without the constant Recruit of this whitish Balsamick
Liquor, the Mass of Blood will in a short time be unfit
to perform any of those good Offices, which a fresh ac-
cession of Chyle qualifies it for.

In a piece of this Spleen we might see, without the
assistance of a Glafs, several round whitish Bodies of a
pretty hard Consistence, and abundance of small white
and fofter specks; but both of the same nature. These,
to me at leaft, appear to be fo many distinct Glands be-
come visible; which in a Natural State are only to be
seen by a fine Glafs, as the curious Malpighius first obser-
v.{Vid. his Treatise de Liene, Cap. V. De quibusdam cor-
poribus per Lienem dispersis. Minima ha glandula, says he,
non aqua facile fese produit in quocunque animalium Liene: i-
mosola Lienis laceratione innotescunt in Bove, Ove, &c. In Ho-
mine vero difficilis emergunt: ft tamen ex morbo universum
 glandularum genus surgeat, manifestiores redduntur, auta
ipsarum
ipsarum magnitudine, ut in defuncta puella observavi: in quâ Lien globulis conspicuis racematim dispersis totus scatbat. Which Case was the very same with mine.

The second Observation. We had still been in the Dark, about the nature of a Luxation of the Head of the Thigh Bone, had we not carefully examined the Part in the dead Body. For by that sort of enquiry, the common mistake of Surgeons was detected, and what was esteemed and treated by them as a Luxation of the Head of the Femur, was discover'd to be nothing else but a Fracture of the same Bone, near its Neck; the globular Head being still retained close in its own Socket, called the Acetabulum Coxendicis.

Amongst all the Writers of Surgery and Anatomy, I know but three that were apprised of this mistake: the first was Ambrose Paree, the second Dr. Ruyfch at Amsterdam, and Mr Cheselden, a Member of the Royal Society; whose Observations on this Subject I intend to communicate at another time, together with an account of the true Structure of this Joint; in which I will consider the depth of the Articulation; the wonderful strength of the Muscles that surround it; the many strong ligaments that bind the Head within the Socket; the smallness of the Neck of the Bone; its porous and spongy Substance, which makes it much weaker than the rest; and last of all the disadvantageous oblique position of this Neck, which exposes it the more to outward accidents. From a review of such like Considerations, it will plainly appear that a Fracture can much more easily happen, than a Dislocation in that Part from an external Cause.

This Os femoris belonged to an old Woman turn'd of Fourscore, who only fell from her Chair whereon she was sitting, and thereby suffered this breach of continuity in the Substance of the Bone. She lived three Weeks after it; and tho' it never was reduc'd, yet she complained of very
very little or no pain, which may seem very extraordinary. It is observable that the Fracture is not only Oblique, near the Neck of the Bone; but that each Trochanter, i.e. the two processes near its Cervix, are likewise broke short off; and that they were both drawn up almost as high as the Head of the Bone it self, by the strong contraction of the Glutaei and other Muscles.

IX. An Account of a Book. D I S S E R T A T I O

The Author of this Treatise, as the occasion of it observes that the little God Telephoros had just cause to complain, that so much respect was paid to Dea Febris, and a Book lately publisht de Dea Pedagra, yet no such Honour was done his Mother Teleœœœœœ, (who certainly was more to be esteemed, than all the Tribes of Diseases). Upon this Conceit, he took what Books he had in his reach, of the antient Latin and Greek, and having collected out of them, what he met with relating to this Goddes, put it together, as now it appears in Print.

It consists of VI Chapters: of which the first is Introductory, speaks of Health in general, has, in praise of it, that memorable Ode of Ariphron the Sicyonian, publisht by Athenœœœœœœœœœœœ, and translated by Sennertus; together with a Hymn, said to be composed by Orpheus, on the same Subject;
Subject; he ranks this Goddess among the \textit{Dii Medioxumi}, and gives an account of Her from the Mythologists.

Chap. II. \textit{de Salutis Symbolo}, which he takes to be a Serpent, an Omen of good things, and a frequent Companion of the Gods; as appears from \textit{Virgil}, \textit{Valerius Flaccus}, \textit{Statius}, and \textit{Macrobius}. He mentions another \textit{Symbolum Salutis}, used by \textit{Antiochus Soter}, now to be seen in some of his Coins, and sometimes printed in Physick Books in the following form.

\begin{center}
\includegraphics[scale=0.5]{symbol.png}
\end{center}

The III. Chapter treats of the Temples erected to this Goddess; in which Prayers were offer'd up to Her, sometimes for the Health of private Persons, and often for the welfare of the Publick: of which many Instances are here produced. To the account of Temples is subjoined the Divination, known by the name of \textit{Salutis Augurium}, which is often mentioned by Roman Authors, as \textit{Dion Cassius}, \textit{Tully}, and \textit{Tacitus}.

In the IV. Chapter, the Statues of this Goddess are consider'd. Some of these represent her and \textit{Æsculapius together, tamquam Thea Συρέ Επιγεύς. Pliny, Pausanius, Lucian, Plutarch and Monfacon afford instances of this kind.}

Coins relating to this Goddess, come next in view, Chap. V. These either express her \textit{Effigies}, or her \textit{Worship} under some Symbol or other. Of the first order, one out of \textit{Fulvius Ursinus} has the Head of the Goddess, with \textit{SALUS} inscribed. Another like this, is in \textit{Gevarius}. Some, together with this Goddess have also her Father \textit{Æsculapius}; as a Coin of \textit{Trajan}; and in one of \textit{Aurelius Antoninus}, struck in memory of the Remedies reveal'd to him in a Dream, which cured the Emperor of...
a Sputum Sanguinis and Vertigo. As indeed most of these Coins were (in all likelihood) struck on some such occasion, viz. the Recovery of some great Person. A noble Expression of Gratitude, fit and worthy of imitation.

Of the second Order is the Coin of Dossenus, having an Altar with a Serpent, taken from Urfinus. Another of Tiberius, with an Altar and S A L. A U G. Another of Nero in which is a Serpens Tortuosus; with many others.

The Gemma of the Antients, according to Leonardus Augustinus, are of use to set forth the Sacrifices made of old to this Goddess. One of these Gemma represents Æsculapius, his Daughter-Hygieia and Grand-Son Telephorus to call'd áτο τη τίλος φέρας, à Valetudine post morbum confirmat. This God, being Young and Tender, had (I suppose, by the care of his Mother Hygieia) a Bardocuculus, or Cloket, to keep him from taking Cold. These three Gods are represented in one Figure, with the following Inscription under them, EΤΖΕΤΕ MΕ, i. e. Salvēr me Jubete, which Augustinus happily conjectures to have been a Form of Prayer offer'd up to them.

In the last Chapter come the Inscriptions, which are taken out of Gruter and Reinefus. They are chiefly to Æsculapius and Hygieia; but to confirm the Divinity of Telephorus the little God of the Pergameni, he is mentioned in one of their Inscriptions dug up at Verona.

The Author makes no manner of doubt, but there are many more Coins and Inscriptions relating to this Goddess to be found in other Books. But these being all, or most of such as came in his way, and enough to give a Specimen of the Devotion paid by the Antients to this Goddess, he has contented himself with this small Number; leaving it to others to make such Additions, as from greater Opportunities and Abilities, they shall think fit.

PHILOSOPHICAL TRANSACTIONS.

For the Months of Octob. Novem. and Decem. 1716.

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Abisque Observationibus accurate institutis, irrato fane conamine verae motuum Cælestialium Theoriae Calculusque cælo conformis expeterentur. Quod autem tam diu latuerit Veritas Astronomica ex eo factum est, quod Veteres Artifices, & imprimis Ptolemaeus, nulla omnino nobis tradiderint Observationa, præter ea quibus ad flabiliendas Hypothese Tabulasque suas Principiis usi sunt, Cum tamen multo magis ex re suiss Timocharidis, Aristilli, Hipparchi suasque Ptolemai ipsius thesairis debità fide ad nos transmisisse, & numerorum suorum à Cælo diffensus ingenue annotasse (ad exemplum magni Hippocratis, cui minime pudori fuit sub cura suæ mortuos a que ac sanitati restitutos Posteriati consignasse) potius quam vanà quàdam gloriae specie, Sphalmata suas sibi ipsis perspecta tacuisse; celatis ite. Observationibus ipsis quibus Tabulas suas male respondentes ex- perti sunt. Hoc autem ante Tychonem Brahe omnium pene gentium Astronomis commune vitium.

Ex quo autem à sagacissimo Keplero adinventà est genuina motus planetariori Theoria, ejusdemque etiam Geometricâ veritas à Newtono magnò nuper patescìta, nonnullos ingens incessit cupidò calculum Astronomicì cum Cælo omnino consentaneum exhiberì posse; & in eum finem, præter Observatores Principiorum, Privati etiam plurimi apud nos observandis Sideribus inhiant: Horum autem nemini arte vel industriâ secundus Rever. D. Jacobus Pound, Reg. Soc. Socius, Observationes sequentes
sequentes à se nuper apud Wansted habitas: Tubisque
longissimis & Micrometro quantum fieri potuit ac-
curatè captas, cum publico communicandas obruit.

Anno 1715. Augusti 21°. 8h. 25', Temp. æq. Mars
præcedebat. Scil. Aleœsione rectâ, Mediam frontis Scor-
pii (Bayero 8) 6°. 54" Borealior Fixâ 9°. 47".

Sept. 18°. 7°. 30'. Mars præcedebat Claram in pede
Serpentarii (Bayero 0) 17°. 48", & eandem habuit De-
clinationem accuratè.

Novemb. 30. 18°. 8'. Saturnus præcedebat y, &
secundam Alæ Virginis 23°. 19" erat Fixâ Australi-
ior 25°. 3". Decembris autem 4°. 17°. 25', præcedebat
cam 10°. 50' & Australior erat 29°. 00".

Anno autem 1716. Feb. 22°. 7°. 23°. T. æq. Mars
præcedebat θ Piscium; five Sequentem trium clariorum in
Lino Aust. Piscium 3°. 35" eademque Australior erat
1°. 23", quam proinde obtegere debuit ante bihorium,
forfan corporaliter.

Juni 22°. 8°. 52°. T. æq. Venus sequebatur Cor
Leonis 34°. 50", & fixâ Australior erat 7°. 23°.

Aug. 14°. 15°. 00'. Jupiter præcedebat Propoda uno-
tantum minuto, cum Declinatione Bor. minore 14°. 26".

Aug. 19°. 13°. 2'. Jupiter præcedebat fixam Tele-
scopicam, quæ vocetur θ, 50°. 06' eandem habens De-
clinationem accuratè.

Aug. 24°. 12°. 19'. Jupiter Micrometro distabat a
præictâ b, 5°. 54", simulque ab aliâ Fixâ clariore a
7°. 17°. Distantiæ fixarum 12°. 31°. Tunc minor Jovis
diameter 0°. 38".

Sept. 12°. 17°. 00' Venus recens a Statione secunda
sequebatur Telescopicam 17°. 40", eãque Australior
erat 5°. 30°. Hæc autem Fixa tunc occupavit Ω 27°.
44° cum Lat. Aust. 5°. 39.

Octob. 15°. 17°. 12°. Venus distabat Microm. a Fixâ
in Cruce Leonis 27°. 55°.
Novemb. 20°. 6h. 18°. Jupiter regressus est ad stellas a & b, ad quas observatus est Aug. 24°. & distabat a b 6°. 21°, ab a vero 11°. 36°.


Nov. 30. 5h. 41°. praeced. Propoda 12°. 36°. Australior 7°. 36°.
Dec. 4. 6°. 01°. Sequemb. eam 22°. 49°. — — — 7°. 47°.
Dec. 5. 6°. 0 Repet. — — 31°. 35°. — — — 7°. 50°.
Dec. 6. 6°. 0 Repet. — — 40°. 30°. — — — 7°. 52°.
Dec. 7. 6°. 0 Iterum. — — 49°. 15°. — — — 7°. 54°.


N. B. Stellas illas Telecopicas a & b vocatas, haberi in Catalogo Fixarum Britannico, D. Flamsteedii, ubi ipsi a Locus datur, ad annum Seil. 1690 inuentem, II 27°. 54°. 29°. cum Lat. Aust. 021°. 55°; alteri vero b II 28°. 5°. 24°. cum Lat. Aust. 28°. 5°. Neque aliam novimus Fixam a corpore Jovis occultatam & ab invento Telecopio observatam, praeter jam dictam Stellam a; ad quam olim arctissime applicabatur Jupiter, ante annos 83. Decembris nono St. nov. Anni 1633. Vesp. cum Gassendus Dinia vidit Jovem huiusFixae conjunctum, nec nisi quinque semidiametris corporis sui superiores. Unde calculo debite inito, constabit Nodos hujus Planetae Planumque Orbis ejus, situm in Sphaera Fixarum servare immobilem, vel taltem lentissimo motu cieri. Vide Gassendi Observationes Tom. IV. p. 162.
II. A Description of that curious Natural Machine, the Wood-Peckers Tongue, &c. By Richard Waller Esq; late Secretary to the Royal Society.

THE Ficus Martius or Wood-Pecker has several particularities in the structure and mechanism of its whole body, which may deserve a nice and accurate observation and description; all which are wisely contrived and adapted, either for catching the food and sustenance of the individual, or continuing the species.

That this bird makes a round hole even in found and hard trees, such as the oak, horn-beam, beech and the like, is commonly observed; and that within these holes, the hollow being enlarged, the nest is made, the eggs laid and hatch'd; and the young brood fed, as by other birds.

For this purpose, that he may be enabled to perform such hard work, the muscles of his neck, breast, and thighs are exceeding strong in proportion to the bigness of the bird: he has also a very firm strong sharp bill, his legs are strengthened with very strong tendons; and his toes, which are two before and two behind, (as it is in some other birds) are provided with sharp strong hooked claws or talons: Besides this, his tail consists of ten very stiff large and strong quills, firmly set into a robust strong Uropygium or rump; so that when he has fastened his claws and feet into the clefts and inequalities of the bark of the tree, he claps his strong tail-feathers against the body of the tree; and so stands with his head erect, to give the strokes with his bill with the greater force.
That he is of the Insectivorous kind is certain, and lives not only upon Insects catcht creeping on the outside of Trees, but also on such as are under the Bark between the Bark and Wood, as likewise on those in rotten Wood; and as I am very confident on Worms and other Insects in the Ground: for I have frequently observed the Roots of their Bills very dirty, as it is in Crows and Rooks, &c. Whence I suppose he strikes his long sharp Bill into the soft Earth to take the Worms out of it. I have also found their Craws full of small Ants.

But the Contrivance and Mechanism of the Tongue in this Bird being the most Remarkable, I shall presume to lay before this Illustrious Society some few Remarks of this curious Contrivance of Nature, with some Figures I have drawn by the Parts themselves, in order to explain the whole.

This Bird is known to throw out a long, slender, round Tongue, to a considerable distance beyond the End of his Bill; and to draw it in again very quick into his Mouth or Bill, with the caught Insect spitted on the Tip of it.

The Chameleon indeed darts out its Tongue to a considerable length; and having intangled the Fly in the glutinous Matter at the End of it, draws it into its Mouth, together with the Prey; but the Mechanism in that Animal is wholly different from that of the present Subject: as may be seen by the Account the Gentlemen of the Academy Royal give thereof, in their Memoirs for a History of Animals.

The Protrusion therefore of the Tongue to the length even of three or four Inches in this Bird, being very extraordinary, and the Mechanism of the several Parts for that end no less Curious; several learned and diligent Enquirers have attempted to explain it; but I am of opinion
opinion they have been, in some Particulars at least, mistaken. I shall mention some of these.

The learned and curious Enquirer into Nature, Monf. Perault, describes it after this manner *

This long Tongue he throws out by the means of two small bony Cartilages, about seven Inches long, and of the thickness each of a middling Pin, which are perfectly Smooth and Slippery. These two Cartilages are united at the End, and being in this place covered with Flesh make the fore-part of the Tongue. The rest of these Cartilages are separated from each other, and pass turning round under the Ears; and then rising up behind the Head, where they meet again, they pass over the Top of the Head, and so extend themselves to the Root of the Beak. These Cartilages which make the hinder part of the Tongue, are also inclosed in a Channel Fleshy on the out-side, and whose inside is covered with a very smooth slippery Membrane.

Now these Fleshy Channels, which incompas and keep in these Cartilages, are the Muscles by which the Tongue is moved: for having their origine at the Larynx, and their insertion at the extremities of the Cartilages, it comes to pass, that when those Muscles of the two Fleshy Channels, which make the hinder part of the Tongue are shortned, they force the fore-part of the Tongue out of the Beak, by drawing the posterior or farthest end nearer to the Larynx: and on the contrary, when the Fleshy Channel which makes the anterior Part acts, it draws the fore-part of the Tongue into the Bill towards the Larynx.

This Mechanism of making a hard part, such as the bony Cartilages are, to come out and return into another,


K k k k 2
such as the Canals are, by the means of Cords drawing them, which are the Muscles, is made use of in Coaches to pull up the Glasses of the Doors; for the String, being fasten'd to the lower part of the Glass-Frame, makes it rise when drawn, which resembles that action of the Muscles by which this Tongue is moved.

Of these Cartilages and other Parts, and of the Head of the Bird, Mr. Perault gives the Figures.

Either the Wood-pkeesers in France are different from ours in England; or this Figure of the Head is very ill designed; it being much too broad and large, and the Beak too short. Besides he makes the two Cartilages to come to the Root of the Beak separately, one on one side, the other on the other side of it; whereas in all the Wood-peckers Heads I have met with, the two Cartilages joyn close together about the Top of the Head, and thence proceed joyned, tho' not fastned to one another, a little slanting towards the right Nose-hole, where they end together.

Besides upon viewing and examining several Subjects, I could not find them agree in divers particulars with his Account and Explication. For the Muscles which are fastened to the end of the Cartilages at the Root of the upper Beak, are not inersted at the Larynx, but pass on and are fastened to the lower Bill. This pair I take to be the Muscles chiefly concerned in forcing the Tongue out of the Bill. There is another pair of Muscles, which, being fastened to the place where the two bony Cartilages are articulated with one single Bone in the fore-part of the Tongue; (as will be shewn in the 4th Figure) is, as I apprehend, the chief pair concerned in the drawing the Tongue with its Prey into the Mouth. These proceeding from that articulation of the Cartilages as far as the Larynx, (each of them sending a Branch to the Cartilago Scutiformis) from thence go on along with the
the Neck, (tho' not fastned to it) till they come within the Cavity of the Thorax, where they are inferted under the Clavicula or Merry-thought-bone, as 'tis called. This pair is represented by k. k. in the second Figure; and by q. q. in the First.

There is likewise a very slender white Thread; (whether Tendon or Nerve, I am uncertain) which accompanies this Muscle its whole length; and which drawn gently, (for fear of breaking) pulls in with it the end of the Tongue. As there is such another all along the Vagina to the End at c.

Volker Coiterus, as he is mentioned by Gerard Blásius, in his Anatome Animalium, Cap. 24. p. 64. treating of the Tongue of this Bird, makes it to be made of three bony Cartilages, round, and as he says bound together, (invicem colligatis) which is a Mistake; for tho' reckoning the two bony Cartilages for Officula, yet the third is not bound up with them, but articulated to the End of them. The same Person says the Tongue may be thrust out to the length of an Inch and a half, whereas when drawn in, it is scarce half an Inch long; when in reality it may be thrown out near four Inches; and I believe cannot be drawn in, so as to be less than an Inch and quarter, viz. to that place where the two Cartilages are articulated with the single Bone. Besides he makes the use of the long flat Muscle running over the Top of the Head, to be (if I rightly apprehend his meaning) to draw the Tongue to the upper Jaw, whereas their use is for thrusting the Tongue out of the Birds Mouth.

But this Person having given no Figures, has render'd what he says less intelligible; tho' indeed he mentions two pair of Muscles, as there are so many chiefly concerned, yet there are at least two other pair, that assist the Performance.
Wherefore I shall leave him, and proceed to the Account given by Alphonhus Borellus in his Treatise de Motu Animalium, part. 2. pag. 24. which is in several Respects likewise unsatisfactory, and the Figure given by him to explain it very defective and ill designed.

He makes the pair of Muscles concerned in thrusting the Tongue out, to be fastened indeed as they are to the lower Beak towards the Point; but then he makes their Insertion at their other End to be at the extremities of the two Ossa Hyoidea; whereas they really reach to the very end of the long Cartilages that go round the Head: These by another Mistake, he makes to be the Rettactors of the Tongue, and joins another pair as Assistants in the same Action, which he makes to be twisted spirally about the Trachia. None of all which agree with the Subjects I have met with, as will be seen by the descriptions of my Figures.

In the History of the Academie Royale des Sciences, publish'd in Latin by Monf. du Hamel, 1698. Lib. 4. Cap. 5. There is another Description of this admirable Contrivance of Nature, by Monf. Mery, read at a Meeting of the Academie, November 16, 1695.

In this he differs from both Perault and Borelli, taking the Hornie End and Bone to which it is joined, to be only the Tongue properly so called, and that the next two Bones answer the Hyoïdes with the long Cartilages annexed to them. But even in this he seems to me not to be so clear; confounding, as I apprehend, the two Bones with the Cartilages. He describes the Vagina, in which the Bones and part of the Cartilages are encompassed, and which is fastned to the Hornie end, and is protruded and drawn back with the Tongue: he takes notice of the little sharp Points or Prickles on the Hornie Part being moveable, and with their Points bending towards the Throat; but I apprehend it is a Mistake to make the
Mucous matter Glutinous which is furnished by the two Pyramidal Glands; for I take the use of that Mucus to be chiefly, if not only, to lubricate the Passage in the Vagina, for the more easy slipping of the Cartilages therein.

He describes the Muscles for exerting the Tongue, and extends them from their Insertion at the lower Beak, to the End of the Springy Ligaments, as he terms what I call Cartilages; to which he adds another small Ligament capable of Extension, at the End of the other two next the Nose, which when the Tongue is thrust out is relaxt and stretch'd. He also describes the pair of Muscles fastned to the Root of the Tongue and Os Hyoïdes, serving to draw the Tongue into the Mouth: these he makes to be wound round about the Aspera Arteria once or twice, in which I think there is some Mistake; being of opinion the Mechanism for this Action of drawing in of the Tongue, is different from what is here described, as in the Explication of the Figures I shall endeavour to shew. But not to insist on all the Particularities mentioned in this Description, which, for want of more Figures to explain the several Parts in so curiously contrived an Organ, is not so clear as might be desired (there being but One, and that a wooden Cut of the Head, Tongue, Bones, Muscles, &c.) I shall now proceed to the Explication of the several Draughts I made, with what exactness and care I could, in 8 or 10 several Subjects.

FIGURE the First.

Represents the Head with part of the Neck of this Bird, the Skin being taken off, in which,

A. The Scull, having two Shallow Grouves or Channels, or rather one broad one with a small Rising in the midst, on the Sinciput or Back part, from each side of the Neck to the Top of the Head, where they unite into
one, which passes slanting towards the right Side, and
ends at the Hole for the Nostril on that side at c.

b. Is the Hole or passage for Hearing.

d. A large white Gland, containing a glutinous Li-
quor, almost like Cream as to Colour and Consistence; which
empties itself into the Mouth; I suppose to lu-
brate the Cartilages.

e. The Eye, which has a Bony Ring, encompassing
the Iris.

f. Part of the Tongue, which in this Figure is re-
presented as almost all drawn into the Mouth, of which
more when I come to describe the Cartilages, &c. In the
2d Fig.

g. Part of the Neck, which is large and furnished
with very strong Muscles.

h. The Oesophagus, opening very wide at the Fauces,
and wholly Musculous.

i. i. i. A long, but thin and flat Muscle in respect of
its breadth, which is about \( \frac{1}{2} \) of an Inch, reaching from
the end of the Cartilage at c, to the under Bill or
Beak at k, to the inside of which it is very firmly fast-
ned; as is such another on the other side.

k. The under Bill very strong and sharp pointed, ar-
ticulated with the Scull a little behind the Ear-hole b.

l. l. l. The Cartilage on one side, the other being
exactly the same. This Cartilage is Round, very Smooth,
Even and Slippery, about the size of a pretty large Pin;
and reaches, when the Tongue is drawn in and the
Muscle i. i. relaxed, from the Root of the upper Beak
at c, to the Root of the Tongue properly so called, or
to the Bones of the Tongue where they are articulated,
being bent like a Hoop as in the Figure, slipping very
freely in a Sheath or Membranous ductus fastned on the
outward or convex Edge of the flat Muscle i. i. i., which
Muscle accompanies it from its end at c, almost to the
end
end of the Canal or Sheath, which opens at a Hole a little before the Larynx; (as will be shown in the third Figure) and thence the Muscle proceeds to its Insertion into the lower Beak at \(k\). From the concave Edge of this Muscle there is a thin and transparent but very strong Membrane, strained like a Drum-head to the Scull at \(m\), where it is very strongly fastned; this Membrane is furnished with Capillary Veins and Arteries, and doubtless is Nervous. \(n\). \(n\). represents this Membrane. This Cartilage, when the Tongue is exerted, parts about half an Inch from the Root of the Beak at \(c\).

\(o\). \(o\). A pretty large Vein and Artery.

\(p\). \(p\). A Muscle reaching from one Jaw to the other, under the Throat, serving as a Bandage to keep in the Cartilages, and the Root and \(os\) Hyoïdes of the Tongue, as I may call it, from starting out at that Part where are the articulations of the Cartilages with the Bones, when by the Muscles, inserted into the Sheath at or near \(p\) and thence passing to the end of the Tongue, it is drawn into the Mouth.

\(q\). \(q\). One of the last mentioned Muscles, which is round, of the size in the Figure, and fastened to the Breast of the Bird, cut off at \(r\).

\(s\). The Aspera Arteria consisting of perfect Rings.

\(t\). \(t\). A Muscle accompanying the Aspera Arteria.

**FIGURE the Second.**

A. A. The under part of the lower Bill.

B. b. The Tongue.

b. The Place where the two Cartilages and two Bones represented by \(f\). \(f\). in Fig. 4. are brought into and included in one Tube or Membranous Sheath.

C. C. Two Glands displaced in this Figure.

c. c. Two Muscles attending these Glands, and fastened near the end of the Bill.
d. d. The two Bony Cartilages, bent, and passing on each side of the Neck, but united at b.

e e e, e e e. The pair of Muscles, one attending each Cartilage from the End of it at the upper Beak, and firmly adhering to the Vagina, in which it slips, till about f f:

f f: The place where these Muscles leave the Vagina, and pass on to the inside of the Bill, where they are inserted. Their Action is to thrust the Tongue forward, or out of the Mouth.

g g. A pair of Muscles fastned a little below the Larynx, to the Musculous part of the Aspera Arteria, at i; the other end of them going up to the place b at the Root of the Tongue, whence they go on encompassed by the Vagina to the articulation of the Cartilages with the two Bones. I take their Action to be to draw the end of the Tongue towards the Larynx.

k k: Two Muscles fastned at one end within the Thorax, under the Merry thought or Clavicula; and at the other Ends to the articulation of the Cartilages with the two Bones of the Tongue, marked f f, in Fig. 4. These have the forementioned Nerves accompanying them. I take these to be chiefly concerned in drawing in the Tongue; each of these sends a Branch to the Grisle at the Top of the Aspera Arteria at n.

l l l l. Two Muscles running along and fastned to the Sides of the Aspera Arteria, from the Thorax to the place where they are united, where each of them sends a Branch, which binding over the Bones and Cartilages goes on to the Fauces, where they are inserted.

m. Part of the Gula.

n. A Cartilage at the Top of the Aspera Arteria.

o o. The Aspera Arteria.

p. The Neck, bending like an S. The Wind-Pipe
and Guila in this Bird pass always on the right side of the Neck.

**FIGURE the Third.**

_A. A._ The two long flat Muscles represented by _i. i._ in the first Figure. These join close to one another at the Top of the Head, and so pass on together to the end of the Cartilages; to the end of which, as I take it, they are fastned: from whence a slender weak kind of Ligament reaches to, and is inserted at, the right Nose-hole, at the Root of the upper Beak. This Ligament is relaxt when the Tongue is thrust out.

_b. b._ The Cartilages running in their _Vagina_ on the out side of the said Muscles.

c. The _Larynx_ or passage to the _Aspera Arteria._ I observed no _Epiglottis._

d. d. Two Articulations or Joints in the under Beak or Bill.

e. The Hole or Passage, whereby the Tongue in its _Vagina_ comes out and is drawn in again.

_f._ What I call the Tongue, in the inside of which the two Cartilages are brought together, till they are both articulated to one Single Bone, at the end of which is the Horny barbed Tip.

g. One of the Pyramidal Glands.

_b._ The lower Bill.

**FIGURE the Fourth.**

_A._ That part which I think may most properly be called the Tongue; a small Bone running thro' it: This, as far as _c_, is Flat and Thin at the Sides. It is cut away at _d_, to shew the Bones within it.

_b._ The Horny Tip of the Tongue, about a quarter of an Inch long, strong and sharp, furnished with four or five Barbs on each side; (not with an infinite Number as _L I I I 2_ Coiterus
Coiterus says) These Barbs are sharp and moveable, like the small Teeth at the Root of the Tongue, and beginning of the Gula, in the Pike and Jack-Fishes, in that of Eagles and the like; so as to let the Prey slip easily on, but not so easily get off again.

c. The End of the Bone of the Tongue where the two bony Cartilages are articulated.

d. The place where the upper part of the Tongue is cut away to shew the Bone.

e. Several small Tendons, or rather, as I take them to be, Nerves running thro' the Tongue. Of these some go to the End of the Cartilages, others accompany the Muscles to the Neck.

f. f. Two Bones or Cartilages, which in the Bird, are united by a thin Membrane as far as the next joint, so as to open asunder to some distance, but not to separate quite. These two Bones seem to answer to the ossa Hyoidea in other Creatures. At the Place marked g. g. the Muscle that draws the Tongue into the Mouth is fastned, or rather leaves the Tongue at that place; it having its Insertion near to the End of it. This Muscle is represented by q. q. in the first Figure.

b. b. The two bony and springy Cartilages running on each side of the Neck; which being joynd close together on the Top of the Head, pass so joynd to the Nostril, or Nose-Hole on the right Side.

From the Consideration and comparing of these four Figures, the true Mechanism and Motion of the Tongue, seems to be in short thus: The two long Muscles inserted near the End of this lower Beak, and reaching to the End of the Cartilages, being contracted, the round Hoop of the Cartilages is drawn up, from each side of the Neck, close to the Pyramidal Glands; and at the same time the Muscles that draw the Tongue into the Mouth being relaxed, and the Articulations at e and
and \( g \), \( g \) in the 4th Figure, brought near to a straight Line, the Tongue is thrown out to the length of 4 or 5 Inches.

But when those long Muscles are relaxed, the pair of Muscles represented by \( k \), \( k \) in the second Figure, being contracted, draw the Articulations \( g \), \( g \) where they are fastned, down into the Throat or wide loose Skin of the Neck; and at the same time the Cartilages opening into a wide Hoop, the whole Tongue is drawn into the Mouth.

**Figure** the Fifth:

- **A.** The Scull.
- **b.** The shallow Crena or Groove, for the Cartilages.
- **c.** The Place of their Ending at the right Noie-Hole.
- **d.** The Orbite of the Eye.
- **e.** The Hole for the Optic Nerve.
- **f.** A Hole or passage thro’ from one Orbite to the other.
- **g.** A Bone covering the Hole to the Ear.
- **h.** The lower Jaw and Bill.
- **i.** A Ridge or Proccuss in the Scull, beginning at the Root of the upper Bill, and keeping the two Ends of the bony Cartilages in their place on the right Side.
- **k.** The Os Jugale.
- **l.** The upper Bill.

**Figure** the Sixth

Represents the right Leg and Foot, in which there are two Digitii before, and two behind. The Strength, Largeness, and Sharpness of the hooked Claws or Talons are remarkable.
FIGURE the Seventh.

A. The Oesophagus.
B. The Ingluvies or Crop, partly Muscular, and lined with a Glandulous Coat. This I found quite filled with small black Pilvires; as also
C. The Venticulus or Gizzard, which joyned close to the Ingluvies.
  d. d. d. The Intestines nearly of the same bigness for the whole Length.
  e. The beginning of the Rectum.
  f. The Pancreas.

FIGURE the Eighth.

One of the middle pair of Feathers of the Tail, in which the great Strength of the Quill for so small a Feather, and its bifurcate End, are very remarkable.

FIGURE the Ninth.

The Roof of the Mouth, where 'tis observable, that the Rima or Passage for the Air to the Nostrils, is beset on each side with a Row of 10 or 12 little sharp Teeth, with their Points standing inwards, towards the Gula. These take the Prey from the end of the Tongue whole Barbs or Prickles are moveable, and are to keep it from going out of the Beak again with the Tongue, and from hence it is conveyed to the Swallow.
III. The Natural History and Description of the Phœnicopterus or Flamingo; with two Views of the Head, and three of the Tongue, of that beautiful and uncommon Bird. By James Douglass, M. D. Reg. Soc. S.

N O M E N.

It was the famous Comical Poet Aristophanes, that first makes mention of this Bird by the Name of Φοινικόπτερος (a), and not long after it is called ὄρνις Φοινικόπτερος (b) by Philostratus in his Life of Apollonius Tyaneus. Apicius, Plinian, Suetonius, Juvenal, and other Latin Writers, retain the Greek Word, and call it Phœnicopterus. Bellonius (c) says, that in French it is named le Flamant or Flambant; Scaliger affirms, that in Provence they call it Flamant: And (d) Gesner says it may be called Avis Rubra per excellentiam: (e) Aldrovandus writes, that in Sardinia it goes by the Name of Flamingo; and de Laet tells us, the Spaniards in the West-Indies call it Flamenco. Dr. Charlton and Dr. Grew convert the Greek Appellation into English, naming it the Phœnicopter: And Sir Hans Sloane, in his Catalogue of Jamaica Birds annexed to Mr. Ray's Synopsis Avium, stiles it the Flamingo. (f) Du Tertre calls it le Flamand, which differs but little from the Name given it by Bellonius: And

Etymologia.

All these differing Names may be easily accounted for, from the Colour most predominant in its Wings. Thus Martial (Epigram 58. Lib. III.) says of this Bird:

Nomencque debet qua rubentibus pennis.

And again (Epigr. 71. Lib. XIII.) he makes it give the true Derivation of its own Name;

 Dat mihi penna rubens nomen.

The Greek Name is compounded of two, viz. puniceus, ruber, and ἀλα, a Wing, quod sit rubentibus Alis; which thing in different Words is expressed as follows, by the several Authors I have consulted. Bellonius says tis called in French Flambant, not only from the Date-Colour of its Wings, a Daffylorum colore, i.e. a Scarlet or light red, like the Fruit of the Palm or Date-Tree, called in Greek φοινίξ; but also from the Luster of the Colour resembling Flame. or as Aldrovandus has it, quod velut ignis instar ejus rubedo emicit. The Words of Gesner are, Ego Gallicum nomen a rubro & flammeo rostri, crurum, pennarumque in aliquibus partibus colore indicum esse conjecturum: aut fortasse quoniam ex Flandria bymne ad Narbonensis Provincia maritima volat; nam Flandum Galli Flammant appellant: vel a corporis proceritate, quales solent esse Flandri. Mr. Willoughby (b) says the French name it thus rather from the flammrous Colour of the Wings and Feet, than that it comes in the Winter Time


from
from Flanders. For he believes there was scarce ever seen in Flanders a Bird of that kind; so far are they from being common there, and flying from thence into other Countries. (k) Dr. Grew believes it named in Greek from the Scarlet Colour of its Wings; and Flamment in French for the same Reason. Du Hamel explains its Name Becharu by Aratri-rostrum, (quasi Bec-Charrie) quoniam rostrum ejus aratri instar inflectitur.

Genus.

All Authors, from Aristophanes down to Aldrovandus, have accounted the Phænicopterus a Bird of the Palmipede or web-footed kind; and tho' this last named Author will not allow it to be so, yet he is forced to own that it is not a true Fisipede or digitated Fowl; nam & membrana digitos sepientis quoddam habet rudimentum, are his own Words. Dr. Charlton only, among all the later Natural Historians, has approved of his Division, and accordingly ranked the Phænicopter in the Class of Aquatick Fissipedes. But that it is a Water-Fowl all agree; Aristophanes calls it λυμβας, i.e. palustris; and Aldrovandus says of it, Avis est aquas amans: not to mention others.

Differentiae.

I find Authors are silent as to the different Sorts of this Bird, only Aldrovandus gives us two Figures thereof that are not alike.

Locus Natalis.

This Bird is found in three of the Principal Parts of the World, that is, in Africa, America and Europe. Heliodorus (Ethiop. Lib. 6.) calls it Νελον φαυνακωτης, a Bird of the Nile; and the old Scholastic upon Juvenal (Sat. xi. ver. 139.) affirms, that abundans est in Africa; and Du Hamel's Words are, Inter animantes

qua sua mole commendantur, Avis illa ex Ægypto allata est, quam Veteres ob plumas in Alis rubeas Phœnicopterum dixere. John de Laet writes, that there is an Abundance of them in the Island of Cuba, as also at the Isle called Rocca, lying on the Coast of the Province of Venezuela in South America; and Rochfort saies the same thing of the Island of St. Domingo.

(l) Dampier saw some few of them at Sal, one of the Cape Virde Islands: He hath likewise seen some of them at Rio la Hacha; also at an Island near the Main of America right against Queirifao, call'd by the Privateers Flamingo-Key, from the Multitude of these Fowls that breed there; and he never saw of their Nests and Young but there only.

Tho' these winged Creatures live for the most part in those hot Countries, yet they sometimes visit us here in Europe, and so may be accounted amongst the Migratory Kind, or Birds of Passage, which is confirmed by the Testimonies of several Authors: For,

Bellonius told us long ago, migrant ultra mare, and are often taken in Italy, and oftner in Spain.

(m) Gaffendus says they are frequently caught in the fenny Grounds and Marshes about Arles in Provence, upon the Rhone.

Gesnerus. Quidam mihi retulit avem hanc non procul à Monte-Pesfulano capi. He says in another Place, that they swim in Flocks not far from the Shore in Mediterrando Mari Gallico.

Willoughby writes, that in hard Weather in the Winter Time, it comes over to the Coast of Provence (and is often taken about Martiquez, a Sea-Port Town in that Country) and in Languedoc, and is frequently found about Montpelier: But whence it comes and where it is bred.

to me, says he, is unknown. *N.B. This Passage is not in the Latin Edition of his Works, but added to the English which was published two Years after the first. However, he says positively, that they don’t come from Flanders, where they are so far from being common, as some allege, that there never was one seen in that Country.

(n) Dr. Charleton informs his Reader that he was presented with the Skin of one of these Birds, well stuffed and dried, by a Gentleman at his Return from the University of Montpelier, near which Place it had been taken. *Hujus exuvias rite conditas infertasque mihi dono dedit prænobilis Juvenis D. Thomas Crew, Eq. Aurat. (o) Dr. Lister says, *Frequens est Phœnicopterus in paludibus maritimis ad mare Mediterraneum Provinciae & Languedocie. Whether this Bird were known to Aristotle is a Question; for all our Writers of Natural History agree, that the Phœnicopterus is no where mentioned by Name by the Philosopher; yet they can hardly believe that he was ignorant of a Bird so clearly described by his Contemporary Aristophanes. *Mirum est, says Gesner, *hujus tam pulchra & eximia Avis nomen ab Aristotele taceri, cum Aristophanes, qui vixit eadem atate, meminerit. Sed Graecis etiam raram esse hanc avem puto.

Bellonius thinks that Aristotle described this Bird under the Name of Glottis or Lingulaca, as Theodorus Gaza translates it. *Aldrovandus is of the same Opinion, but Gesner and Scaliger are not; for the first says, *Ego vero is quas Gallinulas aquaticas nostri vocant avibus Glottidem ad-numero, que omnes fissipedes sunt: And the latter in his Commentary upon this Passage says, *Glottis autem quæ fit nondum mihi constat. *Ridiculum quod quidam de Phœnicoptero ausus est pronunciare.

Vieus Ratio, Nidificatio, Volatus, &c.

Gesner says, circa lacus & paludes victitat, and that it feeds on Periwinkles and Fish: And by Dampier's Account we learn, that they delight to keep together in Flocks, and feed in Mud and Ponds, or in such Places where there is not much Water; that they are very shy, and therefore it is hard to shoot them; that they build their Nests in shallow Ponds, where there is much Mud, which they scrape together, making little Hillocks, like small Islands, appearing out of the Water, a Foot and an half from the Bottom: They make the Foundation of these Hillocks broad, bringing them up tapering to the Top, where they leave a small hollow Pit to lay their Eggs in. And when they either lay their Eggs or hatch them, they stand all the while, not on the Hillock, but over it, with their Legs on the Ground in the Water, resting themselves against the Hillock, and covering the hollow Nest upon it with their Wings: For their Legs are very long, and building thus, as they do, upon the Ground, they could neither draw their Legs conveniently into their Nests, nor sit down upon them otherwise than by resting their whole Bodies there, to the Prejudice of their Eggs or Young, were it not for this admirable Contrivance, which they have by natural Instinct. They never lay more than three Eggs, and seldom fewer. The young ones cannot fly till they are almost full grown; but will run prodigiously fast. Thus far Dampier.

Du Tertre, in his History of the Isles, &c. gives these further Circumstances. Ces oyseaux, dit il, ont le ton de la voix si fort, qu'il n'y à personne, en les entendant, qui ne creust que ce font des trompettes qui sonnent. Ils sont toujours en bandes, et pendant qu'ils ont la teste cachée, barbotant dans l'eau comme les Cygnes, pour trouver leur mangeaille, il y a toujours un en sentinelle tout de bout, le col éten-
du, l'oeil circonfpect, & la teste inquiète. Si tost qu'il appercoit quelqu'un, il sonne la trompette, donne l'alarme au quartier, prend le vol tout le premier, & tous les autres le suivent. Ils volent en ordre comme les Grues; que si l'on les peut surprendre, ils sont si faciles à tuer, que les moindres blessures les font demeurer sur la place. Ils sont rares & ne se voyent jamais, si non dans les salines le plus éloignées du Peuple.

On les écorche, & de leur peau on fait de fourrures, que l'on dit être tres utile a ceux qui sont travailliez des froideurs & debulité d'estomac.

(Rochfort likewise informs us, That Ils ont l'Ouse & l'Odorat si subtile, qu'ils eventent de loin les chasseurs, & les armes à feu. Pour éviter aussi tout surprise, ils se posent volontiers en dés lieux découverts, & au milieu de marécage, d'où ils peuvent appercevoir de loin leurs ennemis; & il y en a toujours un de la bande qui fait le guet. Ils sont gras & ont la chair assez délicate. On conserve leur peau qui est convert d'un mol duvet, pour être employe aux mesmes usages que celles du Cygne & du Vautour.

De Laet observes, that these Birds are so accustomed to Salt Water, that the Indians, when they tame them, mix Salt with the fresh Water for them; elle they pine away and die. And though Aristophanes says it is ἐ τὸν ἅκραω, or not used to be tame; yet Gassendus writes, that M. Varius, President of the Parliament at Aix in Provence, and a great Friend of M. Peirese, used to divert himself with feeding them with Bread moistened with Water, which they commonly eat in the Night and not in the Day Time. The same learned Person observed, that they could discern the Approach of cold Weather, and would come to the Fire, so as sometimes to burn their Feet; and that when one Foot pained them, they would go upon the other, using their Bill instead

(p) Histoire des Antilles.
of the burnt Foot; That they slept standing upright on one Foot, with the other drawn up to their Breast among their Feathers: And lastly, that very little Sleep served their Turn.

Usus.

This beautiful and scarce Bird was much esteemed by the Romans, and frequently made use of in their costly Sacrifices and sumptuous Entertainments. Thus Suetonius (q) describing the exquisite Sacrifices which were appointed to himself by the mad Emperor Caligula to be offered to himself as a Divinity, says of them, Hostiae erant Phoenicopteri, Pavones, Tetraones, Numidicae, Meliagrides, Phasianae, quae generatim per singulos dies immolarentur. And the same Historian relates further (r), that this Emperor pridie quam periret sacrificans respersus est Phoenicopteri sanguine.

That the Tongue of this Volatile was much commended, and in great Esteem, for its excellent Taste and most delicious Relish, will appear from the following Quotations. And first we read in Pliny (s), that Apicius said the Tongue of this Bird was a delicious and savory Bit, Phoenicopteri linguam pracipui esse saporis Apicius docuit, nepotum omnium altissimus gurges.

The Poet Martial says the same thing in the aforecited Epigram:

Dat mihi penna rubens nomen: sed lingua galosiss
Nostra sapit;

And Juvenal (t) in that Satyr where he exposes the extravagant Luxury and Gluttony of the Romans, men-

(q) Suetonii Caligula, § 22. (r) Scaleg. § 57. (s) Plinii Nat. Hist. Lib. X. cap. 48. (t) Juvenal, Sat. XI.
tions this Fowl, amongst some others equally rare, that they made use of in their Feasts.

Et Scythice volucres & Phænicopterus ingens.

We read in Suetonius how the Emperor Vitellius had them often served at his Table, with a great many more Varieties brought from the most distant Parts of the Universe; his Words are, (u) In hâce Scarorum jocinera, Phæsanorum cerebella, linguas Phænicopterûm, Muranarum lactes à Carpathio usque freatoque Hispania per Navarchos ac Triremes petitarum commiscuit; hoc eft. ab extremis imperii finibus Orientem versus & Occidentem. And Heliogabalus, another of the Roman Emperors, as Lampridius writes, treated his Courtiers with sumptuous nice Dishes made of the Inwards and Brains of Phænicopters, exhibuit Palatinis ingentes dapes extis & cerebellis Phænicopterorum refertas.

What is related by Gassendus, in the Life of that learned Nobleman Peiresktus, is no Argument against the excellent Relish of the Tongue of this Bird: For his Friend Varius, who therein seems to contradict the received Opinion, was at that Time just upon the Recovery from a long Illness; he had no Appetite, loathed all Sorts of Meats, and mended but very slowly; so that its no Wonder if he did not perceive all the Relish of that nice Bit, for which of old it was so much commended. Besides, his Answer is not as to the Tongue, which was owned to be much sweeter than that of a Kid, but to the Flesh of this Bird, (as will appear from the Original.) Rogatus subinde suit de sapore carnis Phænicopteri. Exceptit autem mirari se, cur illam Apicius apud Plinium, & Imperatores Caligula & Vitellius apud Suetonium, Heliogabalus apud Lampridium, & nonnulli alii tantis in deliciis habuissent. Esse enim cam inuscundam, aut saporis certe non exquisiti, aquaticarum aliarum instar, cum etiam piscem oleat, unde 

(u) Suetonii Vitell. § 13.
The Way to dress the Phænicopter, and how to make a Sauce fit for it, we may read in Apicius's Book de Obsoniis & Condimentis, & in de Arte coquinaria, Lib. VI. c. 7.


Philostратus Punicam Avem, i.e. Phænicopterum, inter mensarum delicias numerat, Lib. VIII. Vite Apoll.

(x) Wormius. Linguam hujus avis veteribus Romanis in deliciis olim suisse docent cupediarum magistrî Apicius & alii.

Dr. Grew. The Tongue of this Bird, as Apicius said, was a delicious Morsel amnest the Romans.

N.B. In the Treatise de Obsoniis & Condimentis, that goes under the Name of Apicius, there's no mention made of the Tongue of this Fowl: For as Dr. Lister well observes, Apicius noyster hic filet de lingue præcipuo sapore: Which is a pretty convincing Proof, that this Book de re coquinaria, is only a Collection made by some modern Roman; the Name of the old Apicius, that great Master of the Art of Eating, being only prefixed to it, for the Benefit of the Bookseller.
Dampier. The Flesh of both young and old is lean and black, yet very good Meat, tasting neither filthy nor unsavory: A Dish of Flamingo's Tongues being fit for a Prince's Table. They are large, having a large Knob of Fat at the Root, which is an excellent Bit.

Du Tertre. La chair en est excellente, quoq qu'elle sent un peu la marine; mais sur tout la langue passe pour le plus friand morceau qui puisse être mangé.

**Descriptio Partium.**

**Magnitudo.**

According to Bellonius this Bird is of the Bigness of the Fowl he calls Elorius, which is our Curliew.

Scaliger compares it to the Heron, magnitudo ei Ardea.

Gesner says it is as big as a Ciconia or Stork, or rather bigger.

Aldrovandus writes, de magnitudine ejus ego nihil certi asseso, quia Avena nunquam vidit.

Dampier. The Flamingo is a sort of large Fowl much like the Heron in Shape, but bigger and of a reddish Colour.

Du Tertre. Le Flamant est un oiseau gros comme une Oye sauvage.

**Collum.**

It hath an extraordinary long Neck according to Mr. Willoughby.

Du Tertre. Il a le cou rouge, fort menu pour la grandeur de l'oiseau, & long d'une demi Toise.

**Cauda.**

Scaliger. Caudam habet brevissimam ac veluti precisam:

**Rostrum & Caput.**

Scaliger writes, that the Bill of this Fowl is neither freight nor altogether crooked: Rostrum neque rectum N n n n
plane, sed neque aduntum habet, Scythicc arcus partem potius imitatur.

Geßner, who compares this bird to the Crane for Bigness, adds, Rostro septimaltar efe longitudine ad Ciconiae rostrum, superius crasso & tuberculatis quibusdam aspero.

Aldrovandus commends the Account Scaliger gives of the Bill, and then adds, in Rostro autem conformatione non parum luxt Natura; non enim ut Anatum aut Anserum planum est, cum aliqui sit latum, neque ut Ardeearum rectum & rotundum, neque adique ut rapacium Aquilarum aut Accipitrum aduncum; cum tamen sit curvum quidem & dorsum inflexum, sed in medio superioris mandibula notabili extuberrantia insigne, ex digitis longum, intus cavum & canalculatum media sui parte. Superior etiam mandibula inferioris longior est, & in acutissimam aciem definit; contra vero inferior longior crassior.

Du Tertre. Il a la teste ronde & petite, à laquelle est attaché un gros bec, long de quatre pouces, moitié rouge & moitié noire, & recourbé en forme de cuillerière.

Olaus Wormius gives the following Description of the Head and Bill of the Phenicopter, which he had sent him from a Friend, viz.

Caput longitudine uncias octo superabat, ipsum caput, excepto rostro, trium erat. Rostrum ipsum figuram à Scaligero delineatam obtinet, in medio crassum satis sed cavum; superiore ejus parte utrinque ad latera canaliculatum; ad sui extortum duobus grandibus foraminibus olfaetui deputatis préditum, in extremitate aduncum, internè denticulatum cum costa & eae eminentiâ in medio. Pars vero inferior nigra, frequentibus predita stria ad extremum excurrentibus; longitudine vero cedit superiori, sed ampla est & capax, crassa lingue, quam aberat, excipiendo apta.

Dr. Grex has obliged us with a very curious Account of the Bill of this Bird, for which he says it is most remark-
remarkable. The Figure of each Beak is truly Hyperbolical: The upper Jaw is ridged behind, before plain or flat, and pointed like a Sword, with the Extremity bended a little downwards: Within it hath an Angle or sharp Ridge, which runs all along the Middle, at the Top of the Hyperbole, not above a quarter of an Inch high: The lower Beak in the same Place above one Inch high, hollow, and the Margins strangely expanded inward, for the Breadth of above a quarter of an Inch, and somewhat convexly. They are both furnished with black Teeth, as I call them from their Use, of an unusual Figure, soil. slender, numerous, and parallel as in Ivory Combs; but also very short, scarce the eighth Part of an Inch deep. An admirable Invention of Nature, by the Help of which and of the sharp Ridge above-mentioned, this Bird holds his slippery Prey the faster.

Mr. Ray describes the Rostrum to be latinsculum, singularis & insolita figure; mandibula nempe superior incurva, depressa, dentata; inferior casser.

Menippus, the Cynick Philosopher, in a Fragment of his de Homine (which however at this Time is either lost, or at least hard to be come at) affirms this Bird to move its upper Jaw, as we find him quoted by (y) Rondeletius, where he is talking of the Crocodile's moving that Mandible: His Words are, sed id non soli ex omnibus animalibus Crocodilo peculiare, nam intra Aves, Phoenicopterus superiorem partem Rostrì movet, ut anotavit Menippus Philosophus, Libro de Homine.

Gesner makes the very same Quotation from Rondeletius.

(z) Cardanus repeats the same thing without Mention of any Author, nam quidam existimant etiam Phœn-

(y) Rondel. Lib. de Amphibiis, Chap. 5.
(z) Cardan. de Varietate Rerum, Lib. 7. Cap. 37.
copteræ aves Mandibulam movere superiorem; but subjoins, sed non adeo manifesta causa est in Ave ut in Crocodilo.

Wormius is of the same Opinion; but with Cardan, he thinks the Cause is not so manifest as in the Crocodile.

Dr. Charleton says, that it was Cardanus that first made that Observation.

Dr. Grew argues for this Movement from the peculiar Structure of the Rostrum; alledging, however, that there can be no Determination of these Matters, without Inspection into the Muscles, and the Articulation of the Bones. As for the Phœnicopter, says he, it must needs be said, that the Shape and Bigness of the upper Beak (which here, contrary to what it is in all other Birds that I have seen, is thinner and far less than the neither) speaks it to be the more fit for Motion, or to make the Appulse, and the neither to receive it.

Crura & Pedes.

Bellonius remarks, that the Legs of this Volatile are very long. And on the contrary,

* Scaliger writes, Crura pedesque sunt adeo breves, ut cùm in Homine Galenus agnoverit longissimos, hunc omnium, quæ nota sunt nobis, animalium brevissimos attribuere potuerit. For this he is severely taxed by Dr. Charlton, hic nobis candidè notandus occurrat error quidam Scaligeri egregius: Is nimirum in Exercit. in Aris. Hist. Animal. (this is wrong quoted, the Place being in Aris. Lib. de Generat. Anim.) peculiares Phœnicopteri notas satis prolixe descriptas, crura et brevissima curtose pedes attribuit, (verbis supra citatis) atque Avem banc & cruribus & suris gradire longissimis, omnium quotquot seu vivam seu mortuam contemplati sunt oculi confirmant. Et quis precor, ullam aliam, ex Aquaticarum fisspedum & piscivora- rum classe, volucrem unquam conspexit brevibus pedibus in-

* Scalig. Exercit. 233. § 2. de Subtilitate, ad Cardanum.
Dr. Grew observes the same Mistake, but in fewer Words, which are the following; when Scaliger therefore saith that this Bird hath the shortest Legs of any Animal yet known, he would have said the longest.

Gesner says, it is Cruribus rubris, ea proceritate qua in Ciconia sunt, vel procerioribus.

Du Tertre. C'est le plus haut monté de tous les oyseaux que j'ai jamais vu en ma vie. Il a les Jambes tout rouges, & les pieds a demi marin.

(y) Rochfort. Ils ont les jambes & les cuisses si hautes, que le reste de leur corps est elevé de terre de deux bons pieds ou environ.

Color Piumarum.

Scaliger thus elegantly expresses the fine Colour of its Wings. Cinereum colorum nobilitant alarum punicea penae.

Aldrovandus. Mirum est cur nigrum alarum colorum non annotavit Scaliger. Cetera tota Avis ex cinereo, phæniceo & albo coloribus misitis spectatur.

Gesner says, Pennis albis parte prona; rubentibus per Collum, Peæs, Ventrem & Alas. And speaking of one taken near Montpellier, he says, tota alba, prater illas in alis partes que nigra sunt in Ciconis.

(z) De Laet observes, that while they are young their Feathers are chiefly white; but as they grow up, they are painted with an Infinity of Colours. Mais ils different en couleur, d'autant qu'ils ont le plumage blanc quand ils

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Stratum: Certè nemo. Neque conuentaneum est, ut quis Natura consilium ad pisces in stagnis & fluviorum vadis, non natae sed grallatoriis veluti gradibus vadando, captan- dos destinauerit, eis crura concedenterur tam necessario ad vi- tum quareendum officio imparia. Ad hæc, non solum fuit quæm à proceritate crurum & colli, quod Phœnicopterus olim à Juvenale ingens dicetur.
fonce jeunes; puis après à mesure qu'ils croissent, ils deviennent de couleur de rose, & en fin quand ils sont d'âgez, il est tout incarnat. Il se trouve de ce mêmes oiseaux, près de Montpellier, qui ont seulement le dessous des ailes & du corps incarnat, & le dessus noir. Il s'en voit aussi aux Isles, qui ont les ailes mélangées de quelques plumes blanches & noires.

Du Tertre gives much the same account. Les jeunes sont beaucoup plus blancs que les vieux, & ils rougissent à mesure qu'ils avancent en âge. Je n'ai vu aussi quelques uns qui avaient les ailes mêlées de plumes rouges, noires & blanches, & je crois que ce sont les mâles.

(†) Constantinus. Rostrum, & crura, & pars alarum, puniceo colore rutilat.

Willoughby says, the Neck and Body is white: The Alarum Remiges, or Quill-Feathers of the Wings, are black; but the Vestitrices, or Covert-Feathers, are wholly dyed with a most beautiful, bright purple, or flame-Colour, unde et nomen.

Dampier. The young ones at first are of a light grey; and as their Wing-Feathers spring out, they grow darker, and never come to their right Colour, or any beautiful Shape, under ten or eleven Months old. When many of them are standing together by a Pond's Side, being half a Mile distant from a man, they appear to him like a Brick Wall; their Feathers being of the Colour of a new red Brick: And they commonly stand upright and single, one by one exactly in a Row, except when feeding, and close by each other.

Color Pedum.

All Authors agree in the red Colour of its Legs and Feet. Thus Scaliger, Crura pedesque alis habet concolores.

(†) Constant. Lexicon Graeco-Latin.
Color Rostri.

Gesner says, Colore rubro infar sanguinis.

Aldrovandus writes, Pars qua spectat frontem ex albo ad Casstanea colorem vergit, cetero nigrom.

Willoughby affirms, that the Tip of the Bill is black, or of a dark blue.

Figure.

The whole Fowl is delineated by Gesner and Aldrovandus; and Dr. Grew has given us the Figure of the Head and Bill, as he found it amongst the Rarities in Gresham-College. N. B. The Figure of the Phœnicopterus in Willoughby, is copied from the second of Aldrovandus. Gesner says the Phœnicopterus whence his Figure was taken, was sent to him by Rondeletius. Aldrovandus had the first of his Figures from Sardinia; and the second, which he calls Phœnicopterus alter rostro lato, was given him by that famous Botanist Carolus Clusius: He owns that he never saw the Bird himself.

In de Rochfort, the Body and Neck of the Flammant is pretty well delineated; but the Legs are not, neither is the Bill, nor the Claws.

(*) Du Hamel gives a very exact Account of this Bird in the following Words, with which I'll close this tedious and prolix Description, collected from all the Authors that have made any mention of the Phœnicopterus or Red-Wing. Collo pralongo, cruribus productis, exili pede sed firmo donatur; oculi itidem angusti sunt & rubei; cylis fellea e parte inferiore hepatis est penulis. Vas ipsum quos suspenditur, quove bilem excipit, amplum est, contra atque in Homine & in Quadrupedibus observatur; in his

enim radices vesicula sunt admodum exiles. Oesophagus in sui initio valde angustus, paulatim latior factus in ingluviium est in saccum ampliorem definit: Ventriculus fere ut in Gallina, tamen ut Granis non vescitur, sed parvis Conchylia, qua ventriculi musculis terrimus ut Granas. This Author tells us that the Phænicopter was dissected by Monsieur Perrault at Paris.

Alæ ejus diducēte colorēm illum rutulum exibent, unde vulgo Flamand dīci solet, non quod in Belgio reperiatur, sed quia ejus Pluma per membranam pellucidam vis colorēm flammaeum praebent: Vix ulla est Avis major: Rostrum ex utraque parte dorsum incurvatum, quod in ea Ave omnino est singular; aratri enim instar inflatitus, unde & vulgo Becharü, quasi aratrirostrum, appellatur. Hac ille.

When I get the Opportunity, which I do very much long for, of dissecting this fine Volatile with my own Hand, I shall then more particularly insist on the Anatomical Part; and, with all the Exadnels I can, give a true Description of each Viscus. I proceed to the Explanation of the Figures, which were drawn from a Flamingo that was sent to Mr. Botley to be stuff'd.

The Explanation of the Figures. Tab. II.

**Fig. I.** Gives a Side-view of the Head and Bill.

**Fig. II.** In this is represented a Front-view of the same Parts.

**Fig. III.** Exhibits the under-side of the Tongue next the under Bill. In which a denotes a Cartilaginous Substance that covers the Tip or extremity of the Tongue; b a Glandulous Substance at its Basis; c the Horns of the Os Hyoëides.

**Fig. IV.** In this the upper side of the Tongue is fairly delineated, upon which we see two Rows of strong Papilli Nerveae; their Apices or Points turning inwards, for the better retention of the Prey.

**Fig. V.**
Fig. V. In this the Tongue is drawn in a lateral view, that we may have a Prospect of the true Figure of these Papilla, which being hooked and turn'd backwards prevent, in a great Measure, the return of any little Animal (swallow'd alive, which they feed upon.

Fig. VI. The Cornua or Horns of the Os Hyoidæum are drawn in this last Figure, as all the other Parts are, as big as the Life.

Advertisement.

WHEREAS some of our Practising Surgeons, as I am inform'd, have taken Offence at a Passage in a Paper of mine, publish'd in the last Philosophical Transactions, viz. By that sort of Enquiry the common Mistake of Surgeons was detected, and what was esteem'd and treated by them as a Luxation of the Head of the Os femoris, was discover'd to be nothing else but a fracture of the same Bone near it's Neck; I take this first Opportunity to explain myself, that by the preceding Words I only meant the Writers of Surgery now extant, as may appear by my Quotations, and not in the least intended any Reflection on the present Practitioners in it. That skilful Society I not only Respect and Esteem for their great Merit and extensive Knowledge in the several Parts of their Profession, but to them am under many Obligations also, for the Honour they have done me in twice choosing me one of their Lecturers. I have this farther to add, that that Observation was publish'd entirely without my Knowledge about a Year after it had been read at a Meeting of the Royal Society.

James Douglas.

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